QUARK MASS DEPENDENCE of BARYON PROPERTIES

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- **Chiral limit of QCD and Effective Field Theory**
- **Quark mass dependence of baryons properties**
- **Quark mass dependence of nuclear forces**
- **Summary & Outlook**

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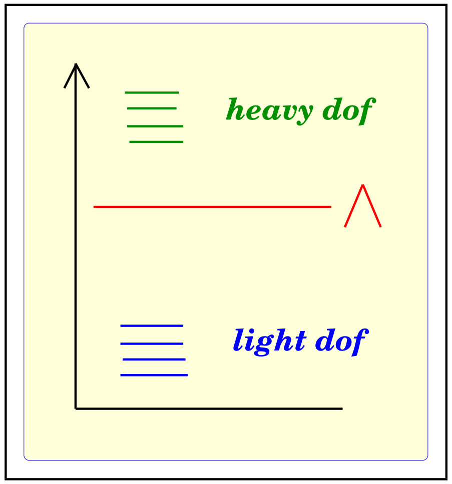
BRIEF REMINDER of EFFECTIVE FIELD THEORY



- Separation of scales: low and high energy dynamics
  - $\star$ low-energy dynamics in terms of relevant dof's (e.g. pions in QCD)
  - $\star$ high-energy dynamics not resolved $\longrightarrow$ contact interactions
- Small parameter(s) & power counting

Weinberg 1979

  - $\star$ Standard QFT: trees + loops $\rightarrow$ renormalization
  - $\star$ Expansion in powers of energy/momenta Q over the large scale Λ

$$\boxed{\mathcal{M} = \sum_\nu \left(\frac{Q}{\Lambda}\right)^\nu f(Q/\mu, g_i)}$$

μ – regularization scale
g_i – low–energy constants

- f is a function of $\mathcal{O}(1)$ – "naturalness"
- ν bounded from below

$\Rightarrow$ systematic and controlled expansion

NB: bound states require non-perturbative resummation

CHIRAL DYNAMICS

- QCD
  - $\boldsymbol{m_u, m_d, m_s}$ light
  - $\boldsymbol{m_c, m_b, m_t}$ heavy

  $\}$ compared to the typical hadronic scale $\Lambda_\chi \sim 1$ GeV

- A THEORETICIANS PARADISE

$$\boxed{\begin{array}{c} m_u = m_d = m_s = 0 \\ m_c = m_b = m_t = \infty \end{array}}$$

$\star$ defines the SU(3) chiral limit of QCD, left/right-handed worlds: $q = q_L + q_R$

$\star$ $\mathcal{L}^0_{\rm QCD}$ contains one parameter, $g \leftrightarrow \Lambda_{\rm QCD}$ – dimensional transmutation

$\star$ $\mathcal{L}^0_{\rm QCD}$ has a high degree of symmetry because gluons are flavor-blind

$$\boxed{\boldsymbol{\mathcal{L}^0_{\rm QCD}[q_L, q_R; G] = \mathcal{L}^0_{\rm QCD}[V_L q_L, V_R q_R; G]}} \quad \boldsymbol{V_{L,R} \in SU(3)_{R,L}}$$

$\rightarrow$ 16 conserved currents and charges: Q^a_V, Q^a_A, $a = 1, \ldots 8$

CHIRAL DYNAMICS cont'd

- The vacuum is invariant under the vector charges — Vafa & Witten
- Nature chooses the Nambu-Goldstone alternative: $Q_A^a|0\rangle \neq 0$
  - The vacuum is not chirally symmetric $\to$ spontaneous symmetry breaking
  - The spectrum contains Goldstone bosons: 8 states w/ $E = \vec{p} = 0$
  - The 8 lightest hadrons are pseudoscalar mesons: $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$
- Real world: π, K, η are not massless

$$\boxed{\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} \underbrace{-\bar{q}_R \mathcal{M} q_L - \bar{q}_L \mathcal{M}^\dagger q_R}_{\mathcal{M}={\rm diag}(m_u,m_d,m_s)}}$$

  - Explicit symmetry breaking, can be treated in perturbation th'y

$\to$ Consequences of the explicit and spontaneous symmetry breaking can be analyzed utilizing an Effective Field Theory - *Chiral Perturbation Theory*

Weinberg, Gasser, Leutwyler

CHIRAL DYNAMICS cont'd

$$\boxed{\mathcal{L}_{\rm QCD}[q,\bar{q};G] \to \mathcal{L}_{\rm EFF}[U,\partial U,\ldots,\mathcal{M},N]}$$

- $\star$ U parameterizes the Goldstone bosons
- $\star$ $\partial U \simeq$ interaction vanishes at $E = \vec{p} = 0$ (Goldstone theorem)
- $\star$ $\mathcal{M}$ parameterizes the explicit symmetry breaking
- $\star$ N denotes the interaction with matter fields
- $\star$ exact mapping via chiral Ward identities   Leutwyler, Weinberg

$\to$ Expansion in powers of small momenta/energies and quark masses:

$$\boxed{E_\pi/\Lambda_\chi\ ,\quad M_\pi/\Lambda_\chi\ ,\quad \Lambda_\chi \simeq 1\ \text{GeV}}$$

standard QFT with tree and loop graphs (order-by-order renormalization)

NB: EFT can be extended to few-nucleon systems   Weinberg, van Kolck, Epelbaum, . . .

APPROACHING THE CHIRAL LIMIT

- S-matrix elements exist in the chiral limit for arbitrary momenta
- The approach to the chiral limit is non-analytic in $\mathcal{M} \to \mathcal{M} \ln \mathcal{M}$

  $\Rightarrow$ *chiral logs* — Dashen, Weinstein, Langacker, Pagels, . . .
- Decoupling theorem — Gasser & Zepeda, Nucl. Phys. **B174** (1980) 445

Leading chiral non-analytic terms stem from pion (Goldstone boson) one-loop graphs coupled to pions (Goldstone bosons) or nucleons (ground state baryons)

⋆ resonances like $\rho, \ldots, \Delta, \ldots$ decouple

⋆ severe constraints for EFTs with resonance fields

⋆ chiral limit of QCD and the large N_c limit do not commute

BARYON OBSERVABLE TO ONE LOOP

- A complete one-loop (fourth order) SU(2) calculation gives

$$\boxed{\mathcal{O}(M_\pi, F_\pi; m_0, g_A; c_i, d_i, e_i)}$$

with:

$M_\pi^2 = (m_u + m_d)\, B_0 + \mathcal{O}(m_q^2)$ Gasser, Leutwyler

$F_\pi = F_0\ [1 + \mathcal{O}(m_q)]$ Gasser, Leutwyler

m_0 = nucleon mass in the chiral SU(2) limit $\rightarrow$ more later

$g_A = g_A^0\ [1 + \mathcal{O}(m_q)]$ Fettes, Steininger, M, Kambor, Mojžiš

c_i, d_i, e_i = LECs of the dimension two, three, four chiral $\mathcal{L}_{\rm eff}$

Bernard, Kaiser, M, Fettes, Steininger, Mojžiš

[NB: complete effective Lagrangian given in Fettes et al., Ann. Phys. 283 (2000) 273 [hep-ph/0001308]]

- Two classes of low-energy constants

$\star$ *dynamical* $\sim \partial_\mu^2, \partial_\mu^4, \ldots \longrightarrow$ phenomenology

$\star$ *symmetry breakers* $\sim m_q, m_q^2, m_q\partial_\mu^2, \ldots \longrightarrow$ lattice

ANALYSIS OF BARYON PROPERTIES

- Quark mass expansion of the N mass in SU(2) and SU(3)
- Quark mass expansion of the baryon octet masses
- Quark mass expansion of the Δ mass
- Quark mass expansion of other nucleon properties

Disclaimer:

– I will discuss continuum extrapolations, no finite lattice spacing or finite volume effects
– I will not discuss (partial) quenching, data are taken as full QCD
– I will be mostly interested in the range of applicability (theoretical uncertainty) rather than precision fits
– I eschew models here

QUARK MASS EXPANSION of m_N in SU(2)

Bernard, Hemmert, M., Nucl. Phys. A 732 (2004) 149 [hep-ph/0307115]

- Expansion in the light quark mass $\hat{m} = m_u = m_d$ with m_s fixed

  $\rightarrow$ mapped onto an expansion in the pion mass

  NB: consider strong isospin violation $(m_u \neq m_d)$ later

- Fourth order calculation:

$$\boxed{m_N = m_0 - 4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{32\pi F_\pi^2} + k_1 M_\pi^4 \ln\frac{M_\pi}{m} + k_2 M_\pi^4 + \mathcal{O}(M_\pi^5)}$$

- Low-energy constants:

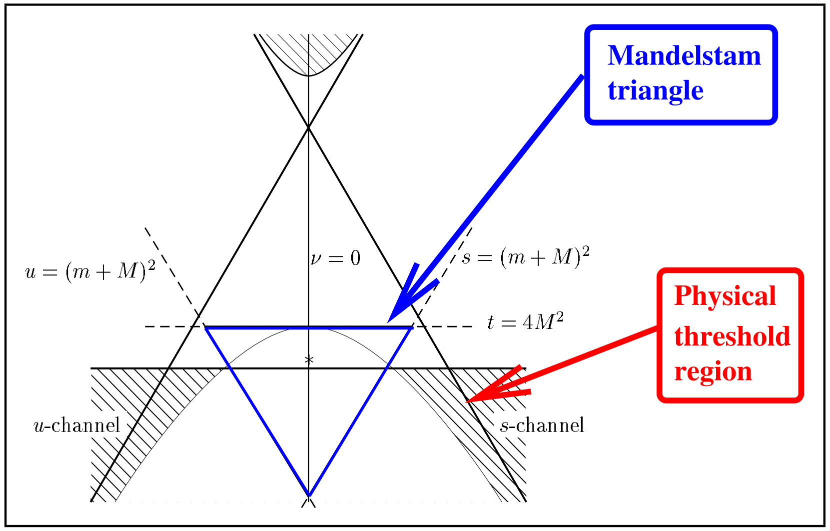
  m_0 – nucleon mass in the chiral SU(2) limit (not equal to the SU(3) c.l.)

$$k_1 = -\frac{3}{32\pi^2 F_\pi^2}\left(-8c_1 + c_2 + 4c_3 + \frac{g_A^2}{m_N}\right)$$

$$k_2 = -4e_1 + \frac{3}{128\pi^2 F_\pi^2}\left(c_2 - \frac{2g_A^2}{m_N}\right)$$

$\Rightarrow$ the LECs (c_1, c_2, c_3, e_1) are strongly constrained from pion-nucleon scattering!

DETERMINATION OF THE LECs



- πN scattering inside the Mandelstam triangle:

$\longrightarrow$ best convergence, relies on dispersive analysis

Büttiker, M., Nucl. Phys. A668 (2000) 97 [hep-ph/9908247]

- πN scattering in the threshold region:

$\longrightarrow$ large data basis, not all consistent

Bernard, Kaiser, M., Nucl. Phys. A615 (1997) 483 [hep-ph/9611253]

Fettes, M., Steininger, Nucl. Phys. A640 (1998) 119 [hep-ph/9803266]

- peripheral phases in NN scattering:

$\longrightarrow$ large data basis, must subtract one-pion exchange

Rentmeester, Timmermans, de Swart, Phys.Rev.C67 (2003) 044001 [nucl-th/0302080]; Epelbaum, Glöckle, M., Eur. Phys. J. A19 (2004) 125 [nucl-th/0304037]

$\Rightarrow$ Resulting values in GeV^{-1}:

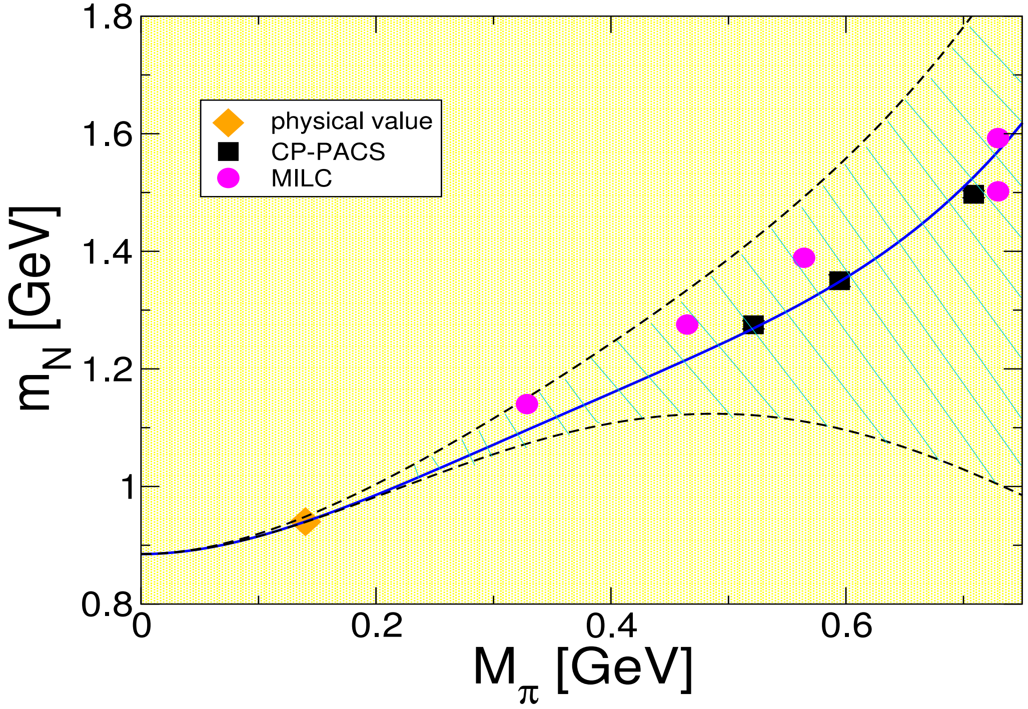
$$\boxed{c_1 = -0.9^{+0.5}_{-0.2}\,,\quad c_2 = 3.3 \pm 0.2\,,\quad c_3 = -4.7^{+1.2}_{-1.0}\,,\quad -2c_1 + \tfrac{1}{4}c_2 + c_3 = -2.1^{+1.3}_{-1.1}}$$

$\Rightarrow$ e_1 only badly determined from $\pi N \to \pi N$, negative, of natural size

Fettes, M., Nucl. Phys. A676 (2000) 311 [hep-ph/0002182]

PION MASS DEPENDENCE OF THE NUCLEON MASS

- LECs close to empirical values: $\boldsymbol{c_1 = -0.9, c_2 = 3.2, c_3 = -3.5, e_1 = -1}$
- $m_N^{\rm phys}$ at physical quark mass $\to m_0 \simeq 0.88$ GeV - chiral SU(2) value
- moderate/large theoretical uncertainty for M_π above 400/500 MeV



Similar work in Hemmert, Procura, Weise, Phys. Rev. D 69 (2004) 034505 [hep-lat/0309020]

EXTENSION TO SU(3)

Frink and M., JHEP 0407 (2004) 028 [hep-lat/0404018]

- More fields and operator structures $\Rightarrow$ more LECs

  nucleon doublet & pion triplet $\to$ baryon octet & Goldstone boson octet

  meson-baryon coupling $\boxed{g_A\,\bar\psi u_\mu\gamma^\mu\gamma^5\psi \to D\,\langle\bar B\gamma^\mu\gamma^5\{u_\mu,B\}\rangle + F\,\langle\bar B\gamma^\mu\gamma^5[u_\mu,B]\rangle}$

  symmetry breaking $\boxed{c_1\,\bar\psi\langle\chi_+\rangle\psi \to b_0\,\langle\bar B B\rangle + b_D\,\langle\bar B\{\chi_+,B\}\rangle + b_F\,\langle\bar B[\chi_+,B]\rangle}$

- Matching conditions

  nucleon mass in the SU(3) limit $\tilde m_0 = m_0\,[1+\mathcal{O}(m_s)]$

  leading axial couplings $g_A = D + F + \mathcal{O}(m_s)$

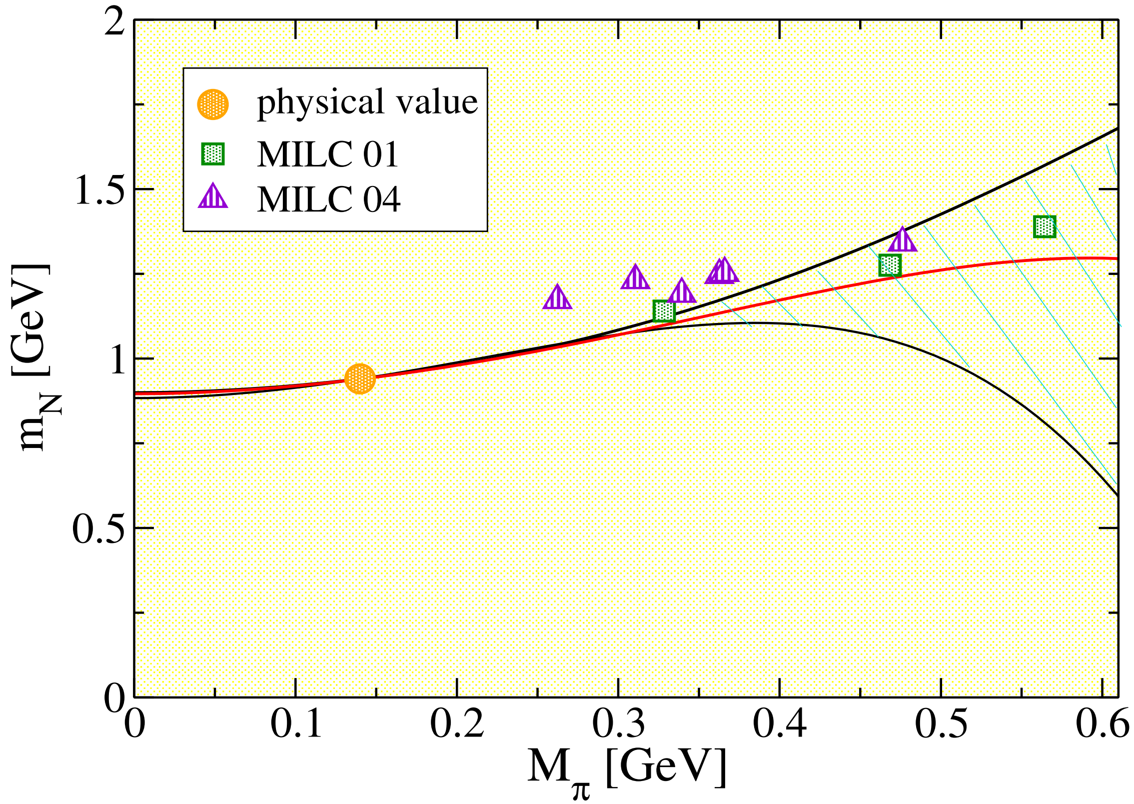
  leading symmetry breakers $c_1 = b_0 + \frac{1}{2}\,(b_D + b_F) + \mathcal{O}(\sqrt{m_s})$

$\boxed{\text{important constraints !}}$

QUARK MASS EXPANSION of m_N in SU(3)

Frink, Scheller & M., Eur. Phys. J. A24 (2005) 395 [hep-lat/0501024]

- dimension two LECs taken from Borasoy and M., Ann. Phys. 254 (1997) 192 [hep-ph/9607432]
- 3 combinations of dim. 4 LECs in m_N, constrain to $m_N^{\rm phys}$ for physical quark masses
- Pion mass dependence based on MILC 01 with constraints



$\star$ Kaon mass dep. and MILC 04

$$\boxed{710 \text{ MeV} \lesssim \tilde{m}_0 \lesssim 1070 \text{ MeV}}$$

$\star$ Pion-nucleon sigma term

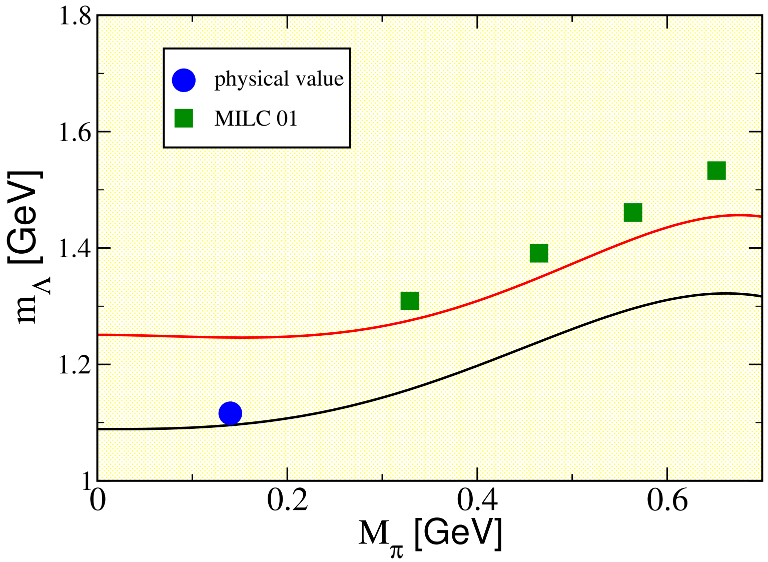
$$\boxed{39.5 \text{ MeV} \lesssim \sigma_{\pi N}(0) \lesssim 46.7 \text{ MeV}}$$

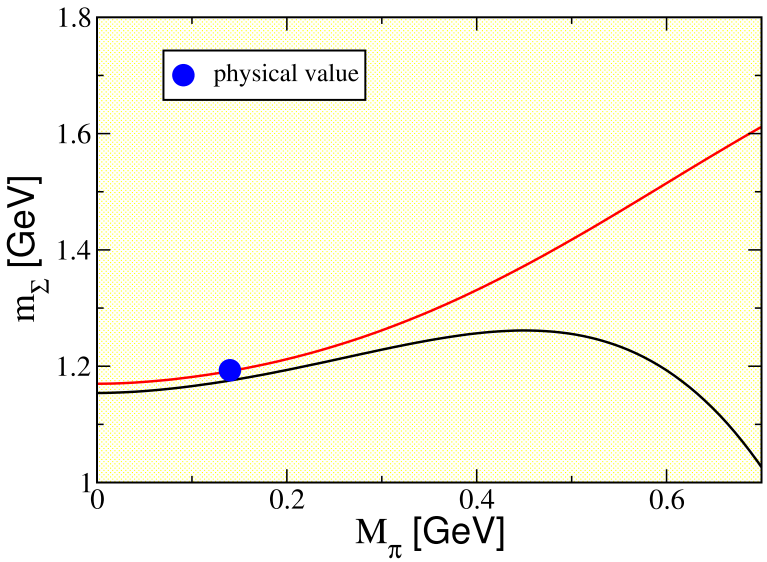
$\star$ Strangeness fraction

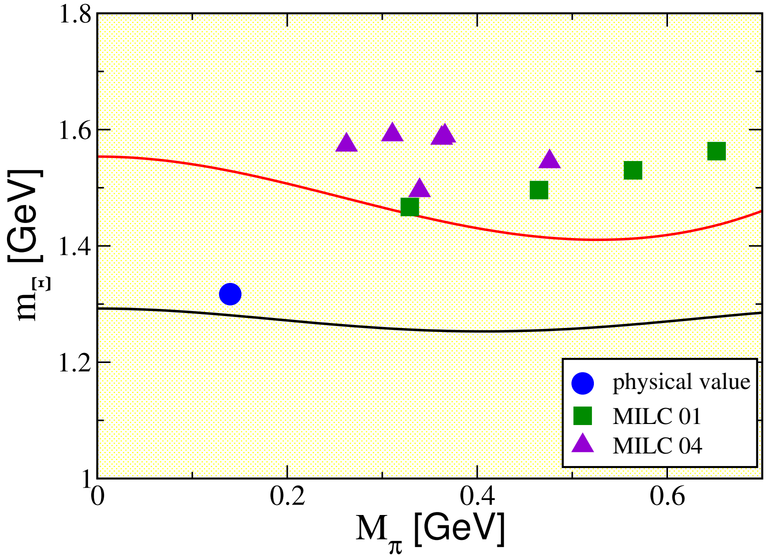
$$\boxed{0.07 \lesssim y \lesssim 0.22}$$

PION MASS EXPANSION in the BARYON OCTET

Frink, Scheller & M., Eur. Phys. J. A24 (2005) 395 [hep-lat/0501024]







LECs d_i from nucleon mass

LECs d_i from octet masses

$\Rightarrow$ what about the cascade?

MILC 01: Bernard et al., Phys. Rev. D64 (2001) 054506 [hep-lat/0104002]

MILC 04: Aubin et al., Phys. Rev. D70 (2004) 094505 [hep-lat/0402030]

STRONG ISOSPIN VIOLATION

Frink and M., JHEP 0407 (2004) 028 [hep-lat/0404018]

- Strong neutron–proton mass difference in SU(2):

$$\boxed{m_n - m_p = 4c_5 B_0 (m_u - m_d) + \mathcal{O}((m_u - m_d)^2)}$$

  - $\star$ c_5 dim. two LEC, operator $\sim \chi_+ - \frac{1}{2}\langle\chi_+\rangle$
  - $\star$ $c_5 = -0.09 \pm 0.01$ GeV^{-1}, use Cottingham sum rule Weinberg; Bernard, Kaiser, M.
  - $\star$ fourth order corrections have been worked out Steininger, Fettes, M., Müller

- Strong neutron–proton mass difference in SU(3):

  - $\star$ $\pi^0 - \eta$ and $\Lambda - \Sigma^0$ mixing $\rightarrow$ $\boxed{\tan 2\epsilon = \frac{\sqrt{3}}{2}\frac{m_u - m_d}{m_s - \hat{m}}}$
  - $\star$ Matching condition: $\boxed{c_5 = b_D + b_F + \mathcal{O}(m_s, m_s \ln m_s)}$

see also recent work in Tiburzi & Walker-Loud, hep-lat/0501018

- *For precise mass splittings, one needs em corrections*

QUARK MASS EXPANSION of the Δ MASS

Bernard, Hemmert, M., Phys. Lett. B565 (2003) 137 [hep-ph/00303198]

Bernard, Hemmert, M., Phys. Lett. B (2005), in press [hep-lat/0503022]

- Covariant EFT w/ explicit $\Delta(1232)$ (spin-3/2) DOFs
- $\varepsilon \equiv (m_\Delta - m_N)/\Lambda_\chi$ as additional small parameter – does not vanish in the c.l.
- more operator structures $\Rightarrow$ more LECs
- must enforce decoupling and work to fourth order in the ε–expansion
- Delta mass:

related work: Zanotti et al., hep-lat/0407039 and refs therein

$$\boxed{m_\Delta = m_0^\Delta - 4a_1 M_\pi^2 - 4e_1^\Delta(\lambda) M_\pi^4 + m_\Delta^{\rm N-loop} + m_\Delta^{\Delta-\rm loop}}$$

with

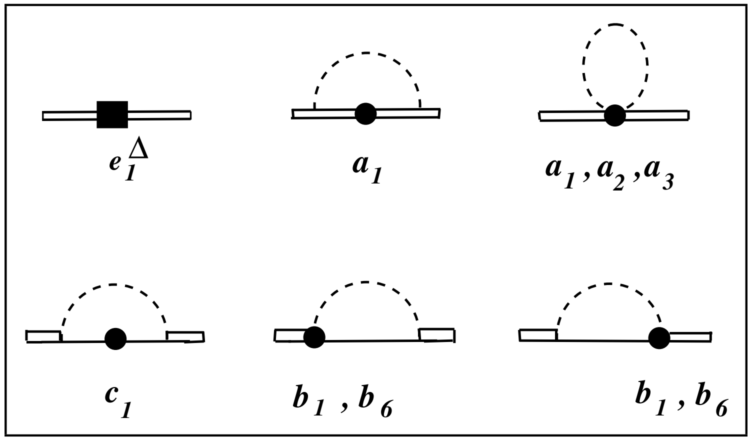
$m_0 = 0.88$ GeV (from phys. value at $M_\pi^{\rm phys}$)

$\Delta_0 = 0.33$ GeV (from complex Δ pole)

$[m_0^\Delta = m_0 + \Delta_0]$

$c_A = 1.1$ (axial $N\Delta$ coupling from complex Δ pole)

c_1, c_2, c_3, e_1 (from $\pi N \to \pi N$ and m_N)



QUARK MASS EXPANSION of the Δ MASS cont'd

– Fit parameters:

$$b_3 = 0.75\,\text{GeV}^{-1}$$

$$b_6 = -0.75$$

$$h_A = 2$$

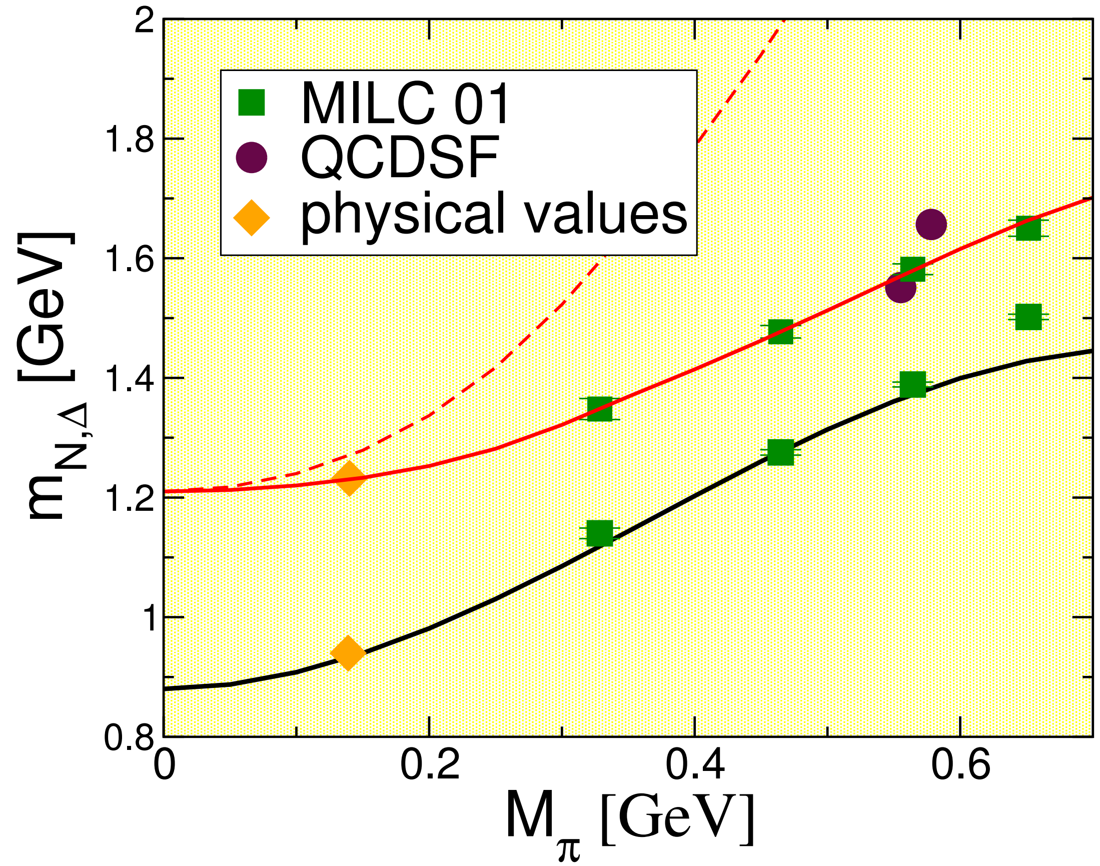
$$a_1 = -0.3\,\text{GeV}^{-1}$$

$$e_1^\Delta = -1\,\text{GeV}^{-1}$$

$$A + B/4 = 0.5\,\text{GeV}^{-1}$$

■ MILC coll., hep-lat/0104002

● QCDSF, G. Schierholz, priv.comm.



Results:

⋆ axial $\Delta\Delta$ coupling h_A not far from $9g_A/5 = 2.28$ as in SU(6)

⋆ leading symmetry breaker a_1 far from SU(6) $\rightarrow$ further study needed

⋆ sigma terms: $\sigma_{\pi N}(0) = 48.9\,$MeV, $\sigma_{\pi\Delta}(0) = 20.6\,$MeV

OTHER NUCLEON PROPERTIES

- Various observables considered in the literature:

  $\rightarrow$ magnetic moments, electromagnetic radii, axial-coupling, ...

  Munich group, Adelaide group, Bern group, ...

- Need global fits at sufficiently small quark masses

  ⋆ LECs relate many observables, are not independent

  ⋆ should incorporate phenomenological input (see mass discussion)

  ⋆ must be in a regime where higher order terms stay sufficiently small

- Results must be independent of the regularization scheme

  $\rightarrow$ these can differ by higher order terms (e.g. HBCHPT versus IR)

  $\rightarrow$ illustrate these facts on the example of the nucleon isovector anomalous magnetic moment $\boldsymbol{\kappa_v}$

ISOVECTOR ANOMALOUS MAGNETIC MOMENT

- Quark mass expansion of κ_v:

Kubis & M, Nucl. Phys. A679 (2001) 698 [hep-ph/0007056]

$$\boxed{\kappa_v = c_6 - 16 m_N M_\pi^2 e_{106} + \kappa_v^{\rm loop,3} + \kappa_v^{\rm loop,4}}$$

  – new electromagnetic LECs c_6, e_{106}
  – c_6 is the anom. magn. moment in the chiral limt
  – the loop contribution depends on c_2, c_4 and c_6

- Study higher order effets by using

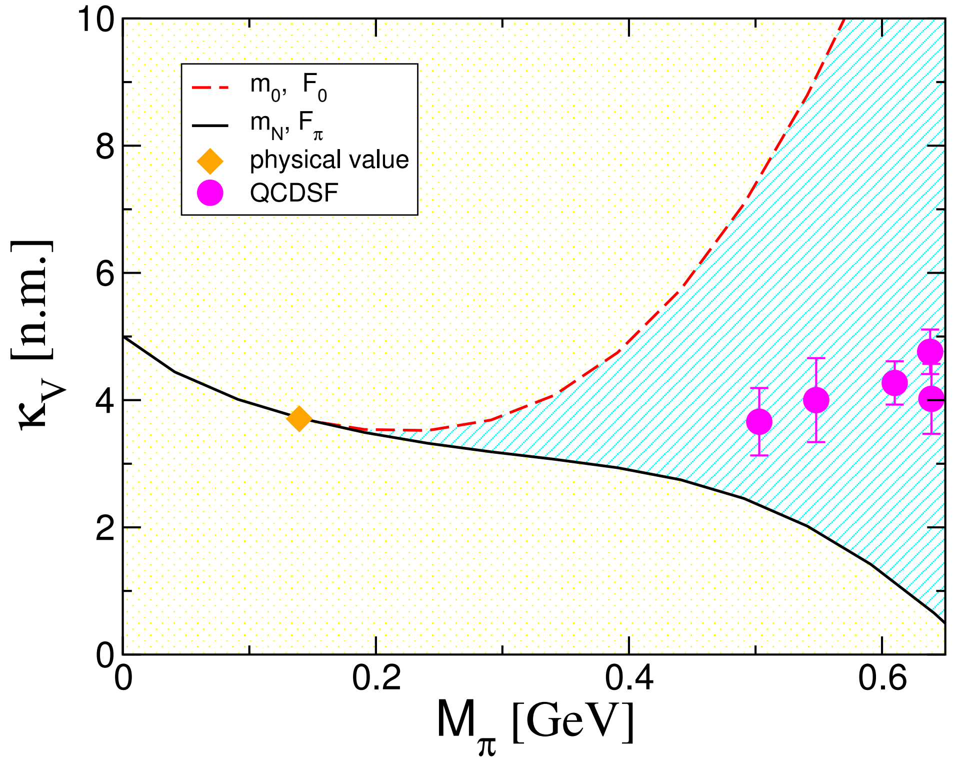
$$\boxed{\{m_0, F_0\} \text{ or } \{m_N, F_\pi\}}$$

$$F_\pi = F_0 \left(1 + \frac{M_\pi^2}{16\pi^2 F_0^2}\left[\log\frac{\lambda^2}{M_\pi^2} + \tilde\ell_4(\lambda)\right] + \mathcal{O}(M_\pi^4)\right)$$

Bijnens et al., Nucl. Phys. B508 (1997) 263 [hep-ph/9707291]

- Results with $c_6 = 5$ and $e_{106} = 0.45$
  – constrain by physical value

● QCDSF Coll., Phys. Rev. D71 (2005) 034508 [hep-lat/0303019]



$\Rightarrow$ this induces large theoretical uncertainty above $M_\pi = 450$ MeV – conservative !

QUARK MASS DEPENDENCE OF NUCLEAR FORCES

WHY?

- Constraints on the variation of fundamental coupling constants

Webb, Flambaum, . . ., Fritzsch, . . ., Langacker, . . .

- Connection to lattice gauge theory results at larger pion masses

HOW?

- Chiral NN-potential @ NLO: $V = V_{\rm OPE} + V_{\rm TPE} + V_{\rm contact}$

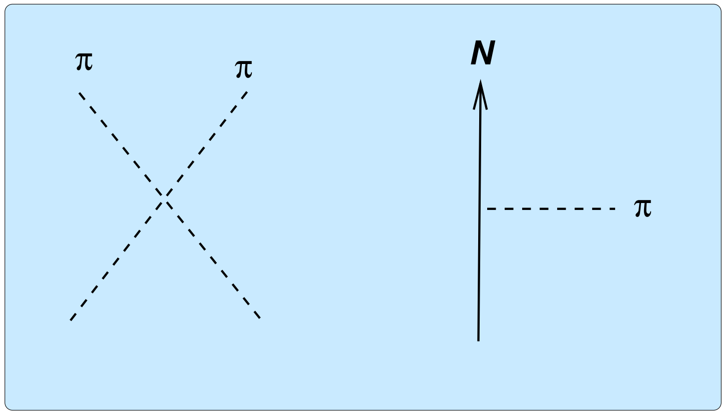
$$\boxed{\begin{aligned} V_{\rm OPE} &= \left(-\frac{g_A^2}{4F_\pi^2} + \ldots d_{16} + \ldots\right) \vec{\tau}_1 \cdot \vec{\tau}_2 \, \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + M_\pi^2} \\ V_{\rm contact} &= C_S + M_\pi^2 D_S + (C_T + M_\pi^2 D_T)\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + \ldots \end{aligned}}$$

$\Rightarrow$ Explicit and implicit pion mass dependence

- d_{16} known from $\pi N \to \pi\pi N$, not very accurately   Bernard, Fettes, M.
- $D_{S/T}$ estimated from dimensional analysis

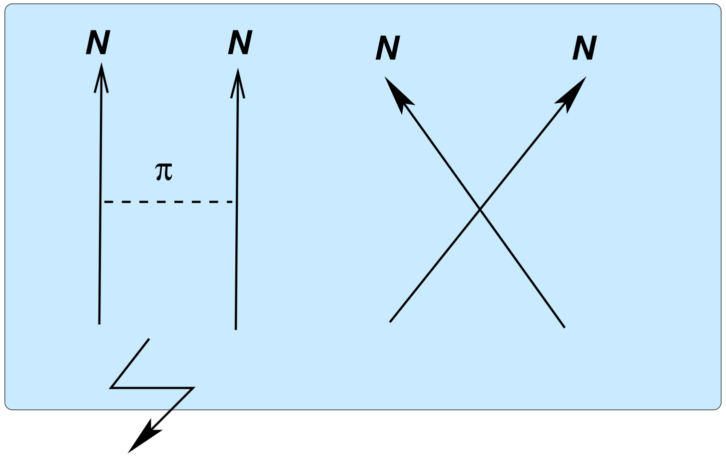
EXPECTATIONS for the CHIRAL LIMIT

$\pi\pi$ and πN system:



NO interactions for vanishing external momenta/energies (Goldstone theorem)

NN system:

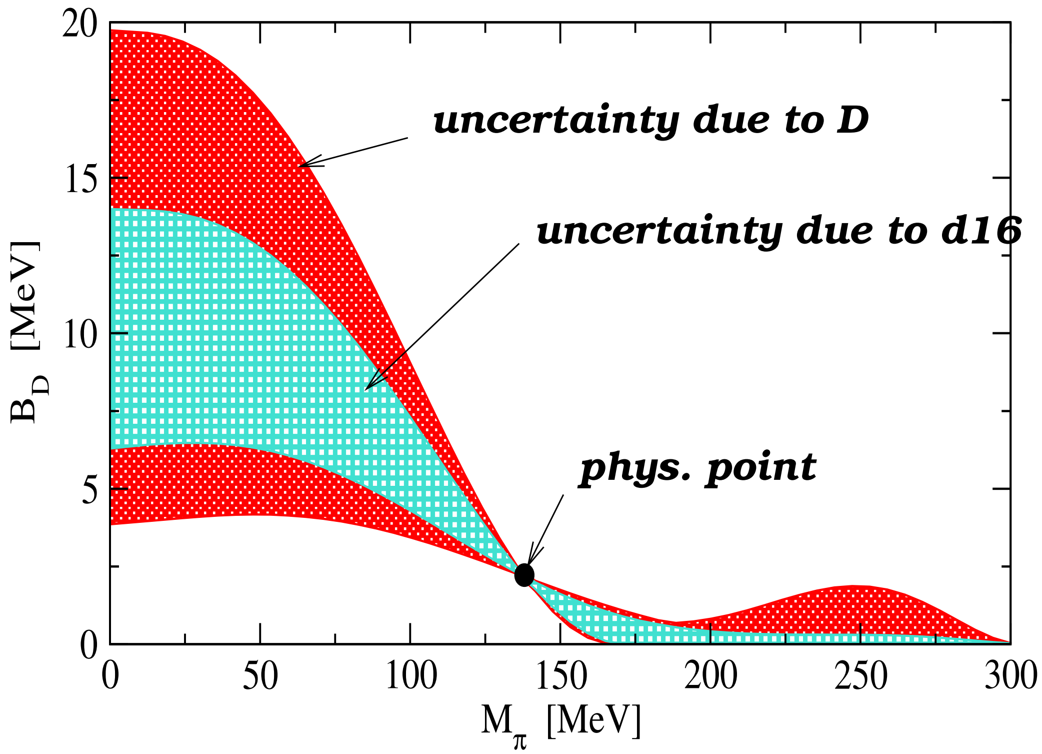


$V_{\rm OPE}^{\rm cl} \sim 1/q^2$ – Coulomb-like
significant scattering for van. momenta
in <u>all</u> partial waves
no more centrifugal barrier

$$\boxed{\delta_\ell(k) \sim k}$$

DEUTERON BINDING ENERGY

Epelbaum, M., Glöckle, Nucl. Phys. A714 (2003) 535 [nucl-th/0207089]; [nucl-th/0208040]



⋆ Deuteron stays bound in the chiral limit. w/ natural BE $\sim F_\pi^2/m_N \sim 10$ MeV

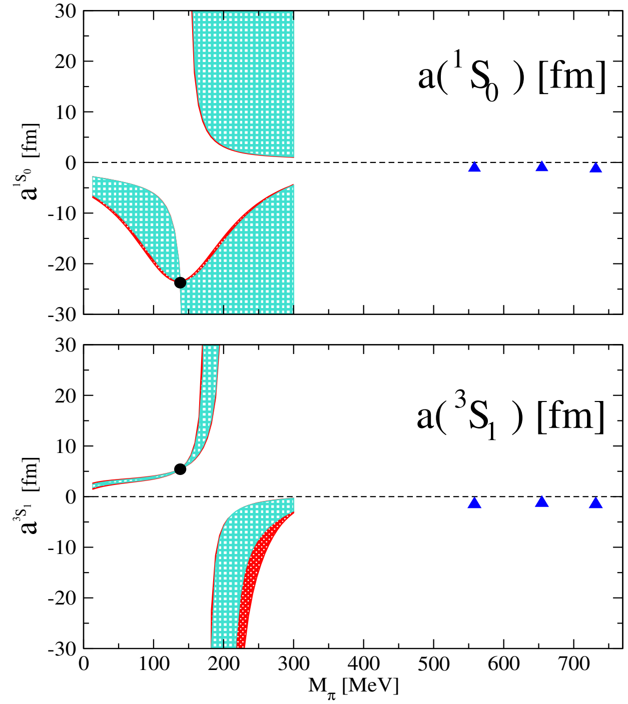
⋆ No other bound states in S- or P-waves

⋆ Larger theoretical uncertainty found in Beane & Savage Nucl. Phys. A717 (2003) 91 [nucl-th/0208021]

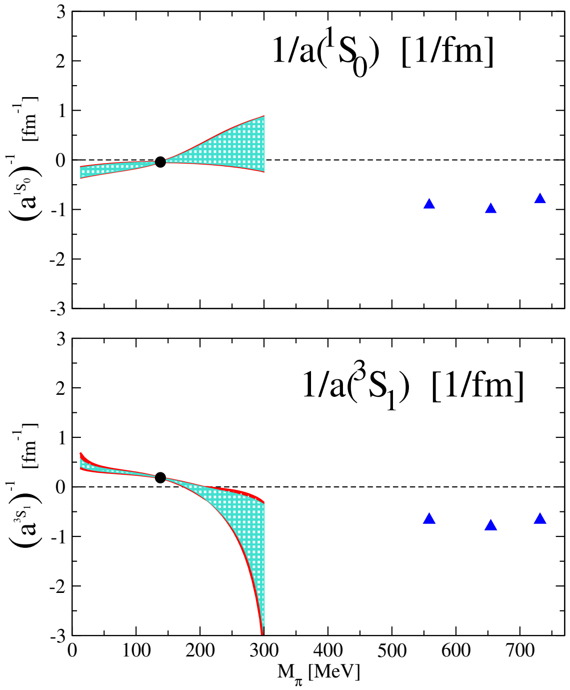
Nucl. Phys. A713 (2003) 148 [hep-ph/0206113]

S-WAVE SCATTERING LENGTHS

scattering lengths



inverse scatt. lengths



⋆ Of natural size in chiral limit: $\boxed{a_{\rm CL}^{^3S_1} = 1.5 \pm 0.4\,^{+0.2}_{-0.3}\ \text{fm},\ a_{\rm CL}^{^1S_0} = -4.1 \pm 1.6\,^{+0.0}_{-0.4}\ \text{fm}}$

⋆ Both inverse scattering lengths vanish for $M_\pi \simeq 175$ MeV → IR limit cycle?

Braaten & Hammer, PRL 91 (2003) 102002 [nucl-th/0303038]

⋆ Lattice "data" ▲ too far away

M. Fukugita, et al., Phys. Rev. D52 (1995) 3003

SUMMARY & OUTLOOK

- Baryon CHPT is a mature field in the light quark sector
  - ⋆ provides unambigouos extrapolation functions for LQCD
- Covariant inclusion of matter fields, spin-1/2 and spin-3/2
  - ⋆ must respect decoupling
- Observables linked by general operator structures
  - ⋆ low-energy constants (LECs) universal
  - ⋆ global fits to observables & input from phenomenology
  - ⋆ SU(3) LECs related to the SU(2) ones by matching conditions
  - ⋆ chiral extrapolation fcts w/ small/moderate theoretical error for $M_\pi \lesssim 400/500$ MeV
  - ⋆ extension to few-nucleon systems, IR limit cycle?

$\Rightarrow$ Need more data at low quark masses for precision studies