THE MINUTE MATHEMATICIAN

By

James Jurin

Edited by David R. Wilkins

2002

NOTE ON THE TEXT

This edition is based on the first and only edition, published in London in 1735, the copy consulted being that in the library of Trinity College, Dublin.

The following spellings, differing from modern British English, are employed in the original 1734 edition: entring, remembred, found, throughly, covert, cloaths.

In section XXIX, first sentence, ‘pottles’ has been changed to ‘bottles’.
In section XXXIV, the spelling of ‘Genleman’ has been corrected.
The section number XVIII has been corrected to XLVIII.

David R. Wilkins
Dublin, June 2002

The Minute Mathematician:

or,

The Free-Thinker no Just-Thinker.

Set forth in

A Second LETTER

to the

Author of the Analyst;

containing

A Defence of Sir Isaac Newton

And the *British* Mathematicians,

Against a late Pamphlet, entituled,

A Defence of Free-Thinking in Mathematicks.

By Philalethes Cantabrigiensis.

It is hard for thee to kick against the pricks, Acts ix. 5.

Thou art weighed in the balances, and art found wanting, Dan. v. 27.

LONDON:

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THE

Minute Mathematician, &c.

Cambridge, June 13, 1735.

SIR,

I Freely own to you, when I sat down to write my defense of Sir *Isaac Newton* and the British Mathematicians, I was not a little moved at the treatment you had been pleased to give to one or two of those Great Men, whom I am proud to call my Masters, and whose memories on that account I shall always reverence and honour. *But now*, you tell me, *I may be supposed cool.* I am so: partly through the length of time that has intervened; and partly by considering the severity of the discipline you have undergone. It has had, I see, a marvellous effect upon you. One may plainly perceive an alteration, notwithstanding your endeavours to conceal it, not only in your sentiments, but in your language, your behaviour, and your very air. You no longer breathe that superiority and contempt of all mankind you were wont to shew. This change in you has greatly mitigated *the passion* with which I was before *overcome.*

In this calm and cool state therefore when I reflect upon what is past, I am not a little startled at my own audaciousness and presumption in entring the lists against so redoubtable an adversary as the Author of the Minute Philosopher. To you, likewise, I find, this presumption of mine appeared so extraordinary, that though you are so good as to qualify it by the softer name of *courage, you could not but admire it, it seemed unaccountable to you, till you reflected on my seeming secure in the favour of one part of my readers, and the ignorance of the other.*

Nevertheless you are persuaded there are fair and candid men among the Mathematicians. I likewise am persuaded not only that there are fair and candid men among the Mathematicians; but that generally speaking Mathematicians are fair and candid men. What should make them otherwise? Are their opinions digested into Creeds and Articles, and established by Law? Has the Publick thought fit to bestow dignities and large possessions on them, which are not to be obtained without embracing those opinions, nor to be retained without persevering in them? Were even this the Case; yet surely there would be found among them fair and candid men. I am sure I know many such among another set of men in these very circumstances.

Ay, but this other set of men, you will say, have nothing but truth to defend. I grant it. And I take this to be the case of the Mathematicians likewise. They are at least as good reasoners as any other set of men whatsoever, and consequently are as likely to know truth, when they meet it, and have nothing to hinder them from embracing it. It is therefore on their judgment, not their favour, that I depend.

But you speak of *the ignorance of the other part of my Readers*. Alas! Sir, of what advantage can that be to me? To me *Philalethes*, who aim at truth alone, who have no interest in deceiving them?

Were I indeed the Author of the Minute Philosopher: *Had* I an *other end to serve than truth*: Were I master of assurance enough to mislead my reader at the instant that I call out to him to mind his way; to desire him to examine, while I am misinforming him; to throw dust in his eyes, and bid him see: then undoubtedly much might be done. But these are arts I neither need nor practise. I content my self with the plain honest way of giving my Reader the best light I can, neither misleading him my self, nor suffering him to be misled by others. With this view it is that I divide my reply into the same number of sections with your defense, and confine my sections to the same matter with yours. This will indeed make what I have to say somewhat less methodical, but then it will enable the Reader more easily to compare us together, and to make a more certain decision between us.

The taking this method, Sir, will make it plainly appear, that what I aim at is only manifesting the truth; and consequently that the reason of my courage in encountering you, is my being verily persuaded that I have truth and justice on my side. I know and am aware of your superior accomplishments: But *Philalethes* is my name: And truth will prevail against the pens of men or angels. Your vanity has engaged you in a difficulty, from which all your abilities shall never extricate you.

Verte omnes tete in facies, & contrahe quicquid
Sive animis, sive arte vales.

Your arms are wedged in the oak you have presumptuously attempted to rend: Your strength is no longer of any use to defend you: A woman, a child may be too hard for you.

II. Your second section teaches us, *that things obscure are not therefore sacred; and that it is no more a crime to canvass and detect unsound principles or false reasonings in Mathematicks, than in any other part of Learning.* I agree with you. I go farther. It can never be a crime, but on the contrary is highly laudable, to canvass and to examine the principles and reasoning made use of in any science whatsoever, and that with the utmost freedom and impartiality. All ingenuous minds will be pleased with such an examination: They will readily consent that the science they profess, be brought to the severest and strictest trial. Truth can never be hurt by Inquiry: Truth loves the light: But error, falshood and imposture dread and abhor it.

III. I am much at a loss here. You speak of *men who reject that VERY THING in Religion which they admit in human Learning.* Do they *admit that Fluxions are* to them *most incomprehensible Mysteries*? Do they, notwithstanding this concession, *believe them to be clear and scientifick*? Do they, notwithstanding this belief, *entertain an implicit faith in the Author of that Method*? These things seem hard to reconcile.

IV. I do not ask, Why you chose to defame Mathematicians in the month of *March, Ann. Dom.* 1734, rather than at any time before? The only question with me was, Whether Vanity or Christianity were the motive to writing the Analyst. *Quæ relligio aut quæ machina belli*?

I have fully proved there was no Religion, no Christianity in it. It was partly Vanity, partly Machine.

V. Here I would observe, that whoever admires Fluxions, must admire them for something of excellence he sees in the Method of Fluxions, and consequently cannot justly be said to *yield Faith* to the Inventor of that Method. But this whole section seems to me to be matter of secret history and declamation of the worst sort, namely, the defamatory.

VI. More secret history and declamation, partly about what no body denies, and partly about what no body believes. You give us to understand, *you have a right to examine Fluxions, even though Religion were quite unconcerned, and though you had no end to serve but Truth.* No body disputes your right of examining: but surely no good can be expected from the examination of a Person who has any end to serve but Truth, let that end be what it will. But pray what is this other end, this end different from Truth, that you have to serve? It looks as if Religion were meant. But I hope better things of you, a Christian, and a Preacher of the Gospel. Truth and the Christian Religion are one. I profess I am greatly puzzled. *I have taken as much pains* to study this passage as, I sincerely believe, you *have done to make sense of Sir* Isaac Newton*'s principles.* A friend of mine is of opinion the passage has been corrupted either by the transcriber or the printer, and bids me for *Religion* read *Promotion. Ita legendum censeo*, says he, *reclamantibus centum* Tonsonis. I am apt to think he is in the right, partly because I take him to be a very able Critick, and partly because this emendation, though considerably differing from the vulgar reading, serves to confirm the proof I had before given, that your end was neither Truth nor Religion.

You tell me I am very angry, and refer to page 13 and 14 of my Defense. I have looked over those pages to see what signs of anger I have there shown, what injury, what affront I have there offered you. All I can find is, that I have proposed to you the example of our Saviour and of St. *Paul.* I beg your pardon, Sir: I took you for one of their followers.

You will not take upon you to say you know me to be a Minute Philosopher. I am much obliged to you for this tenderness, and should be more so, if it appeared that you knew so much as one letter of my name. But it seems, you would not be concerned if others should take me to be such a one. You speak of *my spleen against the Clergy*; and you tell me *the Minute Philosophers make just such compliments as I do to our Church.* Here I apprehend you mistake the compliments I make to yourself, and a few of your credulous friends, for compliments to the body of the Clergy. I assure you, I look upon the body of the Clergy as a body of learned and useful men. I know and am known to a great many individuals among them, whom I highly esteem and honour: I have spoke of some of them in my Defence with singular respect. If I laugh at any, it is at such as think you do service to the Church in writing the *Analyst*: If I dislike any, they are such as are perpetually grasping at dominion and riches. No wonder. I am a Layman. If the Clergy obtain more power, I shall have less liberty: If they will have more wealth, I am one of those must pay to it.

VII. The chief purport of this section seems to be to strengthen the proof you had before given of the infidelity of Mathematicians. You had told us in the *Analyst, you were not a stranger to it: It was known: You were credibly informed.* Now you go farther. *You make no doubt of it: You have seen shrewd signs: You have been* VERY *credibly informed.* Can any thing be plainer? I declare myself fully satisfied with this proof, even without the story told you by Mr. *Addison*, of *a witty man who was an Infidel*, because of *the infidelity of a certain noted Mathematician*. Surely this witty man was in jest; at least he was no wise man.

VIII, IX, X. In these three sections I meet with nothing but declamation. The subject of it is *my passion and injustice, my railing and raging, my rhetorick* and writing *tragedy*; your own sincerity and laudable endeavours to do service to mathematical learning; the *proper respect* you treat Sir *Isaac Newton* with, and the *decency* with which you dissent from him. For which last the reader is desired to have recourse to the *Analyst*, particularly to the thirty-first Query, where Sir *Isaac Newton* is plainly charged with writing nonsense.

As to my *frightful visions and tragical uproars about the Inquisition and the Gallows*, you may laugh at them as much as you please: But I have heard of persons hanged and burnt upon as slender evidence as that which you bring against Mathematicians. And what has been, may be: Especially if the wholsome, ancient discipline should ever be restored, which some persons say is much to be wished. I confess I am not of their mind: And I hope the body of the *British* Laity see too plainly the use that would be made of such a power, ever to trust you Gentlemen with it.

Hoc regnum Dea gentibus esse,
Si qua fata sinant, jam jam tenditq, fovetq;

XI. You say, you heartily abhor an Inquisition in Faith. Upon my word you have a great deal of reason. You have been a grievous Free-thinker in your time: I do not mean in Mathematicks only. As great a *Bigot* as I am, possessed with *the true spirit of an Inquisitor*, I assure you, I should be very sorry that you and I were at the mercy of some men I could name. They seem to me to be singularly well qualify'd to preside in the holy Office, and I doubt they would make us confess that something else existed in Nature besides SPIRIT AND IDEAS.

XII. More declamation about my declaiming, and your own Modesty, and the *compliment* you pay *to Sir Isaac Newton's Understanding*. But, Sir, I don't like that word *Sophism*. It seems not very consistent with the *decency* and *proper respect* you so lately talked of.

XIII, XIV, XV, XVI. You tell me, *The adoration that I pay to Sir* Isaac Newton*, you will pay only to truth: That I may be an Idolater of whom I please; but I have no right to insult and exclaim at other men, because they do not adore my Idol: That I inveigh against you, because you are not guilty of my mean Idolatry.*

——*To deify his power,*
Who from the terror of this arm so late
Doubted his Empire, that were low indeed.

Now give me leave to ask you a question. Do you really and *bona fide* believe that I pay idolatrous worship to Sir *Isaac Newton*, that I make him the object of that adoration which you say you will pay only to truth, and which I will pay only to the God of truth? And thus because I apply to Sir *Isaac Newton*, a Verse which an inferior Poet applied to *Virgil*? Is *adorare vestigia* to be literally taken, think you? What can be meant by these *vestigia*? The mark of his foot in *Crane Court*? Or the truths discovered by him? If the last; to what purpose all this declamation, and ridiculous rant about Idolatry for four sections together?

——*Quo numine læso?*

You seem to dislike my professing that *the highest honour I can ever arrive at, or even desire, is in any the lowest degree to imitate Sir* Isaac Newton*'s example.* You think *It might have suited better with my appellation of* Philalethes*, and been altogether as laudable, if my highest ambition had been to discover truth.* Why so it is. The discovering of truth, and his clear, candid, humane way of making it known to Mankind, is the very thing in which I should desire to imitate Sir *Isaac Newton.*

You say, *I speak of it as a sort of crime to think it possible I should ever see farther, or go beyond Sir* Isaac Newton*. But there are others who think it no crime to desire to know not only beyond Sir* Isaac Newton*, but beyond all Mankind.* You intimate your self to be one of these. Now, Sir, I am for seeing as much beyond Sir *Isaac Newton* as you can be: But first let us see as far as he has done. I agree with you in this desire of knowledge; make it as unbounded as you please. I assure you I think it no crime. The only difference between us is this. You seem to think you have this knowledge already. I am sensible I have it not.

I make no doubt but such a Man as you, or one much inferior to you, may carry a particular point, or many particular points, farther than Sir *Isaac Newton* has done. But that such a Man as Sir *Isaac Newton*, after long consideration of one thing, after touching it and retouching it at different times for above half a Century, after setting it in several various lights, after applying it in an infinite number of examples, after giving several different demonstrations of it, such as had satisfied all the Mathematicians in *Europe*, should all this while have taken error for truth, and given *Sophisms* for demonstrations, and thereby deceived all the world except my dear Friend the Author of the *Minute Philosopher*, is what must be very clearly made out before I believe it.

——*Magnis tamen excidis ausis.*

XVII. You begin this section with addressing your self to me in the following manner. "I have said (and I venture still to say) that a fluxion is incomprehensible: That second, third and fourth fluxions are yet more incomprehensible: That it is not possible to conceive a simple infinitesimal: That it is yet less possible to conceive an infinitesimal of an infinitesimal, and so onward. What have you to say to this?" Truly very little. Only I don't well comprehend, how one incomprehensible can be MORE incomprehensible than another incomprehensible: How it can possibly be LESS possible to conceive one thing than to conceive another thing, which other thing it is not at all possible to conceive.

For clearing up these assertions I have had recourse, pursuant to your directions, to the fourth section of the Analyst, which is the only one relating to fluxions, of the three you refer me to. But all the satisfaction I there meet with is, That *your imagination is very much strained and puzzled* with one thing; That *it seems still more difficult to conceive* another thing; That a third *seems an obscure mystery*; That a fourth *exceeds, if you mistake not, all human understanding*; That *take* another *in what light one pleases, the clear conception of it will, if you mistake not, be found impossible.* This to me seems to amount to thus much. The Author of the *Minute Philosopher* cannot comprehend the principles of fluxions: Therefore no man living can comprehend them. He cannot understand them: Therefore they exceed all human understanding. A notable proof of my Hypothesis, that that Gentleman has too good an opinion of himself, and too mean a one of all other men.

You go on addressing your self to me; *Do you attempt to clear up the notion of a fluxion? Nothing like it.* Very true, nor did I ever undertake it. Sir *Isaac Newton* has done it incomparably well to those who are qualified to read his Works, and thither I refer you. May not I expose your blunders, without pretending to explain his doctrine better than he has done it himself?

But you tell me, *I only assure you (upon my bare word) from my own experience, and that of several others whom I could name, that the doctrine of fluxions may be clearly conceived and distinctly comprehended.* Why pray, Sir, what did you require more? *You appealed to the trial of every thinking reader.* I am one of your *thinking readers.* I have made the *trial* you desired. I acquaint you with the result of that *trial*, and all the return you make me is, *Can you think I will take your word, when I refuse to take your Master's*? I appeal to all my thinking readers whether this be civil usage. You say, you don't understand fluxions. I say I do. I believe you: And yet you won't believe me.

This, Sir, my judgment tells me is all the answer I ought to make to the invitations you so frequently give me upon this head. I am sensible it were better to hold so slippery an adversary to the points we have already in hand, than before these are settled, to go upon new matter. Besides I am afraid of incurring the common fate of Sir *Isaac Newton*'s interpreters, to be less intelligible than my Master. I apprehend likewise that, let me take ever so much pains to satisfy and oblige you, I shall meet with no better usage than when you appealed to me: That all the return I am to expect is, *Alas! I find no sense or reason in what you say.* And yet I am so desirous of contributing my assistance towards your laudable design of *putting this controversy in such a light as that every reader may judge thereof*, that I think I must run that hazard. But I desire it may be remembred that I do not here intend, nor indeed think my self at all qualified to write a complete treatise of Fluxions, that being expected from better hands. All that you require of me is to shew that the principles of Fluxions may be clearly conceived. This therefore is what I shall endeavour to do, and in order to render those principles as intelligible as I can, I shall make use of the plainest and easiest example possible, that I may give my Reader no other trouble than only that of comprehending the principles themselves.

The foundation of the Method of Fluxions I take to be contained in the following

POSTULATUM.

Mathematical quantities may be described, and in describing may be generated or destroyed, may increase or decrease, by a continued motion.

DEFINITIONS.

1. A Mathematical quantity increasing or decreasing by a continued motion is called a flowing quantity.

2. The velocity with which such flowing quantity increases or decreases, is called the fluxion of that flowing quantity.

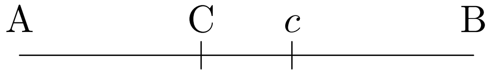
3. A part of such flowing quantity generated in a very small particle of time is called the augment or increment of the flowing quantity, if the flowing quantity be increasing; or its decrement, if the flowing quantity be decreasing.

4. A nascent increment is an increment just beginning to exist from nothing, or just beginning to be generated, but not yet arrived at any assignable magnitude how small soever.

An evanescent increment is the same thing as a nascent increment, but only considered in a different manner, as by a continual diminution becoming less than any assignable quantity, and at last vanishing into nothing, or ceasing to exist.

Explanation of the Postulatum by an example.

If a point as A move in one direction from A to B with a continued motion, it will describe and generate the right line AB. And if the same point return from B to A, it will describe, and may thereby be supposed to destroy or annihilate the same right line AB.



The definitions explained by the same example.

1. While the generating point is in motion either way, the line described by it is called a flowing line. This flowing line perpetually increases, while the generating point is moving in the direction AB, and perpetually decreases, while the generating point is moving in the direction BA.

2. The velocity with which the generating point moves either way, or the velocity with which the flowing line increases or decreases, is called the fluxion of the flowing line. *Ex. gr.* The velocity of the generating point in C is called the fluxion of the flowing line AC; and the velocity of the generating point in B is the fluxion of the flowing line AB.

3. If in a very small particle of time the generating point move from C to c, or from c to C, the small line Cc is in the first case called the increment of the flowing line AC; and in the second case is called the decrement of the flowing line Ac.

4. When the generating point, in describing the line AB, is arrived at the point C, and proceeds from thence towards B: At the instant of time that it sets out or departs from the point C, at that very instant of time an increment begins to be generated, or begins to exist, which therefore is properly called a nascent increment. And as the generating point at that instant of time is supposed to be just setting out, and not as yet to have moved to the least imaginable distance from the point C, nor consequently to have generated the least imaginable increment, it is plain that the nascent increment here considered will be less than any quantity that can be assigned.

In like manner when the generating point returns back from c to C, in order to annihilate the increment cC, that increment will continually grow less and less, will become less than any assignable quantity, and will at last entirely vanish and become nothing by the return of the generating point to the point C. At that instant of time therefore that the generating point returns to C, at that very instant I say the increment vanishes, and therefore is then properly called an evanescent increment.

Behold good Reader, the *difficult*, the *obscure*, the *mysterious*, the *incomprehensible* principles of Fluxions! I am much mistaken if a little attention do not enable thee clearly to conceive them. When thou has done this, then wilt thou be rightly prepared for understanding the following fundamental proposition, upon which Sir *Isaac Newton* has established his Method of Fluxions: the whole business of which Method is, from the proportion between the Fluxions, or between the nascent increments, of flowing quantities, to determine the proportion between the flowing quantities themselves; *& vice versa.*

Proposition.

The Fluxions, or Velocities, of flowing quantities are very nearly as the increments of those flowing quantities, generated in very small equal particles of time: And they are exactly in the first proportion of the nascent increments, or in the last proportion of the evanescent increments.

Demonstration.

1. If the velocities are uniform, it is plain that the increments generated in any equal times must be as those velocities.

2. And if the velocities are not uniform, but are perpetually changing, yet in a very small particle of time their change will be very little, and the increments will be very nearly the same as if the velocities were uniform, *i. e.* the increments will be very nearly as the velocities with which they begin to be generated.

3. And as the first ratio of the nascent increments must be the same, whether the velocities be uniform or variable, it follows that the nascent increments must be exactly as the velocities with which they begin to be generated. *Q. E. D.*

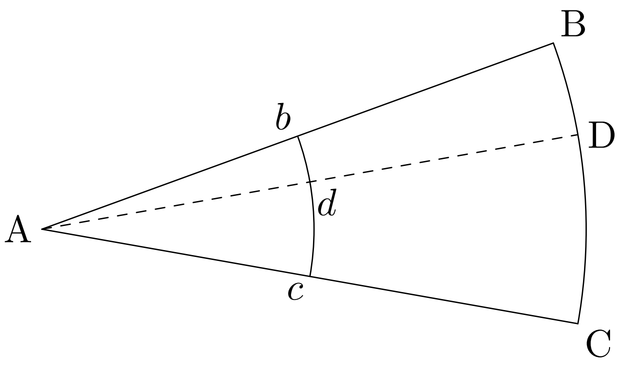
Here, Sir, I must beg leave to observe to you, that if Sir *Isaac Newton* had proceeded no farther than the first part of this proposition, and had contented himself with establishing the proportion between the increments and their velocities very nearly, without going to the utmost exactness, yet his Method had been no less scientifical and no less demonstrative than it now is. Consequently you were very much overseen in charging him and his followers with proceeding blindfold, and not knowing what they were doing, even though you had succeeded in proving that his Method did not come up to the rigor of Geometry.

To prevent cavils, I must farther observe that the third part of this demonstration might easily be put into a more diffusive form, and might be deduced step by step from the *Methodus rationum primarum & ultimarum.* But this at present is no way necessary, especially as you admit the proposition to be true. All that I have to do therefore is to explain it a little more particularly, and this I shall be the more careful in doing, because this proportion of nascent or evanescent increments is what I apprehend, you are so often pleased to call a proportion between nothings. With what justice you do so the Reader may easily judge, if he gives himself the trouble of considering what follows.

In the first place, and above all, it is here to be diligently attended to, that Sir *Isaac Newton* no where settles or determines the magnitude of nascent or evanescent increments any farther than to say it is less than any finite magnitude. On the contrary, he expressly declares that their magnitude cannot be assigned or determined. Nor indeed has he any occasion to determine their magnitude, but only the proportion between them, this being all that is requisite in his Method.

Now the proportion between two evanescent increments is easily to be conceived, though the absolute magnitude of those increments is utterly imperceptible to the imagination. For those increments may be expounded or represented by any two finite quantities bearing the same proportion to one another: And as these finite quantities may be clearly conceived, the proportion between them may likewise be clearly conceived, *i. e.* the proportion of the evanescent increments may be clearly conceived by this means. Of this several examples may be found in Sir *Isaac*'s *methodus rationum primarum & ultimarum*, and one in imitation of him may be seen in this Letter, Sect. XXXII.

This being premised, I come now to illustrate and explain the Proposition, to which end I shall make use of the following easy example.



Let the right line $A\,b\,B$, divided into two equal parts in the point b, revolve about the point A, and with any continued motion, even or uneven, remove into the situation $A\,c\,C$. Then will the points B and b describe the circular arcs $B\,C$, $b\,c$: The velocity of the point B will be always double of the velocity of the point b: The arc $B\,C$ will be double of the arc $b\,c$: And the increment $B\,D$ will be double of the increment $b\,d$. In all this there is no difficulty between us.

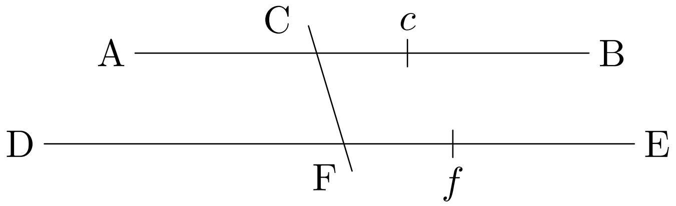
I say farther, The nascent or evanescent increment of the arc $B\,C$ is double of the nascent or evanescent increment of the arc $b\,c$. This you won't understand. I explain it thus, beginning first with the nascent increments.

As soon as the line $A\,b\,B$ begins to revolve upon the point A, and thereby begins to depart from the situation $A\,b\,B$; at that instant of time do the points B and b begin to generate their several increments. And as the velocity of the point B is always double of the velocity of the point b, it is manifest that the increment of the arc $B\,C$ begins to be generated with twice the velocity that the increment of the arc $b\,c$ begins to be generated with; *i. e.* that the nascent increment of $B\,C$ is generated with twice the velocity that the nascent increment $b\,c$ is generated with; and consequently that the former nascent increment is by this proposition double of the latter nascent increment.

To come now to the evanescent increments, let us suppose the line $A\,b\,B$ to have removed into the situation $A\,d\,D$ very near to $A\,b\,B$, whereby the increments $B\,D$, $b\,d$ have been generated. Next let us imagine the line $A\,d\,D$ to return gradually to its first situation $A\,b\,B$, and thereby let the increments $B\,D$, $b\,d$, grow continually less and less, and at last entirely vanish and become nothing. Then as the first of these two increments is double of the second, and decreases twice as fast as the second, it must perpetually bear the same proportion to the second; and consequently the last proportion of these two increments, their proportion at the instant of evanescence will be the same as at first, namely that of 2 to 1. You tell me, when they vanish, they become nothing. I allow it. You say, to talk of a proportion between nothings is to talk nonsense. I agree with you. But their last proportion is not their proportion after they are vanished and are become nothing: It is their proportion when they vanish: It is the proportion with which they vanish.

You will tell me, perhaps, this is unintelligible. I expect it. I ask you therefore, which vanishes first? The increments themselves? Of the proportion between them? I think, Sir, even you will not venture to say, that the increments vanish before their proportion vanishes; or that the proportion vanishes before the increments vanish. If so; we are agreed thus far, that the increments vanish and their proportion vanishes at one and the same instant of time. This proportion therefore which vanishes at the same instant of time that the increments vanish, is the proportion with which the increments vanish, or, in other words, is the last proportion of the evanescent increments.

This, I hope, will appear sufficiently clear to an attentive Reader: But for his farther satisfaction I shall beg leave to lay before him another example.



Let the point A with a given uniform velocity describe or generate the flowing line AB: And let the point D with a velocity continually increasing describe or generate the flowing line DE: Also let both points arrive at the line CF, (cutting the two flowing lines) at the same instant of time, and with velocities exactly equal.

Then it is plain, that if we take two increments Ff, Cc, generated in the same particle of time, Ff will a little exceed Cc. But if we suppose the generating points to return towards the line CF, and their respective velocities in every point of the increments Ff, Cc, to be the very same in returning as they had been before in proceeding from the line CF; it is manifest that the more the increments are diminished by the gradual return of the generating points towards the line CF, the nearer will the proportion between them approach to that of a perfect equality. This is easily conceived, and admits of no dispute.

Farther, if the generating points be supposed to return exactly to the line CF, and thereby the increments vanish and become nothing; the ratio with which the increments vanish into nothing, or the last proportion of the evanescent increments, will be that of a perfect equality. For, as during the time that the generating points are returning towards the line CF, the increments Ff, Cc are continually more and more diminished, and the velocities with which the increments decrease, approach more and more to the ratio of equality; so at the instant of time that those points actually arrive at the line CF, at that same instant the increments entirely vanish, and at the very same instant the velocities with which they decrease and in decreasing vanish, arrive at the ratio of perfect equality: which therefore is the ratio of the velocities with which the increments vanish, and consequently, by this proposition, is the ratio of the evanescent increments.

It is to be carefully attended to, that the proportion here given as the proportion of the evanescent increments, is not their proportion before they vanish. For then Ff will exceed Cc. Nor is it their proportion after they have vanished. For then they are become nothing and have no proportion. But it is their proportion at the instant that they vanish, or the proportion with which they vanish.

I might observe farther, that as the increments do not come to this proportion before they vanish, so neither do they vanish before they come to this proportion: but at one and the same instant of time they come to this proportion and vanish, they vanish and come to this proportion. But I am now afraid I have taken up too much of my reader's time in explaining a point sufficiently clear before.

XVIII. I am not of your opinion, that *every reader of common sense may judge as well of the principles of Fluxions as the most profound Mathematician.* How well the most profound Mathematician can judge, can, I think, be certainly known to the most profound Mathematician only, and I am sure I am not the man. Consequently I cannot take upon me to pronounce upon this point with the same assurance and certitude that you seem to do. But this I well know, that Sir *Isaac Newton* did not write for *every reader of common sense.* He wrote for Mathematicians.

Nor can I agree with you that *the simple apprehension of a thing defined is not*, sometimes at least, *made more perfect by any subsequent Progress in Mathematicks.* It happened to me, and I believe it happens to all or most other Beginners in Geometry, that the definitions of an angle, of a figure, of parallel lines, and of proportion, all become clearer on seeing the application of the things defined in different examples, than upon only reading and considering those definitions with what care and attention soever.

XIX. *I will venture to say that* you *have taken as much pains as (I sincerely believe) any man living*, except a late Philosopher of our University, *to make* non*sense of Sir* Isaac Newton*'s principles.* Your success indeed has been equally bad with his: But that *is not your fault, but your misfortune.* I must needs say, you have done pretty well, considering *you never had a Master in Mathematicks.*

——*Neque ego tibi detrahere ausim*
Hærentem capiti multa cum laude coronam.

XX. I find by this Section as well as by the eighteenth, that you are perfectly well acquainted with what may, or may not be done, *by any progress, though ever so great, in the Analysis, by the best of Mathematicians, by the most profound Analyst.* Such a man as you, one would think, might give one a little light into some very strange things I meet with towards the latter end of this section, such as *velocity without motion, motion without extension, magnitude which is neither finite nor infinite, a quantity having no magnitude which is yet divisible, a figure where there is no space, proportion between nothings, and a real product from nothing multiplied by something.* To me, I must own, these seem to be *Mysteries* utterly *incomprehensible*; but then I take them to be Mysteries of your own making: I can find no more sign of them in Sir *Isaac Newton*'s writings, than of Transubstantiation and some other Mysteries in the New Testament.

XXI. The Picture you here draw is really a very *ingenious portraiture*, but it has no manner of resemblance to Sir *Isaac Newton.* I should sooner have taken it for a picture of *Bellarmine*, or for a handsome likeness of the Author of the Minute Philosopher drawing up an answer to *Philalethes.* A man *driven to arts and shifts* in order to defend his principles, can hardly *take them for true*, must *entertain* more than *some doubt thereof.* For instance, let any man breathing observe *the arts and shifts* you make use of throughout your answer, and he will plainly see you are convinced of your being in an error, but will not own it.

XXII. A new way of passing over a thing is never to have done with it. The reader will easily judge who *colours* most, is most *clamorous*, *reproaches* most and *reasons* least.

XXIII. In the fourth section of the Analyst, instead of fairly giving Sir *Isaac Newton*'s plain, easy, intelligible definition of a second Fluxion, you are pleased to lay down three or four definitions of your own, as obscure, mysterious and absurd as you can possibly devise.

Eripiunt subito nubes cœlumque diemque
Lectorum ex oculis.

After which *you appeal to the trial of every thinking reader, whether the clear Conception of them is not impossible.* This I had taken notice of as a pious art of misleading and confounding your reader, instead of instructing him, and had put the two following questions to you, which I shall here transcribe at large; because with another pious art you have thought fit to truncate the one, and to leave out the other, for particular reasons which I shall by and by lay before the reader. Where, said I, do you find Sir *Isaac Newton* using such expressions as *the velocities of the velocities, the second, third and fourth velocities, the incipient celerity of an incipient celerity, the nascent augment of a nascent augment*? Is this the true and genuine meaning of the words *fluxionum mutationes magis aut minus celeres*?

To these two Questions you are sensible it is incumbent upon you to seem to give an answer, and you are likewise sensible you have none to give. In this *perplexity* it is worthy the observation of a curious reader to see *what arts and shifts a great Genius may be driven to in grappling with an insuperable difficulty.*

In the first place you curtail my first question, cutting off the latter part of it with an *&c.* By this means you hide from your reader one of your definitions, and that the least justifiable of them all, namely *the incipient celerity of an incipient celerity.* There's one difficulty cleverly got over.

In the next place you entirely cut off my second question. In which I find you have two advantages. The first is to avoid giving an answer to it. The second, not to let your reader see Sir *Isaac Newton*'s definition, which I had inserted into that question.

But setting all this aside, after you have proposed my question in your own manner, what answer do you give to it? Do you shew me where Sir *I. N.* uses such expressions? No. You do not pretend to it. What then? Why truly you endeavour to shew, by comparing together two independent Passages taken from two different treatises of Sir *I. N.* that you may justifiably call a second fluxion so and so. Be it so: Though I think otherwise. Yet still this will only shew that a definition of your own may be used; but will not shew it to be Sir *Isaac Newton*'s definition, nor to be equally clear with Sir *Isaac Newton*'s definition. Therefore the *pious art* I at first mentioned, still subsists with the addition of two or three more pious arts to support it, as it generally happens when such arts come to be examined into by any of our family of *Philalethes.*

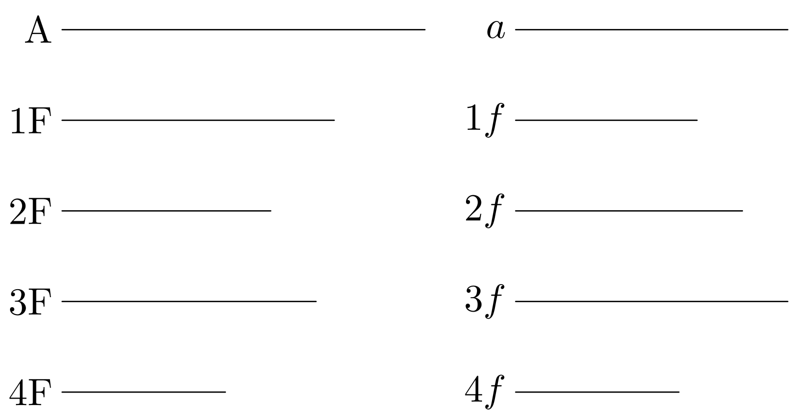
In order to get out of this *Egyptian* darkness in which you have studiously involved the matter in debate, as well as to complete what I had begun in my seventeenth section towards clearing up the first principles of fluxions, I shall now endeavour to give my reader and you too, Sir, if you please, a clear and intelligible conception not only of second, but of third, fourth and fifth fluxions, *&c. ad infinitum*, upon the foot of Sir *Isaac Newton*'s definiton of second fluxions.

Adspice, namq; omnem, quæ nunc obducta tuenti
Mortales visus hebetat tibi, & humida circum
Caligat, nubem eripiam.

That great Man making use of the liberty which has always been allowed to Inventors, of giving new names to new conceptions, and of defining those names as they thought fit, has been pleased to call by the name of fluxion, the velocity with which a flowing quantity increases or decreases. If this velocity do not always continue the same, but undergo any change, the velocity of that change is called a fluxion of a fluxion, or a second fluxion; and as the change is swifter or slower, the second fluxion is said to be greater or less. For instance, if the first fluxion or velocity of the flowing quantity continually increase, the second fluxion is the velocity with which the first velocity increases, and is proportional to the momentaneous increase of that first velocity.

In like manner the third fluxion is the velocity of the change of the second fluxion; the fourth fluxion is the velocity of the change of the third; the fifth the velocity of the change of the fourth, *&c. ad infinitum.*

Here perhaps it may not be amiss to assist the reader's imagination by representing the proportions between fluxions of all the several orders in a sensible manner. I say their proportions: for, as I said before, Sir *Isaac Newton* makes no enquiry into, or ever considers the absolute magnitude of fluxions, or moments, or nascent increments, but only the proportion between them. And this I desire may be carefully remembered.



Let A be a flowing line, and let the velocity with which it flows, be always represented by the line $1\,F$. Then if the line A flow uniformly, that is, if the velocity with which it flows, do never change or alter; the line $1\,F$ will be a constant quantity; and the line A will have only a first fluxion and no second fluxion. But if the line A flow with an accelerated velocity, that is, if the velocity with which it flows, do continually increase; the line $1\,F$ will be a flowing line; and the fluxion of this line $1\,F$, or the velocity with which that line flows, will be the fluxion of the fluxion $1\,F$, or the second fluxion of the line A.

Now let the velocity with which this line $1\,F$ flows, be always represented by the line $2\,F$. Then if the line $1\,F$ flow uniformly, or the velocity with which it flows, do ever change or alter; the line $2\,F$ will be a constant quantity; and the line $1\,F$ will have only a first fluxion, and no second fluxion: And the line A will have a first and second fluxion, but no third Fluxion. But if the line $1\,F$ flow with an accelerated velocity, or the velocity with which it flows, do continually increase; the line $2\,F$ will be a flowing line; and the fluxion of this line $2\,F$, or the velocity with which it flows, will be the fluxion of the fluxion $2\,F$, or the second fluxion of the fluxion $1\,F$, or the third fluxion of the line A: And this fluxion or velocity may be represented by the line $3\,F$.

In like manner $4\,F$ may represent the first fluxion of $3\,F$, the second fluxion of $2\,F$, the third fluxion of $1\,F$, and the fourth fluxion of the line A. And it is visible that after this manner we may proceed *ad infinitum.*

Observing the same analogy, let $1\,f$ represent the first fluxion of a flowing line a: $2\,f$ the first fluxion of $1\,f$, or the second fluxion of the line a: $3\,f$ the first fluxion of $2\,f$, the second fluxion of $1\,f$, or the third fluxion of a: $4\,f$ the first fluxion of $3\,f$, the second fluxion of $2\,f$, the third fluxion of $1\,f$, or the fourth fluxion of a; &c. *ad infinitum.*

Then it is manifest that the proportion between the first fluxion of A and the first fluxion of a, will be the same as that of the two finite lines $1\,F$ and $1\,f$: The proportion between the two second fluxions of A and a, will be the same as that of the two finite lines $2\,F$ and $2\,f$: The proportion between the third fluxions will be that of the finite lines $3\,F$ and $3\,f$: The proportion of the fourth fluxions that of $4\,F$ and $4\,f$, &c. to infinity.

From this methinks it follows, that *second, third and fourth fluxions are* not *more incomprehensible than a first fluxion.*

XXIV, XXV, XXVI. I do not remember to have met with a greater instance of disingenuity and wilful misrepresentation in any controversy I have ever looked into, than what the reader will observe to run through these three sections. You had in the Analyst charged the Mathematicians with unjustly omitting a certain rectangle in their computation of the increment of the rectangle of two flowing quantities, and thereupon had thought fit to represent them as *not proceeding scientifically*, as *not seeing their way distinctly*, as *proceeding blindfold*, as *arriving at the truth they know not how nor by what means*, with abundance of the like compliments plentifully dispersed all over the Analyst.

To this I had replied, First, that this omission, at the worst, could not cause them to deviate from the truth the least imaginable quantity, in computing the most immense magnitude: Secondly, that as they clearly saw and could plainly demonstrate this insignificancy of the omission, they could not justly be said to proceed blindfold: Thirdly that this pretended error or omission of theirs was only a blunder of your own.

In answer to this, you spend three sections in endeavouring to make your reader believe, that the main stress of my defence of Sir *Isaac Newton* and his followers is, That this error of theirs is *of no significancy in practice*; without taking the least notice of the second part of my reply, and barely mentioning the third. Upon this you declaim very abundantly in the style which the Learned call the tautological.

You tell me, and it might have been sufficient once to have told me, that *the application in gross practice is not the point questioned.* I grant it is not. Why then have I said so much about the smallness of the error? I will even tell you the plain truth. Though, as you take notice, I *live in the university*, yet I have been in *London* too, and *am* a little *acquainted with the humour of the times and the characters of men.* Now, Sir, I had observed some of those Gentlemen, who are not greatly pleased that other Persons should be possessed of any learning, which they themselves have not, to be not a little tickled with the rebuke that you had given to the *pride of Mathematicians*: I found them curious to know what this discovery was, that was like to do so much service to the Church: I did my best to give them satisfaction, and to let them see the greatness and importance of it. They see it plainly, and apply the old saying, *Parturiunt montes.*

One of them indeed could make nothing of what I had said about the length of a subtangent, or the magnitude of the orb of the fixed stars; but was fully satisfied by the information given him by one of his acquaintance to the following effect. The Author of the Minute Philosopher has found out that, if Sir *Isaac Newton* were to measure the height of St. *Paul*'s Church by Fluxions, he would be out about three quarters of a hair's breadth: But yonder is one *Philalethes* at *Cambridge*, who pretends that Sir *Isaac* would not be out above the tenth part of a hair's breadth. Hearing this, and that two books had been written in this controversy, the honest Gentleman flew into a great passion, and after muttering something to himself about some body's being overpaid, he went on making reflections, which I don't care to repeat, as not being much for your honour or mine, any more than for that of another person, whom I too highly reverence to name upon this occasion.

XXVII. Now, gentle Reader, we come to the point. You are to be shown the first instance of *my courage in affirming with such undoubting assurance things so easily disproved.* My antagonist *intreats you to observe how fairly I proceed.* I desire you to be upon your guard, to look well about you. After this, if either of us endeavour to throw dust in your eyes, knock him down: Whichsoever of us shall attempt to falsify the words of Sir *Isaac Newton*, or those of his opponent, to the Pump, to the *Thames*, to the *Liffy* with him, pump him, duck him for a Pickpocket. The dispute here is about a matter of fact, and I will endeavour to state the case so plainly, that it shall be impossible either to mistake or to evade it.

In Sir *Isaac Newton*'s demonstration of the rule for finding the moment of the rectangle of two flowing quantities, mention is made of three several rectangles, to each of which the flowing rectangle is equal, at three different times, or in three different states.

This first of these is the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$.

The second is the rectangle AB.

This third is the rectangle $\overline{A + \frac{1}{2}a} \times \overline{B + \frac{1}{2}b}$.

I had observed, Sir, that you were mistaken in taking it for granted, that what Sir *Isaac Newton* was endeavouring to find by the suppositions made in this demonstration, was the increment of the second of these rectangles, the rectangle AB. The reason I gave for supposing you in a mistake, was expressed in the following words. "For neither in the demonstration it self, nor in any thing preceding or following it, is any mention so much as once made of the increment of the rectangle AB." This therefore is a matter of fact that you dispute with me: But how you dispute it, is worth observing. It greatly imports you to contradict me, and yet you cannot, you dare not contradict what I say. Notwithstanding this you will contradict me. Methinks I see my reader stare. I shall be taken for a Madman: And yet I speak the words of truth and soberness. I *affirm*, say you, *the direct contrary.* Contrary to what? To what I have said? No. You cannot, you dare not do it. Your reader would immediately turn to Sir *Isaac Newton* and detect you. But you can first alter what I say, and then contradict me. Instead of my words alone, you can give the reader other words which are not mine, and yet are so intermixed with mine and distinguished by inverted comma's, that every reader shall take them for mine; and then you can *affirm the direct contrary.* You cannot say Sir *Isaac Newton* makes mention of the increment of the rectangle AB: But you can affirm that he makes mention of the rectangle *of such flowing quantities*: That he makes express mention of the increment of *such rectangle*: Of the increment of *that rectangle whose sides have* a *and* b *for their incrementa tota*: That he understands his *incrementum* as belonging to the *rectangulum quodvis.* You go on declaiming about *the words, the sense, the context,* *the conclusion of the demonstration* and *the thing to be demonstrated*:

Involvere diem nubes, nox humida cœlum
Abstulit.——

And when the reader has lost all sight of the point in question, you *refer it to his own eyes.* I refer it to him likewise, and reply to all you have here said, that the first of the three rectangles mentioned above, namely the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, is the rectangle of two flowing quantities, but is not the rectangle AB; is a rectangle whose sides have a and b for their *incrementa tota*, but is not the rectangle AB; is the *rectangulum quodvis* in its first state, but still is not the rectangle AB. The question is, as your self declare, about *matter of fact.* It is not therefore about what Sir *Isaac Newton* means, but what he mentions: Not about what he understands, but what he declares: Not about his sense, but his words. And in all his words throughout this demonstration and every thing preceding or following it, I affirm and aver that he does not so much as once mention the increment of the rectangle AB. Deny it, if you dare.

——*Vim duram & vincula capto*
Tende. Doli circa hæc demum frangentur inanes.

XXVII. You tell me, *I would fain perplex this plain case by distinguishing between an increment and a moment. But it is evident to every one, who has any notion of demonstration, that the* incrementum *in the conclusion must the* momentum *in the Lemma; and to suppose it otherwise is no credit to the Author.* Now Sir, to shew you how little I am inclined to *perplex the case*, I hereby declare that I absolutely and fully agree with you that the *incrementum* in the conclusion is the *momentum* in the Lemma. Let us now see whither this our agreement will lead us.

The *momentum* in the Lemma we both agree to be the *momentum* of the rectangle AB. The *incrementum* in the conclusion is manifestly the excess of the rectangle $\overline{A + \frac{1}{2}a} \times \overline{B + \frac{1}{2}b}$, above the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, *i. e.* the increment of the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$. Therefore we are agreed that the moment of the rectangle AB is the increment of the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$. Consequently you were mistaken in supposing that the moment of the rectangle AB was the increment of the same rectangle AB.

You quote Sir *Isaac Newton*'s words against me to shew that a moment is an increment or decrement. Why Sir! You make me stare. Did not I plainly tell you in my defence that the moment of AB was an increment? Did not I likewise tell you what increment it was, namely the increment of $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$? If you will be pleased to put on your mathematical spectacles, or rather to put on the ingenuity of a Scholar and a Gentleman, (for your eyes are good enough) you will plainly see that the distinction I make, is not between a moment in general and an increment in general, but between a particular moment and a particular increment, between the moment of the rectangle AB and the increment of the rectangle AB, i.e. the excess of the rectangle $\overline{A + a} \times \overline{B + b}$ above the rectangle AB.

Observe me well, Sir, what I have affirmed, and what I still affirm, and what before I have done, I shall prove past a possibility of being denied, is this. The moment of AB is neither the increment nor the decrement of AB; neither the excess of $\overline{A + a} \times \overline{B + b}$ above

$A\,B$; nor the defect of $\overline{A-a} \times \overline{B-b}$ from $A\,B$. This seems to you *a wonderful* assertion. But one of yours, which you call a *very plain and easy one*, is to me much more wonderful.

I asked, which of these two quantities, the increment of $A\,B$, or the decrement of $A\,B$, you would be pleased to call the moment of $A\,B$? Your answer is, *Either of them.* This to me is a very wonderful answer for so great and so accurate a Mathematician to make, and if I have not quite forgot my Logick, I shall draw as wonderful an inference from it. The moment of AB is equal to the increment of $A\,B$: The same moment of $A\,B$ is equal to the decrement of $A\,B$. *Ergo*, the increment of $A\,B$ is equal to the decrement of $A\,B$. That is $A\,b + B\,a + a\,b = A\,b + B\,a - a\,b$, i. e. $2\,a\,b = 0$. Therefore the rectangle $a\,b$, about which the Author of the Minute Philosopher has made such a pother, is by his own confession equal to nothing.

You example in numbers does by no means come up to our case. I shall beg leave to state it a little more pertinently. It is agreed that all numbers are either odd, or even. Upon this you pronounce an unknown number to be even, without giving any reason for it. I represent to you that, since the number is unknown, it may as well be odd as even; and therefore to pronounce it either the one or the other, without any reason for so doing, is no better, and no more like an Arithmetician, than to toss up cross or pile what you shall call it. You may call this mirth, if you please; but the argument is not the less strong against you for this seeming levity.

Nor is the accommodation I proposed in the dispute between an increment and a decrement for the title of moment, at all the less reasonable for being delivered in a ludicrous manner, under which other persons can plainly discern a serious argument, and I perceive you find it much easier to rally than to answer that argument. To say truth, there is no answer to be given to it; it is a demonstration against you as strong as any in *Euclid*, that the moment of the rectangle $A\,B$ is a middle arithmetical proportional between the increment and decrement of the same rectangle $A\,B$. If so;

Redentem dicere verum quid vetat?

XXIX. You are pleased to take notice that *I very candidly represent my case to be that of an Ass between two bottles of hay.* I find by this you are duly sensible of my candour. Had I been less candid, you see plainly I had a fair occasion of representing another person in that perplexity, who might not have had a Ghost so ready at hand to help him out.

The question with me was, Whether the velocity of the flowing rectangle $A\,B$ was the velocity with which the increment, or the velocity with which the decrement, of the same rectangle $A\,B$, might be generated? I could see no possibility of a reason to determine me either way. This led me to fix upon a middle arithmetical proportional between these two velocities, for the velocity of the rectangle $A\,B$: As I had before shewn its moment to be a middle arithmetical proportional between the increment and the decrement. But you, who talk so much of reasoning and logick, and who set up for the great and sole Master of the *ἀκρίβεια Geometrica, are of opinion that either of these velocities may be deemed the velocity* of the rectangle $A\,B$. That is, in your opinion, of two unequal velocities, either the one, or the other, may be deemed equal to a third velocity; or two velocities may be deemed equal and unequal at the same time.

You tell me, *For your part, in the rectangle* A B *considered simply in itself, without either increasing or diminishing, you can conceive no velocity at all.* Nor I neither. But in the rectangle A B considered as flowing, whether increasing or diminishing, I can conceive some velocity or other: And if it flow with an accelerated velocity; I can conceive that velocity to be different in every point of time: And if we suppose the increment of the rectangle to be generated in a given particle of time, and the decrement of the same rectangle to be generated in another equal particle of time: I can conceive the uniform velocity that would generate the increment in the given time, to exceed the uniform velocity that would generate the decrement in the same time: And these two velocities being unequal, I can conceive an arithmetical mean between them; in like manner as I had before supposed an arithmetical mean between the increment and decrement of AB, which mean is the moment of AB: And lastly, while AB flows, I can conceive that the first arithmetical mean is constantly proportional to the last, *i. e.* that the velocity of AB is proportional to the moment of AB.

Upon my asserting that the moment of the rectangle AB is neither the increment nor decrement of that same rectangle AB, you tell me this is *in direct opposition to what Sir* Isaac *himself has asserted* in a passage you quote from him, and you *bid the reader not believe you, but believe his eyes.* Now certainly would any reasonable man, that did not thoroughly know the Author of the Minute Philosopher, conclude that I denied what is expressed in the passage here quoted against me, *viz.* that moments are either increments or decrements; that increments are affirmative moments, and decrements are negative moments. Little would any one imagine from the assurance with which you here express yourself, that all I maintain is, as I said a while ago, That one particular determined moment is not one particular determined increment. But your chicaneries are so many, so gross, and every way so shameful for a Scholar, a Gentleman, and above all for one professing piety and christian zeal, that I grow weary of exposing and refuting them. I solemnly aver, that after I have detected so many, almost in every paragraph of your Reply, I have knowingly and voluntarily passed by many more, particularly those scandalous ones of almost perpetually changing the Words I use, for others that seem to make more for your advantage. One would think your aim was to shew, that whatever care can be used in expression, it shall be no fence against such an adversary as you.

XXX. *You intreat me, in the name of Truth, to tell me what this moment is, which is acquired, which is lost, which is cut in two, or distinguished into halves. Is it,* say you, *a finite quantity, or an infinitesimal, or a mere limit, or nothing at all*? You go on to make objections to every one of those senses. *If I take it in either of the two former,* you say, *I contradict Sir* Isaac Newton. Very true. *If in either of the latter, I contradict common sense.* Very true again. But what then? Can I take it in no other sense, but those four you propose? I assure you I never had a thought of taking it in any one of those senses.

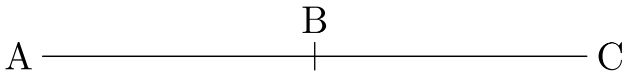
But, in the name of falshood, what is the meaning of this question? Would you have me tell you, what a Moment is? Or, what the magnitude of a Moment is? If the former; I tell you what Sir *Isaac Newton* has told you before, a moment is a momentaneous, or nascent increment, proportional to the velocity of the flowing quantity. If the latter; I have no business at all to consider the magnitude of a moment. * *Neque enim spectatur*, says Sir *Isaac Newton, magnitudo momentorum, sed prima nascentium proportio.* I may tell you farther, that the magnitude of a moment is nothing fixed nor determinate, is a quantity perpetually fleeting

* Princ. Lib. II. Lemm. 2.

and altering till it vanishes into nothing; in short, that it is utterly unassignable. * *Dantur ultimæ quantitatum evanescentium rationes, non dantur ultimæ magnitudines.*

You seem much at a loss to conceive how a nascent increment, a quantity just beginning to exist, but not yet arrived to any assignable magnitude, can be divided or distinguished into two equal parts. Now to me there appears no more difficulty in conceiving this, than in apprehending how any finite quantity is divided or distinguished into halves. For nascent quantities may bear all imaginable proportions to one another, as well as finite quantities. One example of this I have already given in sect. 17. where the nascent increments $B\,D$, $b\,d$, bear to each other the proportion of 2 to 1; and consequently the nascent increment $b\,d$ is equal to one half of the nascent increment $B\,D$. And by dividing the revolving line $A\,b\,B$ into any other assignable parts, it is very easy to conceive what number one pleases of nascent increments bearing any assignable proportions to one another.

Is is possible you may be so exceedingly scrupulous as to object that, though a moment as $b\,d$, is here shewn to be equal to half of another moment $B\,D$, yet still this does not come up to the case of Sir *Isaac Newton*'s demonstration, where one moment is supposed not only to be double of another, as in this case, but to be actually divided into two equal parts. I am willing to have all possible regard for the tenderness and delicacy of your understanding in conceiving any thing that makes against you, and therefore shall readily you give the best assistance I can towards overcoming this difficulty likewise. And perhaps it may be most easy to your imagination, if we first suppose our moments to be finite quantities, and afterwards to become evanescent, as Sir *Isaac Newton* generally does, and observes to be agreeable to the geometry of the ancients.



Let therefore the line $A\,C$ be bisected in the point B, and at a given instant of time let a point set out from A to describe the line $A\,C$ with any given velocity. It is plain this point will arrive at C in a given time. Let another point at the same given instant of time set out from B with one half of the former velocity, to describe the line $B\,C$. Then will this second point arrive at C in the same given time as the first point will arrive there. Now let us suppose the lines $A\,C$, $B\,C$, to be gradually destroyed by this motion of their respective describing points A and B. It is manifest that these lines will be to one another as 2 to 1, not only at the first, but all the time they are diminishing. And as by the approach of the points A and B to the point C these lines will be diminished *sine fine*, and will at last vanish into nothing by the actual arrival of those points at C; the proportion beforesaid of 2 to 1 will still subsist between them to the instant of their evanescence, and even at that very instant. Here then we have the evanescent line $A\,B$ actually divided into two equal parts, as was above proposed. For this division does not cease before the line vanishes, any more than the line vanishes before the division ceases. The whole line $A\,C$ does not vanish before its half $B\,C$; nor the half $B\,C$ before the whole $A\,C$: But the whole line $A\,C$ and the half line $B\,C$ vanish at one and the same instant of time.

* Princ. Lib. I. Sect. I. Schol.

I am satisfied that what I have here laid before you, in order to assist you in conceiving an evanescent quantity distinguished into two equal parts, will be of little use, unless I clear up the rest of those *strange conceits, if words without a meaning may be called so, which*, you say, *I utter with that extreme satisfaction and complacency*, that *unintelligible account, in which you find no sense or reason, and bid the reader find it if he can.*

And here, I own, you have fairly gravelled me. I am at a stand, at a loss, in as great a perplexity, as when my hunger was equally divided between the two bottles of hay, without seeing any possibility of its being satisfied. Oh for a whisper from another Ghost! But alas! What would even that avail me against a Freethinker in Mathematicks, against a man so hardened in infidelity, that he will not believe, though one should arise from the dead, not upon *the word of a Ghost, how venerable soever*? What then can be done? I had, I thought, rendered that account as clear as words could make it. I had shown not only what a moment was, but to prevent, as far as possible, all mistakes about it, I had most carefully and circumspectly shown what it was not. Since that account was published, I had observed several persons to be greatly satisfied with that paragraph, and some to have rectified their notions by it. What then can be the reason of this *phænomenon*, that the perspicacious Author of the Minute Philosopher cannot comprehend what everybody else so easily understands, cannot see what to others appears as clear as the day? Is it that he has hurt his sight by poring so long upon objects too small to be discerned, as *a triangle in a point*? Or has he blinded himself by gazing upon a light too strong for his eyes, with endeavouring to find spots in the Sun? Or has he crack'd his brain by his meditations upon a science too hard for an * Angel? Hark! Is not that he, exclaiming yonder?

O thou, that with surpassing glory crown'd
Look'st from thy sole dominion like the Author
Of this new Method; at whose sight the Sages
Hang their unfurnish'd heads! To thee I call,
But with no friendly voice, and add thy name,
Isaac! *to tell thee how I hate thy wreaths,*
That bring to my remembrance from what state
I fell, elated far above thy sphere;
Till pride and lust of M—e threw me down,
Warring in vain against thee, matchless Knight!
Ah wherefore! He deserv'd no such return
From me. 'Twas he that taught me all I knew
Of fluents, moments, and of increments
Nascent or evanescent, with his science
Upbraiding none, nor were his fluxions hard.

Bless us! How the poor Gentleman raves!
Hush! He begins again.

O then at last relent! Is there no place
Left for repentance, none for pardon left?

* Essay on Vision. Sect. CLIII to CLIX.

None left but recantation, and that word
Disdain forbids me, and my dread of shame
'Mongst Aaron*'s Lordly Sons, whom I deluded*
With other promises, and other vaunts,
Than to recant; boasting I could subdue
The Analysts. Ay me! They little know
How dearly I abide that boast so vain,
Under what torments inwardly I groan,
While Br—s adore me on the Thr—e of Cl—
With M—e and with Cr—r high advanc'd.

But hold! These circumstances surely can never suit my correspondent; and besides, I remember, he abominates the very found of *Miltonick* verse. I must certainly be mistaken.

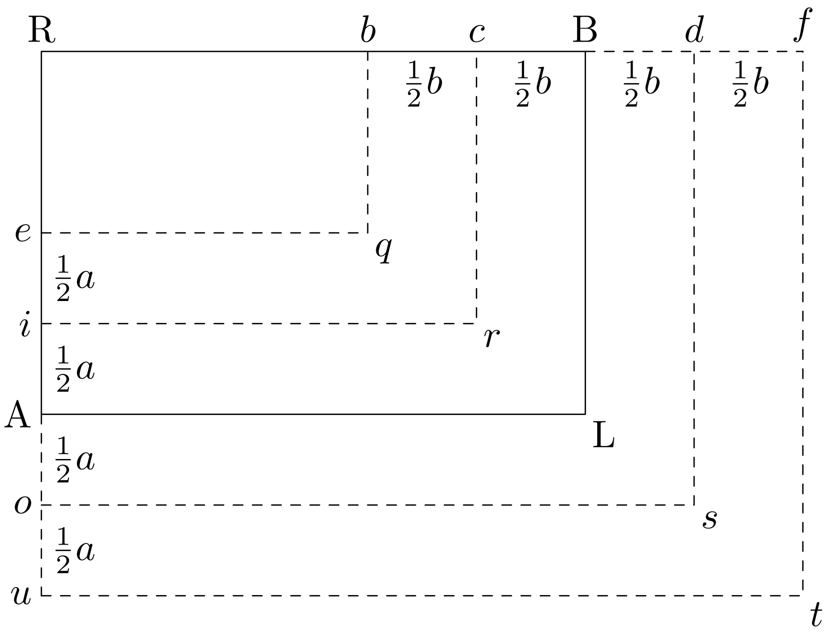
Is it then, that by having been long in the dark, and fixing his attention upon dim and obscure objects which he had not light enough to perceive distinctly, his pupil is so dilated as not to be able to distinguish things in open day? I was going to say, like a Cat that had lost her *Membrana nictitans.* But perhaps this comparison, though with so sagacious an Animal, may give him offence. What then shall I say? I have it. I beg his pardon for these offensive guesses. It was my own fault I was not understood by him. This comes of saving sixpence to one's reader. Had I put a figure in that place, all had been right. But I was resolved to have none. For, I knew, my Bookseller, who understands his business as well as *Jacob Tonson*, would not have failed of clapping on the other sixpence to the price of my performance, which would have disappointed me in my design of making Truth come cheaper than error. But it will be asked, why the want of a figure to that account should be of greater disadvantage to him than to other readers. I answer, this proceeds from an infirmity that I have long observed in him, though every body may not have taken notice of it, and though it is, as I believe, unknown to himself. It is, that his Ideas are almost all sensible. He has few or none of those Ideas which are purely, or partly, intellectual, and which have no sensible images to represent them. But of this disease I may perhaps speak more largely another time; at present I shall endeavour to obviate this defect in him by the following figure.

Let therefore $RALB$, or RL, represent the flowing rectangle AB in Sir *Isaac Newton*'s demonstration; RA the side A; and RB the side B; ei, iA, Ao and ou, each, one half of a; and bc, cB, Bd, and df, each, one half of b; and compleat the rectangles $eRbq$, $iRcr$, $oRds$, $uRft$.

Then will the rectangle $\overline{A-\frac{1}{2}a}\times\overline{B-\frac{1}{2}b}$ be represented by the rectangle Rr; the rectangle $\overline{A+\frac{1}{2}a}\times\overline{B+\frac{1}{2}b}$ by the rectangle Rs; and the difference between these two rectangles, or the moment of the rectangle AB or RL, will be represented by the gnomon rs lying partly within and partly without the rectangle RL.

The rectangle $\overline{A-a}\times\overline{B-b}$ will be represented by Rq; and the difference between this rectangle and the rectangle AB, or the decrement of AB, will be represented by the gnomon Lq lying within the rectangle RL.

Likewise the rectangle $\overline{A+a}\times\overline{B+b}$ will be represented by Rt; and the difference between this rectangle and the rectangle AB, or the increment of AB, will be represented by the gnomon Lt lying without the rectangle RL.



Let us now see if by the help of this figure my *unintelligible account* of a moment can be cleared up.

First then, the moment of the rectangle AB, or RL, is neither the increment from AB to $\overline{A+a} \times \overline{B+b}$; nor the decrement from AB to $\overline{A-a} \times \overline{B-b}$: i. e. rs is neither Lt nor Lq.

It is not a moment common to AB and $\overline{A+a} \times \overline{B+b}$, which may be considered as the increment of the former, or as the decrement of the latter, *i. e.* rs is not Lt, common to RL and Rt, which may be considered as the increment of RL, or as the decrement of Rt.

Nor is it a moment common to AB and $\overline{A-a} \times \overline{B-b}$, which may be considered as the decrement of the first, or as the increment of the last: *i. e.* rs is not Lq common to RL and Rq, which may be considered as the decrement of RL, or as the increment of Rq.

But it is the moment of the very individual rectangle AB itself, and peculiar to that only; and such as being considered indifferently either as an increment or decrement, shall be exactly and perfectly the same: *i. e.* rs is the moment of RL, and peculiar only to RL; and if RL be considered as an increasing quantity, rs may be considered as an increment; if RL be looked upon as decreasing, rs may be considered as a decrement. But whether rs be considered as increment or decrement of RL, it is one and the same quantity.

And the way to obtain such a moment, (*viz.* such as being considered either as an increment or decrement of the rectangle RL, shall be exactly the same, such as is not common to RL and some other rectangle, but peculiar to RL only) is not to look for a moment lying between AB and $\overline{A+a} \times \overline{B+b}$, i. e. between RL and Rt; nor to look for one lying between AB and $\overline{A-a} \times \overline{B-b}$, i. e. between RL and Rq: Not to suppose AB as lying at either extremity of the moment, but as extended to the middle of it, *i. e.* not to suppose Lt to be the moment and RL lying at the inner extremity of it, nor to suppose Lq to be the moment and RL lying at the outer extremity of it, but to suppose rs to be the moment, and RL extended to the middle of it; as having acquired rL the one half of the moment, and being about to acquire the other half Ls; or as having lost Ls the one half of the moment, and being about to lose the other half $L\,r$.

I hope, by this time, Sir, you may have discovered some *sense and reason in what I say* in my account of a moment: but if you cannot or will not discover any, I flatter myself the reader both will and can. And having now a figure before me, I shall take the opportunity of shewing you, that there is some *reasoning* couched under what you are pleased to take for *mirth and humour*, in the proof that I have given, *pag.* 45, 46. of my Defence, that the moment of the rectangle $A\,B$ is not the increment or decrement of $A\,B$, but a middle arithmetical proportional between them.

After proposing to you what by your own confession is the increment, and what the decrement of the rectangle $A\,B$, I ask, you say, *with an intention to puzzle you*, which of these you will call the moment of $A\,B$. I supposed it impossible for you to give any answer to that question, and therefore I decided it my own way. You now say, *Either of them*: And you call this a *plain and easy answer*. My question was, What is the moment of the rectangle $R\,L$? You answer, *EITHER* $L\,t$ or $L\,q$. I ask again, How can I take *EITHER* $L\,t$ or $L\,q$, for the moment of $R\,L$, when $L\,t$ and $L\,q$ are unequal? If the moment be equal to $L\,t$, then must $L\,q$ be less than the moment: And if the moment be equal to $L\,q$, then must $L\,t$ be greater than the moment. Which then must I take for the moment, since each of them can never be equal to the moment? All the Answer I can get out of you is, *EITHER of them*.

Things standing thus, I offer this argument to your consideration. Since, according to you, I may take $L\,t$ for the moment of $R\,L$; and since, according to you I may likewise take $L\,q$ for the moment of $R\,L$; it is manifest that, according to you, I may take $L\,t$ and $L\,q$ added together for twice the moment of $R\,L$. Consequently, according to you, I may take the half of $L\,t$ and the half of $L\,q$ added together for the moment of $R\,L$, i. e. I may take $r\,s$ for the moment of $R\,L$. I hope I may now be allowed to say, "Believe me there is no remedy, you must acquiesce."

——*Frustra cerno te tendere contra.*

I suppose, Sir, you may now comprehend my meaning, when I say, that the moment of $A\,B$ is not the increment of $A\,B$, tho' I allow the moment of $A\,B$ to be an increment, agreeable to Sir *Isaac Newton*'s definition of the word moment. But still it is possible, you may doubt whether the sense I assign to the word moment, be Sir *Isaac Newton*'s sense of the term, or a new one that I have affixed to it in opposition to you. This is the next point to be cleared up.

And here I beg leave to observe in the first place, the presumption is strong in my favour, that by the moment of $A\,B$ Sir *Isaac* means something different from the increment of $A\,B$. For if these two words signified precisely the same thing, it is probable he would have used them indifferently, sometimes the one and sometimes the other. Whereas the fact is, that, after he has done with defining his terms, he never mentions the word increment but in one place, and then he does not speak of the increment of the rectangle $A\,B$, but only of the increment of the rectangle, *i. e.* of the flowing rectangle taken at large. But where he names his rectangle, or other flowing quantity, as $A\,B$, $A\,B\,C$, A^2, A^3, &c. He never mentions the increment of $A\,B$, of $A\,B\,C$, of A^2, &c. but always the moment of $A\,B$, the moment of $A\,B\,C$, the moment of A^2, of A^3, &c. And when such a writer as Sir *Isaac Newton* chuses constantly to use one term, rather than another seemingly of the same signification, it is to be presumed he has some reason for so doing.

But farther, we are to take notice that, according to Sir *Isaac Newton*, the moment of a flowing quantity is ever proportional to the velocity of the same flowing quantity. Let the velocity and the moment of a flowing quantity vary as they will, yet if any instant of time be taken, these three things will be given, such as they are at that same instant, namely, the rectangle itself, its velocity, and its moment. And this velocity and moment are always proportional. If therefore it shall be shown, that the moment of a flowing quantity, such as I suppose it, is proportional to the velocity of that same flowing quantity; it will follow that what I suppose to be the moment, is the same with the moment intended by Sir *Isaac Newton*.

In order therefore to render the conception of this point as easy and as clear as possible, I shall once more have recourse to that well known and familiar instance of a flowing quantity I have so often made use of, *viz.* that of a line described by the motion of a point.

Let x represent the time, in which a flowing line is generated, in all the following cases, and since time flows uniformly, let the constant quantity $\dot{x}$ represent the moment, or increment, (for in this particular case they are both one) of the time x.

Case 1. If the velocity of the generating point be uniform, the flowing line will be as the time in which it is generated, and consequently the line may also be represented by x, and its moment or increment may be represented by $\dot{x}$. In this case therefore the moment $\dot{x}$, being constant, must be proportional to the velocity, which is likewise constant.

Case 2. Let the velocity be equably accelerated, as in the case of a falling body according to *Galileo*'s Theory. Then will the velocity be as the time, and consequently the velocity likewise may be represented by x. And the flowing line being as the time and velocity jointly, that line may be represented as x^2. Now the supposed moment of this line x^2, is $2x\dot{x}$, and I say, $2x\dot{x}$ is proportional to x, the velocity of the flowing line. For since $\dot{x}$ is a constant quantity, it is evident that $2x\dot{x}$ is as $2x$; and $2x$ is as x. Therefore $2x\dot{x}$ is as x. In this case therefore the supposed moment is as the velocity of the flowing quantity.

Case 3. Let the velocity be as the square of the time, and be represented by x^2. Then will the flowing line still be as the velocity and the time jointly, and consequently may be respresented by x^3, the supposed moment of which, *viz.* $3x^2\dot{x}$, is evidently as x^2, or as the velocity.

Case 4. In general, let the velocity be as any power of the time, and consequently be represented by x^{n-1}. Then may the flowing line be represented by the time and the velocity jointly, or by $x \times x^{n-1}$, *i. e.* by x^n. And the supposed moment of this line will be $xx^{n-1}\dot{x}$, which is manifestly as x^{n-1}, that is, as the velocity.

The moment therefore supposed by me is ever proportional to the velocity, and consequently is the moment supposed by Sir *Isaac Newton*.

While I am upon this consideration, it may not be amiss for a more compleat illustration of what we have been talking of, to consider a little more particularly the second case, or that of the flowing quantity x^2, answering to the rectangle AB of Sir *Isaac Newton*. I shall therefore take the liberty of laying before my reader in one view, the decrement, the moment, and the increment of the flowing quantity x^2, together with the several velocities that would generate them respectively, with an uniform motion, in a given time.

The Decrement.	Moment.	Increment.
$2x\dot{x} - \dot{x}\dot{x}$	$2x\dot{x}$	$2x\dot{x} + \dot{x}\dot{x}$
Velocity.	Velocity.	Velocity.
$x - \dfrac{\dot{x}}{2}$	x	$x + \dfrac{\dot{x}}{2}$

Here it appears that as the moment is a middle arithmetical proportional between the decrement and increment; so is the velocity of the flowing line, or the velocity that would generate the moment in the given time $2\dot{x}$, a middle arithmetical proportional between the velocities that would respectively generate the decrement and increment in the same given time. And this proportion equally holds, whether the moments be evanescent, or finite quantities of whatsoever magnitude.

Whence I infer, that although it were not possible to conceive an evanescent moment divided into two equal parts, yet as finite ones may be conceived to be so divided, that demonstration of Sir *Isaac Newton*'s which you object against, will still hold firm and entire, by substituting finite moments in the room of evanescent moments. Which is a secret you were not aware of.

One more observation, and I have done. You would have us take the increment of $A\,B$, or in this case the increment of x^2, for the moment of x^2; that is, you would have us take $2x\dot{x} + \dot{x}\dot{x}$, and not $2x\dot{x}$, for the moment of x^2: And yet you allow that the moment of x^2 is proportional to the velocity of x^2. But the velocity of x^2 is x; and the quantity you give us as the moment, namely $2x\dot{x} + \dot{x}\dot{x}$, is not proportional to this velocity x. Therefore by your own concession, that quantity is not the true moment. But the quantity that Sir *Isaac Newton* assigns, namely $2x\dot{x}$, has just now been shown to be proportional to x, the velocity of x^2, and therefore is the true moment. Now therefore I may safely repeat my question, and *ask with my accustomed air*, "What say you, Sir? Is this a just and legitimate reason for Sir *Isaac*'s proceeding as he did? I think you must acknowledge it to be so."

But hark you! Why all this outcry about Ghosts and Visions? Pray who first introduced them? If I brought in one, you might consider it was to a very good purpose, to help my self out, or rather to help you out, at a dead lift. Whereas you had before needlessly introduced an innumerable multitude of *Ghosts of departed quantities*, for no other intent or purpose in the world but your own diversion.

In consideration of which I hope I may be pardoned for bringing in one more, though I can give no better reason for it, than that the Apparition runs strongly in my fancy.

See where the Phantom comes, a sable wand
Before his decent steps! Of regal port,
But faded spendour wan: His flowing hair
Circled with golden Tiar: A gorgeous vest,
Dyed *Meliboean*, from his shoulders broad
Hangs graceful down: In sable armour clad,
Sable his body, but in whitest mail
His sinewy arms refulgent: Such the bird
Majestick treads the albent cliffs, or wings
The air *Roystonian*. Passion dims his face

Thrice chang'd with pale, ire, envy and despair.
His gestures fierce, and mad demeanour mark!
His form disfigur'd more than can befal
Spirit of happy sort: For heavenly minds
From such distempers foul are ever clear.
The thought both of lost fame and lasting scorn
Torments him; round he throws his baleful eyes,
That witness huge affliction and dismay,
Mixt with obdurate pride and steadfast hate;
And breaking silence, horrid, thus begins.

Fall'n from what height! So much the stronger prov'd
He with his Moments*: And till then who knew*
The force of those dire arms? Yet not for those,
Nor what the potent Victor in his rage
Can else inflict, do I repent, or change,
Though chang'd in outward lustre, that fixt mind,
And high disdain from sense of self-weigh'd merit,
That with proud Newton *rais'd me to contend,*
And shook his Throne. What though the field be lost?
All is not lost. Th'unconquerable will,
And study of revenge, immortal hate,
And courage never to submit, or yield,
As at the head of battle still defies him,
Undaunted, since by Fate the wings of Ganders,
And Sepia *sable-blooded cannot fail.*

So spake th'Apostate Analyst, though in pain,
Vaunting aloud, but rack'd with deep despair.
Frowning he ended, and his look denounc'd
Desperate revenge, and battel dangerous
To less than *Philalethes*; when upstood
One next himself in crime, in strength superior;
Nisroc, of principalities the prime,
And to that eminence by merit rais'd;
Nisroc, the strongest and the fiercest Spirit,
That fought in this bad cause, the strongest far,
The fiercest once, now broken with despair.
His trust was with great *Isaac* to be deem'd
A match in strength, and rather than be less
Car'd not to be at all. Grown humbler now,
As one, he stood, escap'd from cruel fight,
Sore toil'd, his riven arms to havock hewn,
Mangled with ghastly wounds through plate and mail.

Clouded his brow, deep on his front engraven
Sat meditation silent, in his eye
Shone piercing contemplation, thought profound,
And princely counsel in his face yet shone,
Majestick, though in ruin. Sage he stood,
With *Atlantean* shoulders fit to bear
The weight of loftiest Theories: His look
Drew audience and attention still as night,
Or summer's noontide air, while thus he spake.

O Prince, O Chief of many wronghead Powers,
That led th'imbattled Increments *to War*
Under thy conduct, and with dreadful blunders,
Brainless, endanger'd Newton*'s deathless Fame;*
And put to proof his high Supremacy,
By chance upheld, or science; and that strife
Was not inglorious, though th'event was dire:
The dire event too well I see and rue,
That with sad overthrow and foul defeat
Hath lost thy fame, and all this muddy Host
In horrible destruction laid thus low,
As far as Ghosts *and* shadowy Entities
Can perish.

The Vision would lead me a great deal farther, and I might proceed to relate in heroick verse the rebuke given by the fallen Chief to this his Associate, for pusillanimity in abandoning the noble undertaking.

If thou beest he: But O how fall'n! How chang'd
From him, from that sworn Friend, whom mutual league,
United thoughts and counsels, equal hope
And hazard in the glorious Enterprize,
Join'd with me once, now misery hath join'd
In equal ruin——

But I am afraid, Sir, you begin to be tired. Possibly this vision of mine may give you as little pleasure, as the Ghosts you introduced some time ago afforded to any of your readers. I shall therefore stop here, and hope from your known candour, that if you chance to spy any inconsistencies, or any little marks of vanity in this my vision, you will be so just as to consider there are but few visions, apparitions, dreams, or castles built in the air, that are not liable to some objection.

XXXI. It is now so evident even to your self, that the moment of the rectangle $A\,B$ is not the increment of the rectangle AB, that I expect to be complimented upon my civility in charging you with want of caution only, in putting the one for the other. You have indeed *replied, that this charge is as untrue as it is peremptory.* But that was in your state of blindness, and I forgive you without your asking pardon. You say in your justification, Sir *Isaac Newton*, in the first case of this *Lemma, expressly determines it to be an increment.* Yes, he determines it to be an increment. But an increment of what? An increment of $A\,B$? Methinks I see the good old Knight hold up his finger and cry *Cave.* It is the increment of $\overline{A-\frac{1}{2}a}\times\overline{B-\frac{1}{2}b}$.

You say, *take it increment or decrement as you will, the objections still lie, and the difficulties are equally insuperable.* Very true, if I take it for increment or decrement of $A\,B$. But I will not take it for either, and then all the difficulties and objections vanish before me, *they become nothing*, there are no difficulties, no objections, I meet with nothing in my way but *the Ghosts of departed* difficulties and objections.

XXXII. Before I proceed to vindicate that assertion of mine which makes the subject of this section, I crave leave to observe, that this assertion, true or false, is no way material to the point in debate between us. You were fully answered before I laid down that assertion: And all the subterfuges you have since made use of, are clearly removed before I vindicate it. Why therefore did I make that assertion? Dear Sir, the true reason is a secret. I see plainly it never entered your *Pericranium*, any more than that of some other persons much superior to you in this part of science. In due time it may come out. In the mean while all I shall say is, it was made to guard, not against present, but future objections.

Do not mistake me, Sir, I am not going to excuse that assertion, much less to give it up. I intend to vindicate it to the last drop of my pen. Like *Mackbeth* in blood,

> ——*I am in ink*
> *Stept in so far, that should I wade no more,*
> *Returning were as tedious as go o'er.*

My assertion was, That the moment of the rectangle $A\,B$, determined by Sir *Isaac Newton*, namely $a\,B+b\,A$, and the increment of the same rectangle determined by your self, namely $a\,B+b\,A+a\,b$, are perfectly and exactly equal, supposing a and b to be diminished *ad infinitum*; and this by *Lemma* I. *Sect.* I. *Libr.* I. *Princip.*

You answer, *If* a *and* b *are real quantities, then* a b *is something, and consequently makes a real difference; but if they are nothing, then the rectangles whereof they are coefficients, become nothing likewise; and consequently the* momentum *or* incrementum*, whether Sir* Isaac*'s or mine, are in that case nothing at all.*

By giving this for an answer to my assertion, it is plain you have no notion of what Sir *Isaac Newton* means by a quantity being infinitely diminished, though he has so fully and clearly explained himself in the *scholium* of that section of the *Principia*, which I so often refer you to.

Suppose a given line to be gradually diminished, during a given time, by the continued motion of a point, so that at the end of the given time the line would entirely vanish and become nothing. Then if the motion of the point, and the gradual diminution of the line consequent thereupon, be supposed to stop before the expiration of the given time, it is plain that the line will not as yet have been diminished *ad infinitum*; it will still be something, it will be a real quantity, it will be a finite quantity. But if the motion go on, without stop or stay, to the end of the given time, it is manifest that the line must be diminished *sine fine, sine limite*, it must be diminished *ad infinitum*, it must vanish, it must become nothing.

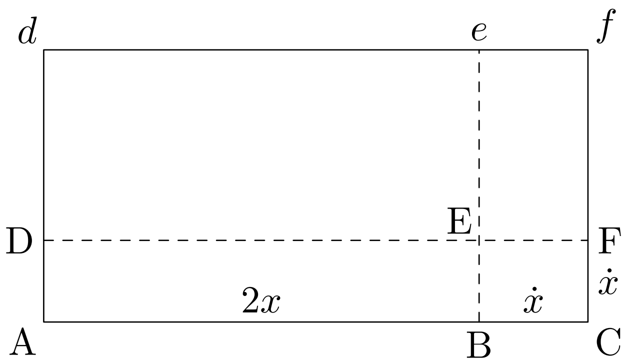
The end of this diminution *ad infinitum*, the vanishing of the line, and its becoming nothing, these three must all happen at one and the same instant of time, namely at the expiration of the given time. So that an instant before the expiration of the given time, or before the quantity becomes nothing, it cannot truly be said to be actually diminished *ad infinitum.* Therefore while a and b are real quantities, they are not yet diminished *ad infinitum*, they may be farther diminished. And consequently the first part of your answer is quite beside the purpose: It tends only to shew that there is a real difference between the moment and the increment, before the instant of time when I suppose them to become equal; that while they are unequal, there is a difference between them. A great discovery, and undoubtedly true!

You proceed in your answer, *If they,* i. e. a *and* b*, are nothing, then the rectangles whereof they are coefficients, become nothing likewise: and consequently the* momentum *or* incrementum*, whether Sir* Isaac*'s or mine, are in that case nothing at all.*

This likewise is undoubtedly true. But it is so far from contradicting Sir *Isaac Newton*'s doctrine, that it is perfectly agreeable to it. What he says, and what I contend for, is this.

Though so long as a and b are real quantities, their rectangle $a\,b$ is a real quantity, and there is a real difference between the two quantities $a\,B + b\,A$ and $a\,B + b\,A + a\,b$: Yet, when by a continual diminution *ad infinitum* a and b vanish, their rectangle $a\,b$, or the difference between the two quantities $a\,B + b\,A$ and $a\,B + b\,A + a\,b$, vanishes likewise, and there is no longer any difference left between those quantities, *i. e.* those quantities are equal. But you say, when $a\,b$, when the difference between these two quantities vanishes, the quantities themselves do likewise vanish. I agree with you. Their difference therefore vanishes when they vanish: And they vanish when their difference vanishes: Or, The quantities themselves, and the difference between them, vanish at one and the same instant of time.

You see I agree perfectly with you, that the moment and increment vanish at the same instant that their difference vanishes. All I contend for is this, That the moment and increment vanish with a ratio of equality, and that they do so, I am going to demonstrate after Sir *Isaac Newton*'s manner.



Let the rectangle $A\,E$ represent $2x\dot{x}$, the moment, and let the rectangle $A\,F$ represent $2x\dot{x} + \dot{x}\dot{x}$, the increment of the flowing square x^2. I say when $\dot{x}$ vanishes, the moment $2x\dot{x}$, and the increment $2x\dot{x} + \dot{x}\dot{x}$, will vanish with a ratio of equality.

DEMONSTRATION.

Produce the lines $A\,D$, $B\,E$, $C\,F$, to the distant points d, e, f, and draw the right line $d\,e\,f$ parallel to $D\,E\,F$. Then will the rectangles $A\,E$, $A\,F$ be proportional to the rectangles

$A\,e$, $A\,f$. Now let $C\,B$ be diminished *ad infinitum*, and vanish into nothing by the coincidence of the point C with the point B. At the instant that these points coincide, the lines $C\,F\,f$, $B\,E\,e$ will likewise coincide, *i. e.* the rectangles $A\,e$, $A\,f$, will coincide and become perfectly equal, and at the same instant the rectangles $A\,E$, $A\,F$, i.e. the moment and increment, will vanish. But at the instant that the rectangles $A\,e$, $A\,f$ become equal, the rectangles proportional to these, $A\,E$ and $A\,F$ must likewise become equal. Therefore these rectangles vanish and become equal at one and the same instant of time, or vanish with a ratio of equality. *Q. E. D.*

I am so desirous of leaving both you, Sir, and my reader without any scruple upon this point, that I cannot content myself with only demonstrating, that in fact the thing is as I say, unless I likewise shew you by what means it comes to be so. The case is this.

While $\dot{x}$ is gradually diminished, the ratio between the increment and moment is likewise perpetually diminished, and tends to a certain limit which it can never pass, and can never arrive at till $\dot{x}$ is diminished *ad infinitum*, and vanishes into nothing. That limit is equality.

Likewise while $\dot{x}$ is gradually diminished, the increment and moment are perpetually diminished, and tend to a certain limit, which they can never arrive at till $\dot{x}$ is diminished *ad infinitum*, and vanishes into nothing. That limit is nothing.

So that the ratio of the increment and moment, and the increment and moment, do both arrive at their several limits, *i. e.* at equality and at nothing, at one and the same instant of time. That is, the increment and moment become equal and vanish, vanish and become equal, at the same instant.

Methinks, Sir, you and I are now so far agreed, that it is pity there should be any difference between us about the *Lemma* I quoted to you. But as it may be of some service to you, and may possibly save trouble to us both another time, I am willing to take a little farther pains for your information; though I greatly fear it will be lost upon you, and that you will make no better use of it, than you did of the friendly advice I gave you in my last letter, to weigh very well what Sir *Isaac Newton* says, before you censure him. For I see my hypothesis about the cause of your errors still holds good: You have too good an opinion of your own understanding, to think you can ever be mistaken. Else how was it possible for you to say, when such a man as Sir *Isaac Newton* was laying down the foundation of the Method of Fluxions, *That his very first and fundamental* Lemma *was incompatible with and subversive of the Doctrine of Fluxions? that it seemed the most injudicious step that could be taken? That it was directly demolishing the very doctrine I would defend*? Pray let us see what this *Lemma* is.

LEMMA I.

Quantitates, ut & quantitatum rationes, quæ ad æqualitatem tempore quovis finito constanter tendunt, & ante finem temporis illius propius ad invicem accedunt quam pro data quavis differentia, fiunt ultimo æquales.

In this *Lemma* are manifestly contained the following suppositions.

1. That the quantities or ratio's of quantities, tend to equality.
2. That this tendency to equality constantly holds during a given time.
3. That they come nearer to equality than to have any assignable difference between them.

4. That they come thus near to equality before the expiration of the given time.

Upon these suppositions Sir *Isaac* affirms and demonstrates, that the quantities do at last become equal, *i. e.* do become equal at the end of the given time.

We come now to see what you object to this; you, I say, who *have long since consulted and considered this* Lemma; you who *very much doubt whether I have sufficiently considered this* Lemma, *its demonstration, and its consequences*; you *who have taken as much pains as (you sincerely believe) any man living to understand that great Author, and to make sense of his principles*; you, *on whose part, you assure me, no industry, nor caution, nor attention have been wanting: So that, if you do not understand him, it is not your fault but your misfortune.* I am going to take my candle and lanthorn, as *Harlequin* did a while ago at *Paris* to look for the complete victory at *Parma*; and shall make a diligent search after your industry, caution and attention in considering this short *Lemma*. It certainly deserves all the caution you can use, since it contains, according to Sir *Isaac Newton*, the foundation not only of the method of fluxions, but of the *Principia* themselves, of that book which is the admiration and astonishment of all mankind, except the Author of that greater and more stupendous work, *The Minute Philosopher*.

You suppose Sir *Isaac Newton to argue, that quantities must be equal, because they have no assignable difference.* Is this then the only supposition he makes, that quantities have no assignable difference? Does he not plainly make the first, the second and fourth supposition above-mentioned, as well as the third? Are the following words, *ad æqualitatem tempore quovis finito constanter tendunt, & ante finem temporis illius*, left out of your copy? If not, where were your eyes, that you overlooked them? Or your integrity, that you suppressed them? Might not the most orthodox Father of the Church, or the great Apostle St. *Paul* himself, be proved an errant Heretick by such a proceeding? For shame go and look over that *Lemma* again, read it diligently, consider it throughly, understand it if you can, and till you have done so, never dare to take the venerable name of Sir *Isaac Newton* within your lips, much less to condemn him.

XXXIII, XXXIV. We come now to the method for obtaining a rule to find the fluxion of any power of a flowing quantity, which is delivered in the introduction to the Quadratures, and considered in the Analyst. *And here*, say you, *the question between us is, whether I have rightly represented the sense of those words*, evanescant jam augmenta illa, *in rendering them, let the increments vanish*, i. e. *let the increments be nothing, or let there be no increments*? And so, Sir, you would have the Reader believe that this is the whole of the question between us: That we have each of us spent four or five pages, and may possibly spend twice as many more before we have done, in wrangling about the translation of four *Latin* words. If so, methinks his best way will be to let us wrangle by our selves, and to translate those four words himself as he thinks fit, without ever troubling his head about us.

But I take the question between us to be of a little more extent, and of somewhat more importance. What I have endeavoured to establish the sense of, is not those four words alone, but the whole passage taken together, *i. e.* in the style of divines, the text and the context. The whole passage is, *Evanescant jam augmenta illa & eorum ratio ultima erit*, and I have endeavoured to settle the meaning of this whole passage taken together, by comparing it with an equivalent passage, but expressed in such terms as not to be liable to any sophistication, *Nascantur jam augmenta illa & eorum ratio prima erit.*

The question therefore between us is not barely how those four words may be translated: If they stood alone, they might be translated twenty diferent ways: But the question is, how these four words ought to be translated in conjunction with the other words that follow; how the whole passage ought to be translated, so as to let the Reader understand the meaning and design of Sir *Isaac Newton* in that passage. His design is manifestly to consider the proportion between the evanescent augments, or to consider the proportion with which the augments vanish. He plainly makes two suppositions in this passage. The first is, that the augments vanish, or become nothing. The second, that the augments have a last ratio. And his business is to determine what this last ratio is: Now the question between you and me is, when, at what instant of time Sir *Isaac Newton* supposes the augments to have this last ratio? You will needs have it, the that he supposes the augments first to vanish, to become nothing, and then considers the proportion between those nothings. I maintain, that he considers the proportion between the augments, not after they are vanished, but at the instant that they vanish, in the very point of evanescence. And I am justified by his own words, where he more fully explains himself, * *intelligendam esse rationem quantitatum non antequam evanescunt, NON POSTEA, set quacum evanescunt.* You see therefore, Sir, the hard words, you say, I have used, do not fall upon my Friends, but fall where I intended them. The blunder of making the quantities first become nothing, and then settling the proportion between those nothings, stands just as it did. It puts me in mind of an Evidence, who was instructed to swear that a certain will was made just as the Testator was dying, and was therefore subsequent to another will made some time before his death. This person resolved to make sure work, and swore positively that this was the last will, for it was made after the Testator was dead.

You see likewise you had no reason to despair of making me acknowledge, that vanishing and becoming nothing were equivalent terms with Sir *Isaac Newton*. Indeed, how was it possible to think otherwise? A nascent augment must have been nothing before it began to exist, and an evanescent augment must be nothing after it ceases to exist.

As it is my business chiefly to keep upon the defensive, and I have hitherto had very little occasion to act offensively, I did in my first Letter consider your importent *Lemma* and reasoning upon it, no farther than was necessary to justify Sir *Isaac Newton* against the consequences you draw from that *Lemma*. But now, as you are pleased to shew more than ordinary arrogance in this and the following section, I hope the reader will excuse me, if I step out of my way to call you a little to account. A vigilant General, who is assaulted in his entrenchments by an overbearing and insolent Enemy, may sometimes observe that Enemy in the heat of his attack, to lay himself so open, as to give a fair opportunity of sallying out and chastising him.

And it may not be amiss to shew, that Mathematicians are not the only persons, who *falsely imagine their rational faculties to be more improved than those of other men, which have been excercised in a different manner, and on different subjects.* That there are other persons, who *erect themselves into judges and oracles, concerning matters*, which they have *never* sufficiently *considered nor comprehended.* And if this appear, it will surely *furnish a fair* argumentum ad hominem *against men, who reject that very thing in* Geometry *which they admit in* Logick. It will be *a proper way to abate the pride, and discredit the pretentions of these* Logicians and Metaphysicians, *who insist upon clear Ideas in points of* Mathematicks,

* Schol. Sect. I. Libr. I. Princip.

if it be shewn that they do without them in their own science.

The substance of your *Lemma* is this. If one supposition be made, and be afterwards destroy'd by a CONTRARY supposition; then every thing that followed from the first supposition, is destroyed with it. This being laid down, you proceed thus. Sir *Isaac Newton* supposes certain increments to exist, or that there are certain increments. In consequence of their supposed existence, he forms certain expressions of those increments, with intent to deduce the proportion of the increments from those expressions. He afterwards supposes that those increments vanish, *i. e.* say you, that the increments are nothing, that there are no increments.

I forbear making any remarks upon your interpretation of the word vanish. I admit it to be as you are pleased to make it, that the first supposition is, there are increments; and that the second supposition is, there are no increments. What do you infer from this? The second supposition, say you, is contrary to the former, and destroys the former, and in destroying the former it destroys the expressions, the proportions, and everything else derived from the former supposition. Not so fast, good Mr. *Logician.* If I say, the increments now exist, and, the increments do not now exist; the latter assertion will be contrary to the former, supposing now to mean the same instant of time in both assertions. But if I say at one time, the increments now exist; and say an hour after, the increments do not now exist; the latter assertion will neither be contrary, nor contradictory to the former, because the first now signifies one time, and the second now signifies another time, so that both assertions may be true. The case therefore in your argument does not come up to your *Lemma*, unless you will say Sir *Isaac Newton* supposes that there are increments, and that there are no increments, at the same instant of time. Which is what you have not said, and what, I hope, you will not dare to say.

But perhaps you will still maintain, that whether the second supposition be esteemed contrary, or not contrary, to the first, yet as the increments, which were supposed at first to exist, are now supposed not to exist, but to be vanished and gone, all the consequences of their supposed existence, as their expressions, proportions, &c. must now be supposed to be vanished and gone with them. I cannot allow of this neither.

Let us imagine your self and me to be debating this matter, in an open field, at a distance from any shelter, and in the middle of a large company of Mathematicians and Logicians. A sudden violent rain falls. The consequence is, we are all wet to the skin. Before we can get to covert, it clears up, and the Sun shines. You are for going on with the dispute. I desire to be excused. I must go home and shift my cloaths, and advise you to do the same. You endeavour to persuade me I am not wet. The shower, say you, is vanished and gone, and consequently your coldness, and wetness, and everything derived from the existence of the shower, must have vanished with it. I tell you I feel my self cold and wet. I take my leave, and make haste home. I am persuaded the Mathematicians would all take the same course, and should think them but very indifferent Logicians, that were moved by your arguments to stay behind.

Another example may make all clear. I know a certain Gentleman, who about the first day of *April* 1734, was verily persuaded he saw more clearly into the principles of fluxions, than Sir *Isaac Newton* had ever done. The consequence of this persuasion was, that he published a book, which immediately convinced all mankind of the contrary. He has since had such reasons given him, as have entirely altered his opinion. His former persuasion is vanished and gone; but the book that was the consequence of that persuasion, is not vanished and gone with it. It would have been much for his credit, and for the quiet of the poor Gentleman's mind, if it had.

XXXV. You mistake me, Sir: What I dislike in you is not your *modesty*, but your arrogance. 'Tis your unparallel'd and amazing insolence, to the greatest discoverer of truth, of a mere mortal, that ever appeared in the world.

I am of opinion, that *placing the same point in various lights is of great use to explain it.*

You have not shown Sir Isaac Newton's *various accounts of fluxions* to be *inconsistent.* I find them perfectly consistent, and do again profess my self greatly obliged to him for his condescension, in setting his doctrine in several different lights, without which, *I* still *doubt, I should never have understood it.*

But you seem to think it great vanity in me, to talk as if I understood the doctrine of Fluxions. Why, Sir! I hope Sir *Isaac Newton* wrote so as to be understood by somebody. I have taken pains to understand him, and I suppose many others to understand him likewise: I prefer my self to no body, and I never compare my self with any body but one. It is where I speak of such ordinary Proficients in Mathematicks, as you and me. Even there, you see, I have the good manners to place you first. Had I said, no body understands him, but I; Or, I don't understand him, and therefore no body can understand him, it were unpardonable vanity.

You say, I insult you, in asking what it is you are offended at, who do not still understand him? I neither insult you, nor blame you, for not understanding him: But it is, I think, pretty extraordinary for a man, who so often professes not to understand Sir *Isaac Newton*, to complain that Sir *Isaac* takes too much pains to explain and illustrate his doctrine, by setting it in several different lights. As to your request to help you out of the dark, I have done my best, and hope you see much better than you did. The eye-water I have applied, might possibly given you some pain; but it will do you a power of good. *E cœlo descendit* Γνώθι σεαυτὸν.

XXXVI. I flatter my self, I have already done to your mind what you here request.

XXXVII. If I were to say, there are a hundred mean and low artifices in a certain pamphlet, scarce a section without one or more too scandalous and too trifling to mention: This is plain to me; but I will not undertake to demonstrate it to others: Is this the same as to say, I cannot demonstrate it to others? No. But it would take up too much of my time, it would swell my letter to too great a bulk to demonstrate it. You say below, I neither *will*, nor *can*. You make therefore a difference between the meaning of these two words.

XXXVIII. In this Section you address yourself to me in the following words. "You will have it, that I represent Sir *Isaac Newton*'s conclusions as coming out right, because one error is compensated by another contrary and equal error, which perhaps he never knew himself not thought of: that by a twofold mistake he arrives, though not at Science yet at Truth: that he proceeds blindfold, *&c.*. All which is untruly said by you, who have misapplied to Sir *Isaac Newton*, what was intended for the *Marquis de l'Hospital* and his followers." If this was untruly said by me, I assure you it was not a wilful untruth. You see Mr. *Walton* fell into that mistake as well as I. And I do not know a single person who has read the Analyst, but is in the same mistake. However a mistake it undoubtedly is; no body ought in the least to dispute it, after a person of your character has made the publick declaration just now recited, and has farther assured us, that *this double error doth concern the Marquis alone, and not Sir* Isaac Newton. Far be it from me to call the truth or sincerity of this declaration in question. On the contary, I ask you pardon for my mistake; and to make you all the satisfaction in my power, I do hereby retract, recant and abjure my error, and abandon my picture, my *ingenious portraiture of Sir* Isaac Newton *and* Dame Fortune, to the flames. If you are not yet satisfied, I beg leave to alledge the following reasons in mitigation of my offence.

1. Your discourse seemed to me to be directed to a follower of Sir *Isaac Newton.* And as in Sect. XX. of the Analyst, where this affair of the double error begins, you perpetually address yourself to him in the second person, as *you demonstrate, you are conversant, you conceive, you proceed, you apply, your conclusions, your logick and method*, &c. I too hastily judged that the double error related to this follower, and consequently to his master.

2. As this affair is pursued through eleven Sections, beginning at Sect. XX. and ending with Sect. XXX. I find Sir *Isaac Newton*'s way of notation to be used in three of those Sections, and the *Marquis*'s notation in two. I find Sir *Isaac*'s language and expression, as increments, moments, fluxions, infinitely diminished, vanish, *&c.* to be used in nine of those Sections, and the *Marquis*'s language and expression, as differences, infinitesimals, *&c.* in seven of those Sections. Whence it seemed to me, that Sir *Isaac Newton* was as much concerned in this matter, as the *Marquis.*

3. In one of those Sections, namely Section XXVI. you refer to Sect. XII, and XIII.; in the first of those Sections, *viz.* Sect. XII. I find this Quotation, *Philosophiæ Naturalis Principia Mathematica, lib. 2. lemma. 2.* and Sect. XIII. contains nothing else but your instance of false reasoning taken from Sir *Isaac Newton*'s Book of Quadratures. Likewise in another of those Sections, namely Sect. XXVIII. I find the same thirteenth Section quoted. From all which it seemed to me, that Sir *Isaac Newton* was rather more concerned in this affair than the *Marquis*, whose works I do not find to be quoted in any of those Sections, so much as once.

4. The argument used in the Analyst seemed to me to bear equally hard against Sir *Isaac Newton* and the *Marquis*; so that I could not see how you could condemn the one, and acquit the other, of either of the two errors.

These considerations had so fully possessed my mind, that Sir *Isaac Newton* was supposed by you to be guilty of this double error, that nothing, but my firm persuasion of your veracity and integrity, could ever have removed that apprehension. I must own, I have still one scruple upon my thoughts. If you will be so good as to remove that, my mind will be perfectly easy about this affair. It is this.

The first error in giving $2x\,dx$ for the difference, or $2x\dot{x}$ for the moment of xx, is common to the *Marquis* and Sir *Isaac Newton.*

The *Marquis* makes a second error, which perfectly corrects the first, whence his conclusion comes out right.

Sir *Isaac Newton* makes no second error to correct his first, and therefore his conclusion ought to come out wrong.

And yet Sir *Isaac*'s conclusion comes out exactly right, and is the same with the *Marquis*'s conclusion. The more I consider this, the more it puzzles me: Possibly, for want of the *Philosophia prima*, which you are so great a master of.

XXXIX. As you do not *persist*, nay, on the contrary, desist, and entirely disown your accusing Sir *Isaac Newton* of this double error, methinks there is now no occasion of my producing any evidence to justify him. But you are pleased to *call publickly upon me to produce it*, to *deny as strongly as I affirm*, to *aver, that my declaring I have such evidence, is an unquestionable proof of the matchless contempt that I*, Philalethes, *have for truth.* Why this indeed is matchless—Blindness, or assurance, shall I call it? I beg the Reader will turn to *p.* 70 of my Defence. There he will see I have already produced my evidence, and have named the passages, where these very objections of yours appear to have been foreseen, and to be clearly and fully removed. I have there named the passages, I say, though you have suppressed them, and every Reader, who is qualified to examine those passages, will find what I say to be true; and that the pretence of your first error is fully removed by *Lemma* 7. and that of the second by *Lemma* 1.

XL. I have nothing to say to the principles of the *Marquis de l'Hospital*, I defend nothing but his reasoning. You say, *he rejects infinitesimals in virtue of a Postulatum, and this you venture to call rejecting them without ceremony.* I know of no greater ceremony used by *Euclid*, than to reject a thing in necessary and unavoidable consequence of a *Postulatum.* You tell me, *he inferreth a conclusion accurately true, contrary to the rules of Logick, from inaccurate and false premises.* This I deny: for if his premises be allowed, his conclusion will follows by the strictest rules of *Logick*, though those premises are false. Allow him his first *postulatum*, and then $2x\,dx$ will be equal to $2x\,dx + dx\,dx$. Allow him his second *postulatum*, and then RN in your figure (Analyst, p. 32.) will be equal to RL. And his conclusion must come out right. I seems therefore, that the *Marquis* is acquitted of this double error, as well as Sir *Isaac Newton*, and that it is you alone, who *have acted blindfold, as not knowing the true reason of the conslusion's coming out accurately right, which I shew* not *to have been the effect of a double error*, but of his two *Postulata.*

XLI. To all this declamation I shall need to give no other reply, than one you furnish me with, *p.* 27 of this very answer. *It must be owned*, say you, *that after you have misled and amused your less qualified reader, (as you call him) you return to the REAL POINT in controversy, and set your self to justify Sir* Isaac*'s method in getting rid of the above-mentioned quantity.* I think I have already told you, that I had talked so much of the smallness of the error, only for the information of some great Churchmen, to make them sensible of the consequence of your discovery, in order to induce them the more readily to join in the hymn to your honour.

You say to me, *You affirm, (and indeed what can you not affirm?) that the difference between the true subtangent and that found without any compensation is absolutely nothing at all.* These are not my words. You will perhaps affirm, that they express my sense. I deny it. I neither speak thus, nor mean thus, nor have any meaning like this, but the direct contrary, with regard to the subtangent determined without any compensation, upon the principles of the *Marquis de l'Hospital*, who alone is here referred to.

XLII. Empty, childish declamation.

XLIII. The same, or something worse. I apprehend it was, as you say, *discreetly* done, to fix upon two or three of the main points, and to *overlook* the rest of the *difficulties proposed in the Analyst*, particularly the Queries, threescore and seven in number. You tell me, I am *not* *afraid nor ashamed to represent the Analysts as very clear and uniform in their conceptions of these matters.* Where have I so represented them? I know there is a great diversity in opinions among Analysts: Some follow *Monsieur Leibnitz*, some the *Marquis de l'Hospital*, some, other writers, and some, whom I take to be the better judges, follow Sir *Isaac Newton*, and these are *uniform* so far as they follow their master, and *clear* so far as they understand him.

XLIV. If you have met with all these different opinions, in conversation with Analysts, in ten months time, and some Analysts, perhaps 5 or 6, of every one of those opinions, one would think the Country, where you have resided for those ten months, must be better stocked with Mathematicians than all the rest of *Europe*. I hope they are not all *Infidels*. If they are, it is a mercy they are not very able *Infidels*, at least so far as one can judge of them by their mathematical opinions. Otherwise, I should apprehend Religion to be in great danger there, unless that Country be well stocked with men able *to deal with them at their own weapons, and to shew, they are by no means those masters of reason, which they would fain pass for.*

XLV, XLVI, XLVII, XLVIII. You come now to the point of Metaphysicks in dispute between us, about which you write, contrary to your usual manner, so very inaccurately and unintelligibly, as plainly convinces me *you have some other end to serve than truth.* And upon revisiting what I had before addressed to you upon this subject, I think I neither can, nor need, set that matter in a clearer light, than I have already done. I perceive likewise, my rebukes have had a good effect upon you. You make excuses. *It was not*, you say, *with intent to carp or cavil at a single passage.* You talk no more of *manifest, staring contradictions.* No, you express your self with some modesty, *all this looks very like a contradiction*; with some other signs of grace, that give me hopes, as you are now made to see your errors, you may in time be brought to acknowledge them.

It must be owned in your favour, you have already recanted the principal of them, and that which led to all the rest, as amply and fully, as from you could possibly be expected. You had expressed your self in Art. CXXV of your new Theory of Vision, in the following words. "After reiterated endeavours to apprehend the GENERAL IDEA of a Triangle, I have found it altogether incomprehensible." But now

Ut primum discussæ umbræ & lux reddita menti,

your eyes being opened, (pardon me this vanity) by the arguments I have done my self the honour of laying before you, you are pleased to say, "This implies that I hold, there are no GENERAL IDEAS. But I hold the direct contrary, that there are INDEED GENERAL IDEAS, but not formed by abstraction in the manner set forth by Mr. *Locke*."

I am so much pleased with this piece of ingenuity and candid proceeding, that for the sake of it I willingly excuse all that follows, however inconsistent with this recantation: Particularly your making no difference between a round square, and a space comprehended by three right lines. For the same reason I willingly pass by your supposing, that the words of my definition have no ideas, or conceptions of the mind, joined with them, and consequently that the definition has no meaning. For to make the definition have a meaning, some particular idea, simple or compound, must be joined to every word used in it; and a compound idea, made up of all those particular ideas, must be joined to, and always going along with the whole definition: And these two, the compound idea and the definition are inseparable, if the definition be understood. Methinks therefore, instead of separating these, it were better to make a distinction between this compound idea answering to the definition of a triangle, and the image, or sensible representation of a triangle; two things which I have observed you often to confound, both here and in your other writings. The compound idea is general, but the image, if exactly attended to and adequately perceived by the mind, must always be particular.

XLIX. You here propose some points for the Reader *to reflect* upon *and examine by my light*, when you well know I never endeavoured to give him any light about them. In this second letter indeed I have, at your request, endeavoured to explain some part of them. But there are some others, which I so far from being able to explain, that I never heard of them before, and cannot possible imagine what you mean by them. Possibly they may be some *arcana* of the Boeotian Analysis, explicable only by the *Philosophia prima*.

L. As these Queries are not proposed to me, I leave it to the consideration of my learned Friends of this University, whether they deserve or need any answer.

I am, Sir,

Your most Obedient Servant,

PHILALETHES CANTABRIGIENSIS.

FINIS.