ON THREE PROBLEMS OF SMALL DIVISORS

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We present three problems of Small Divisors which were "foklorically" analogous, and for which we have today a rigourous link.

The first one is the problem of the local dynamics of a germ of holomorphic diffeomorphism with an indifferent fixed point.

It is related to the second problem : The local structure of an holomorphic holomorphic vector field in $\mathbf{C}^2$ near a singularity in the Siegel domain. This relation was first explained by an holonomy construction by Mattei and Moussou (1980). In the resonnant case, the complete equivalence of the two problems was stablished by Martinet and Ramis (1982). Finally, the general equivalence was proven by Yoccoz and the author (1993).

The third problem is the study of the dynamics of an analytic circle diffeomorphisms. In 1991 in the setting of germs of holomorphic diffeomorphisms, the author has stablished the existence of Siegel compacts : Compact, connected, non trivial, totally invariant sets containing an indifferent fixed point. Then the dynamics on the exterior of such Siegel compacts corresponds exactly to the dynamics of an analytic circle diffeomorphism on one component of the exterior of the circle embedded in the complex plane. This construction has many applications. The main one links the linearization results in one problem to the other. For instance, Siegel-Brjuno linearization theorem for holomorphic germs can be recovered fron Arnold-Russman-Yoccoz local linearization theorem for circle diffeomorphisms.

Using these correspondences, we can answer a question of Moussu (suggested by Dulac) in the setting of singularities of holomorphic vector fields. Such a singularity of an holomorphic vector field in the Siegel domain is non-linearizable if and only if there exists a solution (distinct from the two trivial ones) accumulating the singularity. Recently (1995), using this correspondence the author has solved a question of Dulac (1904) : If the singularity of the holomorphic vector is in the Siegel domain and non resonnant, then there are no other solution accumulating the singularity than the two trivial ones.

It is amazing to note that there is no other way of proving these results other than to reduce it to a problem on circle diffeomorphisms by the correspondences explained before.