SOME REMARKS ON MR. WALTON'S APPENDIX CONCERNING MOTION AND VELOCITY

By

John Hanna

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Edited by David R. Wilkins

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NOTE ON THE TEXT

The text of the *Some Remarks on Mr. Walton's Appendix* has been taken from the pamphlet published in Dublin in 1736.

In the paragraph beginning 'As for the Moments of a Rectangle in Page 50. . . ' a formula originally printed as '$Ab + B + a.ab$' has been corrected to read '$Ab + Ba + ab$'.

Some obvious typographical errors have also been corrected.

The following spellings, differing from modern British English, are employed in the original 1736 edition: dependance, interveening, encreasing, inertnes.

David R. Wilkins
Dublin, June 2002

SOME

REMARKS

ON

Mr. *WALTON*'s Appendix,

Which he wrote in Reply to the AUTHOR of

The Minute Philosopher;

CONCERNING

MOTION and *VELOCITY.*

By JOHN HANNA, M.A.

Semper ego auditor tantum nunquam ne reponam?
JUV.

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SOME
REMARKS
On Mr. *WALTON*'s Appendix, Concerning *MOTION* and *VELOCITY.*

It may be reckoned improper or imprudent in me, to meddle in a Controversy which hath occasioned so much Heat, but as I have no *Dependance* on, nor *Expectation* from the Parties engag'd, I may venture to give my Thoughts; and the rather, that I am persuaded *Free-Thinking* in mixt Mathematicks (such as Sir *Isaac Newton*'s Principles contain) not only allowable, but necessary, in order to know Truth, and *Jurare in Verba Magistri*, or to receive Things on the Testimony and Opinion of any Man, tho' never so learn'd argues a mean submissive Capacity, which is worse in the Common-Wealth of Learning, than passive Obedience in a Kingly Government. Moreover, he who is not willing to have his Works try'd and examined, is either conscious of some Error, or weak and impatient of Contradiction, or proud and afraid of missing the designed Applause; but on the other Hand, he who searches impartially for Truth, will be thankful to him that discovers his Error, tho' it should be publickly by Writing, especially seeing he thereby honours him, thereby declaring that it's worth one's While to answer him. And truly I should not have undertaken such a Task, had I not considered that By-standers may sometimes see more than Gamesters; and 2*dly*, That the Clashing of Arguments is like that of two Flints, that give thereby a Spark of Light. And 3*rdly*, I thereby satisfy some Unlearned, that want to know my Thoughts, and tho' I would not have them judge thereby; yet it may be a Grain in the Ballance to determine their Judgment that is grounded upon Hear-say.

But then I took Care to write but little. For, 1*st*, I avoided, as much as possible, repeating what was said before. And 2*dly*, I kept to the *proton Pseudos* or false Foundation on which a Superstructure was built, which must fall on the Removal of the former. And, 3*dly*, Few have Patience to read much, but so say no more by Way of Preface. I come to observe, that the Author is about giving the Meaning of *Velocity*, which he might soon have done in other Words, but I suppose he meant the Nature of it, by giving a Definition, or at least, a Description of it, which I think can never be done without taking in *Time* and *Space*, the necessary Ingredients that constitute *Velocity*, for little Space and much Time, or little Time and much Space, make Motion either slow or swift, and for the Quantity of Motion, we must take in the Body moving, in which every Part hath a Motion; and therefore, the more Parts in any determined Magnitude, there is the more Motion, tho' the Velocity be the same, and the greater the Velocity, the greater the Motion, tho' the Matter be the same; for $\mathrm{M} = \mathrm{Qv}$. Hence we have *a Measure of Velocity*, and know how great it is, by dividing the Motion by the Quantity of Matter, but then that doth not give us the Nature of Velocity, more than a Yard of Wood gives the Nature of the Cloth it measures, tho' they are both the same as to

Quantity, yet in other Respects, very different. Supposing then $\frac{\mathrm{M}}{\mathrm{Q}}$ to be only a Measure of Velocity, it's plain it includes both Time and Space, for both these are included in Motion, represented by the Letter M; and if Force produce Motion, and F be put for M, then $\frac{\mathrm{F}}{\mathrm{Q}}$ or $\frac{\mathrm{F}}{\mathrm{W}}$, *i. e.* Force divided by Quantity of Matter or Weight will likewise include Time and Space, the Idea of F having with it that of M.

But I see not why he should make F and M the Force and Motion always proportional to one another, for the Motion that comes from the same Force may vary, and be either lessen'd or increas'd. 1*st*, It may be lessen'd, for the Body mov'd communicates its Motion to the Particles of the Medium, which by their direct accidental Collision against one another, will lessen or destroy the Motion in some Measure, and if it be said they are Proportional at first, it's to be considered, that there is Force where there is no Motion consequent, the *vis resistentiæ* being greater, and also Motion when there is no Force as the Cause of it, as when a Body moves by the Will of a free separate Spirit, which is sometimes the Original of Motion. Seeing then they are so different, F cannot be substituted for M; moreover, Motion from the same Force may be increased to any Degree, as in Elastick-Balls in Geom: Progression placed in a straight Line, the Motion communicated to the First, is augmented in the Last; therefore, Force, and Motion its Effect, are not in this Case proportional no more than than the Flame of a Candle bears any certain or determin'd Proportion to the Flame of a House set on Fire by it.

Again, $\frac{\mathrm{F}}{\mathrm{W}}$ is called a Measure of Velocity where there is substituted W for Q, or Weight for Quantity of Matter; but there is also to be considered the Density, Superfice of the Body and the Medium, all which may retard or hasten the Motion of heavy Bodies, and so $\frac{\mathrm{F}}{\mathrm{W}}$ will not always give the Velocity of a Body. Yea, in many Cases it will not; for the Pressure of a Stone on the Ground (which is one of his three Forces) divided by its Weight would give its Velocity when it does not move, which is absurd.

As to his other two Forces, which generate Motion they are much to be doubted; as for the First, *viz.* A Blow that only transfers the Motion from one Subject to another, and if Water flow from one Part of a River to another, that doth not generate a new River, and if a Blow be given by an animate Body, there is then only a transferring the Motion from the Fibres and Muscles to the Body moved, the Spirit in the animate Body, changing the Determination of the Animal Spirits in the Body, which causes no new Motion. As for the other Cause of Motion, *Weight*, by which, I suppose is meant, an attractive Force in Matter; on it may be made these few Queries.

1*st*, How can Matter that is purely Passive, move other Matter? And, whether may it not as well move itself as other Matter?

2*dly*, Whether or not is it to be counted a Miracle, for a Body at Rest to cause a Body in Motion, seeing Rest and Motion are as opposite to one another, as Water and Fire?

3*dly*, How can a Body act at a Distance, when there is no interveening Medium supposed as between the Earth and the Moon that are said to attract one another?

4*thly*, Whether or not should a Philosopher explain the Laws of Nature mechanically? and whether did ever any One attempt to explain Attraction mechanically, except Sir *Isaac*

Newton Himself, by a Medium infinitely Rare, and infinitely Dense, which Mr. *Whiston* in a Pamphlet I have read, endeavoured to expose him for.

5*thly*, If it must be resolved immediately to the Will of GOD, why may I not as well say, The Planets are retain'd in their Orbits, and a Stone falls down by the Will of GOD, as well as say, they are done by Attraction, which is said to be by the Will of GOD; especially, when that Quality agrees not to all Matter; for tho' Glass is said to attract, yet other Matter, as Air, hath a repelling Force, directly contrary to it, and the Sun is said to have both, so that there is no general Law but the Will of GOD, which is a good sure Rule the Vulgar take to explain all the Phænomena of Nature by; but Philosophers pretend to know something more, and indeed it were better for one to own his Ignorance, than have Recourse to an occult Quality, like to that of Rays of Light, taking a Fit sometimes of reflecting, and sometimes refracting. Thus far we know that GOD is the first Cause of all Motion, which is transferred thro' the Universe from one Subject to another mechanically, the Will of Man only altering the Determination of the Animal Spirits in the living Body; but where there appears to us no Relation between the Cause and the Effect, as when Water came out of *a dry Rock*, or when Spirits move Bodies by their Will, or heavy Bodies descend, these cannot be from any Quality in the suppos'd Cause, but from the Appointment of the Author of the Universe, who is not ty'd to any Law, and can as easily make the Earth revolve about the Sun, or a Stone descend, without an Attraction, as with it. But I forget that herein I am opposing a greater Man than Mr. *Walton*; and indeed if Attraction be taken as an Hypothesis, it will solve several Phænomena in Nature: But I must ask once more, how the Velocity of the Hand that gives the Blow can be measured by $\frac{\mathrm{F}}{\mathrm{W}}$ or $\frac{\mathrm{F}}{\mathrm{DB}}$. I can measure it by $\frac{\mathrm{S}}{\mathrm{T}}$ or Space divided by Time, but I cannot by applying the Force to the Weight of the Hand, and far less to the Density, multiplied by the Bulk, for the Velocity depends not on the Weight, or Density of the Hand, but on the Will of the Mover, by the Appointment of Him by whom we live and move.

Again in his Page 37, I read thus; *Hence it appears that a Body in Motion will have a Velocity inherent in it self, during the whole Time of it's Motion, and consequently, there must be a Velocity, wherever the Body is, exclusive of Time and Space.* By the Words *Wherever the Body is*, he would prove Velocity in a Point. But, 1*st*, There is no indivisible Point of Matter or Space, and therefore no Velocity in it; and as for a divisible Point, it's a *Contradictio in terminis*; and as for an assignable Part of Space, however small, there is a Motion and Velocity from the Beginning to the End of it, and as for the Words *exclusive of Time and Space*, which are brought into the Consequent, they were not in the Premisses; Yea, Time and Motion (and therefore Space) are expressly mentioned in the Premisses, and therefore not to be excluded in the Consequent.

In Page 38, his Argument to prove Velocity in a Point, supposes the Thing in Question, his words are, *If the generated Velocity be not the same in any two different Points of the described Space, a Velocity must exist in every Point*, but he ought first to prove that it's in a Point. And then, 2*dly*, That it's not the same in every Point. One might thus prove the Motion of the Earth by saying, it moves not alike swift in every Point of the Ecliptick, therefore it moves in the Ecliptick.

In the same Page it's said, *The Velocity of a Body is the Degree of Tendency forward,* but for Answer, Bodies that are pressed, have, in some Cases, a Tendency to move, when they do not move; and thus they will have Velocity, when they have no Motion, which is inconceivable; for if we suppose two Bodies that are Elastick lying, the one on the other, they then have both a Tendency to move, tho' both at rest, Pressure or Tendency forward, being no Motion, but only *conatus ad motum.*

In Page 38 it's said, *That Velocity is a Part of Motion.* If so then that Part Velocity must have a Velocity, for every Part of Motion hath some Degree of Velocity, and that Velocity again another Velocity *&c.* I should rather have called it a Quality, Adjunct, or Property of Motion; and it's known to Logicians, that Qualities are no Part of the Subject, for they may be increased or diminished, while the Subject is the same in its Parts. Thus in the same Quantity of Motion, there may be more or less Velocity, by lessening the Quantity of Matter, and encreasing the Velocity, or encreasing the Matter, and lessening the Velocity.

In the same Page 39, it's said, *Sir* Isaac*'s Velocity, is that of a Body existing in some one certain Place*; if so, he must understand it of the Velocity of a Body at rest, for to exist in one certain Place, and to be at rest, are the same, which never could be the Meaning of a Man of his Learning, for this would expose him as one who excluded all *Space* from the Idea of *Velocity*, when it's known he makes $\mathrm{V} = \dfrac{\mathrm{S}}{\mathrm{T}}$.

In Page 40, it's said, *Velocity is retained by the Inertnes of Matter, and becomes thereby the Cause of a Body's going forward.* But I think it's rather destroyed, or transferred to another Body, by the Inertness of Matter in the Medium, to which the Motion and Velocity is communicated, as the Motion and Velocity of a Ship is retarded by the Inertness of the Matter that it passes through, but forwarded by its Motion the same Way; moreover, *if Velocity be retained by the Inertnes of Matter and Velocity, be also the Cause of a Body's going forward*, as is alledg'd, then the Inertnes of Matter causes the Body go forward, or rest is the Cause of Motion, which is absurd. It's likewise said, That the continual Translation of a Body into a new Place, is an Effect of Velocity; it's indeed the Effect of Motion, or rather Motion it self; but it cannot be the natural and necessary Effect of any particular Degree of Velocity, for a Translation may be without that Degree of Velocity, and therefore I would have said, The Translation of a Body in a certain Time, is the Effect of a certain Degree of Velocity, or rather its Concomitant; I speak of a particular Degree of Velocity, for *Velocity in general, is but an abstract Idea of the Mind.* As for Velocity existing in a Body before it moves, it's inconceivable; I may as well say, it moves before it moves, and tho', as is said, Causes may be conceived to exist prior to their Effects, yet not all Causes, and tho' the Sun may be conceived before its Heat, yet we are not to conceive it existing before its Heat, for that were false; it's indeed prior *Natura*, but not *Tempore.* Moreover, if a particular Degree of Velocity be prior to Motion, it being but an Accident, it may be said the Accident exists before its Subject, contrary to a logical Axiom.

In Page 41, he speaks of *The Ratio of the Space to be described*, but I think of that which hath no Existence, but is only to be, there *can be no Ratio*, except in *Mente*, for as it's said in Page 49, *to consider the Ratio or Proportion of Things, implies that such Things have Magnitude*, that is, that they exist.

In Page 42, *The Space to be described will begin to exist, no Part of it being yet described with the Ratio of the Velocity*: Now seeing Space described, must thereby have an Existence, the Space to be described, can yet have no Existence, for then described, and to be described, were the same, or what is to be, would really be.

In Page 43. *The general Measure of Velocity, consists in the final Limits, or Extremities of Quantities, actually generated by Motion, tho' as yet no Parts of their isochronal Increments are described.* But, 1*st*, If no Parts of their Increments are described by Motion, why is it said or supposed, that these Quantities are actually generated by Motion? 2*dly*, Why is it said afterwards, there is a Continuance of that Motion, if there was no Motion before, or no Part of the Increment described. 3*dly*, If Velocity be in the Limits of Quantities, Motion must be there too; but I can conceive no Motion, without *Terminus a quo and Terminus ad quem.* Whereas, a Limit, is but one *Terminus*, and there can be no Motion in one *Terminus.*

If *Tendency of a Body to move, be the same with Velocity*, as in Page 47. *then a Stone on the Ground, having a Tendency to move to the Center hath some Velocity, and in a certain Time will reach the Center, tho' it do not move.* And in the same Paragraph, the *Translation of a Body and describing of Space, are the same Thing.* But, if a Fish in the Water, in the Bottom of a Ship, swim precisely as much North as the Ship fails to the South, there is no Part of absolute Space described, and yet the Fish is translated to another Part of the Ship, tho' not of the Universe.

In Page 49. *Its allowable there cannot be a Ratio of a Thing that hath no Magnitude*, and yet it's said afterward, *there may be the first Ratio of a Thing that hath no Magnitude*, which is contradictory; but to explain it, it's said, Page 55, *the first Ratio of the Increments, is not the Ratio of those Increments, or of any Part of them*; as if the Beginning of a Thing were no Part of it; and as for the Illustration of this in Page 55, where he supposeth the Velocity in C and E, to be as 3 to 2, if C and E be indivisible Points, it's already proved there can be no *Motion* or *Velocity* in them, but if divisible, they are the Increments themselves, or Fluxions of the two Lines.

As for the Moments of a Rectangle in Page 50, it's plain, if A and B denote the two Sides, and a and b their respective Increments, or Velocities, to which they are proportional, the increase of the Rectangle will be $Ab + Ba + ab$ if it be said ab will be nothing, then $Ab + Ba$ will be also nothing; for a real Quantity multiplied by nothing, makes Nothing. But if a be Something, tho' never so Little, ab will be more, and that more lost, if the Fluxion be only $Ab + Ba$. If a denote only the First Ratio of the Increment, then the Increment is begun, and there is a Motion from the last Point of the Side, to the first Point of the Increment, and the Tendency to move, is turned to Motion, so that there is something more than Velocity or Tendency to move, which is said to be prior to the Motion, and there seems to be no Difference between the Ratio of a Magnitude infinitely diminish'd, and the last Ratio of a Mgnitude destroyed, for then we may reckon they both vanish and return to their primitive State, so that either the Increment exists, or not; if not, it can have no Ratio if it do, there is a Transition from the final Limit of the Side, to the Increment before the two Points become one, which Transition being a Motion, must be denoted by a, and so ab will constitute an Area, and if there be no Area, there is no Fluxion, and the Rectangle AB as it was.

I must observe also what is said, Page 54, that *he hath not asserted that Increments subsist when they have no Magnitude, but that the first Ratio of the Increments subsist, when the Increments have no Magnitude.* Where he supposes the first Ratio of the Increments, subsist before the Increments themselves. And, 2*dly*, That Increments may have no Magnitude, and if so, they can never afterwards become a Part of Magnitude: But all Increments having Magnitude, these joined, increase the Magnitude.

And further, if the Increments have no Magnitude, their first Ratio is not the first Ratio of any generated Magnitude, but of that which is no Magnitude, or no Part of it, and so nothing; for the Beginning and Ending, being Instantaneous, either the Increment is begun or not; if begun, there is a Ratio of something Real, and tho' never so small, it's an Increment, denoted by a, if not begun, there is no first Ratio of what is not; and therefore I am of Opinion with *Leibnitz*, That there is no Harm in allowing the Fluxion to be some REAL THING, generated, which being indefinitely Small, may be neglected, as we do in the Decimal, for the third Part of any Thing.

In Page 57, it's said, *Two Points which are called* Mathematical, *may be in one Point, which he calls* D, *but* mathematical Points *exist* only in the Mind, *they being abstract Ideas*; they indeed may become One, *that is*, Things which I conceived before as Two, I may conceive them as One: But to say *That Two such Points may be in One*, supposes that they are Two, and *One* at the same Time. And, as for their moving *two*, or *from* D, it's plain such Points can have no such Motion: And if we understand *physical Points*, then stirring *from*, or *to D*, will make the Fluxion something REAL.

In Page 55, it's said, *It's not the Ratio of Magnitudes, but the Ratio with which they begin*; *but if they begin to have Magnitudes*, they have Magnitudes. If a Stone begins to *fall*, it *does fall*; for the Beginning of every Thing is a *Part* of the Thing *begun*; and, as is acknowledged, Page 62, the Moment of Sir ISAAC, is a finite Quantity; being a Product contain'd under the *moving Quantity* and *its Velocity*, which is a giving up the CAUSE. For seeing the *VELOCITY* and the *moving Quantity* may be expounded by Lines, one Line multiplied by another, must produce something REAL, of *a determined Magnitude*; and yet Sir ISAAC says, *Cave Intelligas Quantitates Magnitudine Determinatas.* So I conclude with a Saying I have read, *Nihil est ex omni parte Beatum.*