DIRECT NUMERIC CALCULATIONS OF FLOW AND HEAT TRANSFER IN NON-LINEAR SYSTEMS†

by

Francesc Giralt, J. Herrero, F. X. Grau[1], I. Cuesta[1] and J.A.C. Humphrey[2]
Departament d'Enginyeria Química
Escola Tècnica Superior d'Enginyeria Química
Universitat Rovira i Virgili
Ctra. Salou s/n, 43007 Tarragona, Catalunya.

Abstract

The structural characteristics of three different flow systems, natural convection in a cubical cavity heated from below, viscous flow in an enclosed rotating cylinder pair and turbulent flow in an open channel are examined and discussed. The non-linear equations of motion coupled with the heat transfer equation are solved numerically with the second order accurate in time and space codes 3DINAMICS and CUTEFLOWS to characterise bifurcations and stability in the two former viscous flow and heat transfer problems. Kohonen maps are used to analyse the direct numerical simulation data for the turbulent flow in an open channel reported in the literature.

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† J. Ferré, J. Pallares, D. Puigjaner and R. Rallo have contributed to parts of this work as current doctoral students of the University Rovira i Virgili.

[1] Departament d'Enginyeria Elèctrica i Mecànica

[2] Department of Aerospace and Mechanical Engineering, University of Arizona, Tucson.

The purpose of this work is to study numerically the structural characteristics of two viscous flows, where the non-linear convective terms in the Navier-Stokes equations play a significant role, and of a turbulent flow bounded by a free-slip interface and a wall. The two viscous systems studied are the flow and coupled heat transfer in a cubical cavity heated from below and in a fixed cylindrical enclosure containing a pair of parallel disks attached to and corotating with a vertical hub. In the former system buoyancy has been induced by imposing a moderate temperature difference between the bottom (hot) and the top (cold) plates with adiabatic vertical walls, while in the latter centrifugal and Coriolis induced buoyancy forces are also present. The respective geometrical configurations are shown in Figs. 1 and 2. The present study also deals with the determinations of structural characteristics of the turbulent flow in an open channel. Kohonen maps (Kohonen, 1982a,b) are applied to analyse the instantaneous three-dimensional velocity time-records obtained previously by Lam (1989) and Lam and Banerjee (1992) using a Fourier-Chebyshev pseudospectral method to solve the full Navier-Stokes equations.

1 FLOW STRUCTURES IN A CUBICAL CAVITY HEATED FROM BELOW

1.1 Problem definition and governing equations

The geometry of the cubical cavity heated from below and with adiabatic vertical walls is shown in Figure 1. Both the bottom and the top plates are taken to be isothermal with the lower one held at temperature T_H which is moderately greater than the temperature of the upper wall T_C. The fluid considered in this study is air (Pr = 0.71). Compressibility effects, viscous dissipation and the variation of fluid properties with temperature have been neglected, with the only exception of the buoyancy term, for which the Boussinesq approximation has been adopted.

The reference scales for length, velocity, time and pressure are, respectively, L, α/L, L^2/α and $\rho\alpha^2/L^2$, with L being the height of the cavity, ρ the density and α the thermal diffusivity. The non-dimensional temperature is defined as

$$T^* = \frac{T-T_o}{\Delta T} = \frac{T - (T_H + T_C)/2}{T_H - T_C} \tag{1}$$

while all other variables are converted into dimensionless form using the above reference scales. The non-dimensional governing transport equations in Cartesian coordinates are,

$$\frac{\partial u_i^*}{\partial x_i^*} = 0 \tag{2}$$

$$\frac{\partial u_i^*}{\partial t^*} + \frac{\partial (u_j^* u_i^*)}{\partial x_j^*} = - \frac{\partial P^*}{\partial x_i^*} + Pr \frac{\partial^2 u_i^*}{\partial x_j^* x_j^*} + Ra\, Pr\, T^*\, \delta_{i3} \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} + \frac{\partial (u_j^* T^*)}{\partial x_j^*} = \frac{\partial^2 T^*}{\partial x_j^* x_j^*} \tag{4}$$

where u_i is the velocity vector, P the pressure and Ra the Rayleigh number.

1.2 Numerical method and computational procedure

Numerical solutions of the governing equations in terms of the primitive variables were obtained by using the CFD code 3DINAMICS (Cuesta, 1993; Cuesta et al., 1995). In this code equations (2-4) are discretised following a control volume scheme (Patankar, 1980) with non-uniform three-dimensional staggered grids. The convective fluxes across the surfaces of the control volumes were discretised using a second order QUICK scheme (Leonard, 1979). 3DINAMICS incorporates the non-uniform grid version of QUICK proposed by Perng and Street (1989) to enhance convergence. Second order approximations were also implemented for the diffusive fluxes with a centred scheme. The SMAC algorithm Amsden and Harlow (1970) was applied to compute the velocity-pressure coupling.

A critical step in the pressure algorithm, that affects the global performance of the code, is the optimal solution of the Poisson equation. It is for this reason that a preconditioned version of the Conjugate Gradient Method (CGM) was adopted in the present study. A semi-implicit, second order accurate, ADI method proposed by Peaceman and Rachford (1955) was applied for time-marching integration.

The boundary conditions for the dynamic field are $u_i = 0$ at the walls, i.e., all six walls of the cavity are assumed to be rigid and at rest. The thermal boundary conditions are $(\partial T/\partial n) = 0$ at the four adiabatic vertical walls, $T_H^* = 0.5$ at the bottom plate and $T_C^* = -0.5$ at the top plate. Initially, the fluid was assumed to be at rest and a conductive temperature distribution was assumed. To start the Rayleigh-Bénard (RB) convective regime, a random disturbance with an amplitude of ±0.05 was added to this initial thermal field, as suggested by (Grötzbach (1982). This procedure avoids the predetermination of the final structure. Heat transfer rates at the top and bottom walls are characterised with local and surface averaged Nusselt numbers, Nu = $(\partial T^*/\partial z^*)_{wall}$.

1.3 Results and discussion

The most common approach to visualise three-dimensional structures in unsteady flows is to plot vector fields, streamlines, streaklines and particle lines. In this work the second invariant of the gradient of the velocity tensor (Π) is also used to describe the structural characteristics of the flow (Hunt et al., 1988). In non-dimensional form, this magnitude is defined as

$$\Pi^* = \frac{\partial u_i^*}{\partial x_j^*}\frac{\partial u_j^*}{\partial x_i^*} = S_{ij}^* S_{ji}^* + \Omega_{ij}^* \Omega_{ji}^* \qquad (5)$$

In the cubical cavity with insulated vertical walls the critical Rayleigh number (Ra_c) is 3446 (Catton, 1972). As the Rayleigh number increases, a buoyancy driven flow develops with a corresponding increase in heat transfer rates. Results for Rayleigh number 9000 are depicted in Figure 3 showing the isosurfaces of Π for the four structures obtained (S1-S4). Three of these structures S1 and S2, and S4 were previously reported in the literature by Ozoe et al. (1976) and Hernández et al. (1994), respectively. In the present study an new roll-type structure S3, with different topological and heat transfer characteristics is obtained. The dynamic fields indicate that structures S1 and S3 are formed by single rolling motions with clockwise or anti-clockwise rotation. The axis of rotation of the roll is parallel to the coordinate x or y, indistinctly. These two possible orientations are equivalent due to the symmetry of the enclosure. Their sense of rotation depends only on the random thermal distribution mentioned above. There is a great similarity between S1 and S3 but the structure S3 is more tilted and elongated towards the top and bottom opposite edges than S1. This fact produces a substantial difference in the heat transfer rates for both structures at the top and bottom plates. The second structure S2 is also a single roll but with a diagonal mode of circulation. It shows a greater three-dimensionality than S1 and S3. The topology of S4 over the range of Rayleigh numbers studied is characterised by a cold descending current in the central region of the enclosure. Heating of this current at the bottom plate while moving towards the four lower corners causes the fluid to ascend parallel to the vertical edges and to close the recirculation pattern depicted in Figures 3.

The different type of recirculating motion present in each structure is also illustrated in Figure 4, where the two-dimensional distribution of the local Nusselt number at the top and bottom plates is plotted for Ra = 9000. These two-dimensional thermal fields are given in terms of lines of constant local Nusselt numbers. A maximum absolute value of the Nusselt number corresponds to a region of impingement of an ascending or descending current, while relatively low absolute values correspond to regions of boundary layer flow. Figures 4.a-b-c-d and 4.e-f-g-h show, respectively, the distribution of local Nusselt numbers at the top and bottom plates for the four structures present in the flow. For S1, S2 and S3 (a-b-c and e-f-g), the distributions of the hot and cold plates are perfectly anti-symmetrical, in accordance with the presence of a single rotating roll. The maps for S4 agree with a nearly toroidal configuration with dominating descending currents at the vertical edges and a strong ascending current in the core.

Among all different structures, S4 is by far the less effective for transferring heat between the top and bottom plates, as indicated by the lower values of the local Nusselt numbers given in Figure 4.d and 4.h. On going calculations indicate that a new structure S5, formed by four rolls rotating parallel to the vertical walls, develops at a critical Rayleigh number $Ra_c \approx 6,400$. The sequence of critical Rayleigh numbers determined numerically ($Ra_c \approx 3,500$ for S1, S2 and S3; $Ra_c \approx 6,400$ for S5 and $Ra_c \approx 7,700$ for S4) is in agreement with the values predicted by the linear stability analysis which is presently carried out in collaboration with Prof. C. Simó of the University of Barcelona.

2 FLOW STRUCTURES IN COROTATING ENCLOSED DISKS

2.1 Problem definition and governing equations

The geometrical configuration investigated here consists of a fixed cylindrical enclosure containing a pair parallel disks attached to and corotating with a vertical hub. The top and bottom disks may be at different temperatures, while both cylindrical walls are always adiabatic. A sketch of this geometry, hereinafter referred to as ECDP (Enclosed Corotating Disk Pair), is shown in Fig. 2. Detailed literature reviews on the most relevant prior work in isothermal ECDP flow may be found in Humphrey et al. (1991, 1995). The first analysis of this flow configuration can be attributed to Stewartson (1957), who obtained approximate analytical boundary layer solutions for the flow between corotating disks when a discontinuity in the disk angular velocity Ω is present at a certain radial position; in the context of the present study, at R_2 (Fig. 2). For the case with both disks rotating at the same angular velocity and the fluid at R_2 rotating at an angular velocity $\Omega(1+\delta)$, with δ being small but positive, Stewartson (1957) deduced that there should be a radial inflow along the disk Ekman layers of thickness $\delta/R_2 \approx O(Re^{-1/2})$ with a corresponding outflow about the midplane between disks. In the above and subsequent expressions, the Reynolds number is defined as $Re = \Omega R_2^2/\nu$, where ν is the kinematic viscosity of the fluid and Ω is the angular velocity of the rotating disks. Had Stewartson (1957) analysed the case of negative δ, he would have obtained the cross-stream recirculation characteristic of the ECDP configuration analysed here, consisting of two counter-rotating toroidal vortices driven by an *outflow along* the disk Ekman layers.

The cross-stream flow structure in *steady* ECDP flow was characterised much later in the calculations of Duck (1986) and Chang et al. (1989), as well as by the analysis and measurements of Schuler (1990). Following the methodology of Greenspan (1968), Schuler et al. (1990) provide a comprehensive analysis of the distinct flow regions in the cross-stream (z-r) plane. The strong shear generated at the fixed enclosure wall causes an imbalance between the radial pressure gradient and the centrifugal force in the disk Ekman layer, which results in an outwards-directed motion along the disks. For small aspect ratios, $\Gamma = H / (R_2 - R_1)$, the vortex pair formed in the z-r plane does not span the entire inter-disk space. Near the region surrounding the hub fluid is in solid body rotation.

The cross-stream motion due to the vortex pair driven by the disk Ekman layers presents a pattern similar to that of a cell pair in Taylor-Couette flow (Koschmieder, 1993). It should be noted that while the cross-stream motion in the ECDP *is not* driven by centrifugal instability, the Rayleigh stability criterion written in terms of the circumferential component of velocity v_θ in a fixed frame of reference

$$\frac{1}{r^3}\,\frac{\partial\left(v_\theta\, r\right)^2}{\partial r} \rangle\, 0 \tag{6}$$

cannot be fulfilled everywhere in the flow at any Reynolds number.

In the early experiments of Lenneman (1974) it was already observed that ECDP flow becomes unsteady beyond a certain critical value of the Reynolds number. More recently Abrahamson et al. (1989) reported the existence of circumferentially-periodic axially-aligned vortical structures rotating at about 75% the angular velocity of the disks. They also found that the number of structures, *m*, tended to diminish with increasing Re while it increased with decreasing Γ. Similar conclusions were drawn by Schuler et al. (1990) who also reported hysteresis effects in the transitions between different *m* states when increasing or decreasing the Reynolds number.

As Tavener et al. (1991) pointed out, transition from 2D steady to 3D unsteady ECDP flow takes place through a Hopf bifurcation, which is mathematically realisable even for the axisymmetric case *m*=0. These authors also obtained a complete bifurcation diagram for values of the aspect ratio Γ in the range about $1 \leq \Gamma \leq 4$, considerably higher than that of the present study. The transition Reynolds number, Re_c, was found to increase with decreasing Γ. In the present investigation, with $\Gamma \approx 0.2$, the transition is detected at $Re_c \approx 18{,}000$. The family of super-critical anomalous steady solutions reported by Tavener et al. (1991) for $\Gamma \approx 1$ is not physically realisable in *isothermal* ECDP flow for the present geometrical parameters. Notwithstanding, such anomalous solutions are obtained in 2D numerical calculations at Reynolds numbers larger than about 45,000. Quantitative evidence of the link between the circumferentially periodic vortical structures and the instability of the cross-stream vortices in 3D ECDP flow has been provided by the numerical calculations of Humphrey et al. (1995).

The effect of moderate buoyancy forces on the ECDP geometry of Fig. 2 has been analysed numerically by Herrero et al. (1994a,b) for the Reynolds range $3{,}500 \leq Re \leq 17{,}000$ where the flow is steady. This investigation was restricted to air ($Pr = 0.7$) and to dimensionless temperature differences between disks $\Delta T/T_o \leq 0.1$ where the Boussinesq hypothesis is applicable. In this expression T_o is the fluid average temperature. For the conditions investigated by these authors the Rayleigh number was $Ra = g\beta\Delta T H^3/\nu\alpha \approx 2{,}200$, where g is the acceleration of gravity and β is the thermal expansion coefficient. At this relatively low Rayleigh number the rotational speed of the ECDP system is sufficiently high to inhibit the effect of thermal convection. The contribution of Coriolis buoyancy in the flow, Eqns. (11) and (12), is negligible and only centrifugally induced buoyancy effects are significant. Herrero et al. (1994a) characterised the effects of centrifugal buoyancy on the structure of steady ECDP flow when the bottom disk was set at a uniform temperature higher than the top one (see Fig. 2). These authors found that centrifugal buoyancy favoured a clockwise cross-stream circulation. Moreover, a slow centrifugally induced cross-stream circulation also developed in the inner flow region, originally in solid body rotation with the disks and hub.

The effects of centrifugal buoyancy have been also investigated in other non-isothermal rotating flow configurations. The experiments of Koschmieder (1967) on Rayleigh-Bénard convection in a rotating shallow cylindrical cavity filled with a silicone oil showed that when the acceleration ratio, $Ac = g/\Omega^2 R_2$, attained sufficiently small values ($Ac << 1$) centrifugal buoyancy dominated over gravity-induced buoyancy, thus leading to a kind of cross-stream motion different from the typical roll pattern. Analysis by Barcilon and Pedlosky (1967) and by Homsy and Hudson (1969) confirmed the observations of Koschmieder (1967) and showed that even for relatively large values of the acceleration ratio, $Ac>1$, such centrifugally induced motion will be always present in the absence of Rayleigh-Bénard convection. The numerical analysis carried out by Herrero et al. (1994) on the buoyancy induced flow in the cylindrical cavity sketched in Fig. 2, but with the external cylindrical wall also rotating, confirmed that motion was controlled by the disk Ekman layers. In these layers the radial and azimuthal components of velocity scaled well with $v_E \approx w_E \approx \Omega R_2 \varepsilon$, where ε is the thermal Rossby number defined as $\varepsilon = \beta\, \Delta T$.

Calculations of 3D ECDP flow have been carried out in the present study in the range of Reynolds numbers between 20,500 and 82,400, approximately. Results are shown here for the periodic 3D flow structure that develops at $Re \approx 20{,}565$ ($\Omega \approx 300$ rpm) for the ECDP geometry specified in Fig. 2. At Reynolds numbers larger than about 30,000, the isothermal flow is no longer periodic and becomes increasingly irregular and unsteady. On the other hand, the existence of anomalous solutions that arise as a result of a flow bifurcation is also examined at $Re = 82{,}380$. One of these steady anomalous 2D solutions for isothermal ECDP is compared with the 3D quasi-periodic solution of this kind that is obtained in non-isothermal flow at the same Reynolds number.

2.2 Conservation equations

The fluid is assumed to be incompressible and Newtonian. All buoyancy terms present in the conservation equations are approximated by Boussinesq expressions. In rotating disk systems, the external radius R_2 and the maximum velocity in the disk Ekman layer, $v_E \approx \Omega R_2$, are often employed as length and velocity scales. By defining the modified pressure

$$P^{|} = P - \rho_o g z \tag{7}$$

and the dimensionless variables

$$U = u/\Omega R_2 \quad , \quad V = v/\Omega R_2 \quad , \quad W = w/\Omega R_2 \quad ,$$

$$Z = z/R_2 \quad ; \quad R = r/R_2 \quad , \; \tau = \Omega\, t \;, \; P^{||} = P^{|}/\rho_0\, \Omega^2 R_2^2 \tag{8}$$

the differential equations for the conservation of mass, momentum and energy in a frame of reference rotating at an angular velocity Ω may be written for cylindrical coordinates (z, r, θ) as

$$\frac{\partial U}{\partial Z} + \frac{\partial V}{\partial R} + \frac{V}{R} + \frac{1}{R}\frac{\partial W}{\partial \theta} = 0 \tag{9}$$

$$\frac{DU}{D\tau} = -\frac{\partial P^{||}}{\partial Z} + \frac{1}{Re}\nabla^2 U + Ac\, \varepsilon\, T^{|} \tag{10}$$

$$\frac{DV}{D\tau} = -\frac{\partial P^{||}}{\partial R} + \frac{1}{Re}\nabla^2 V - R\, \varepsilon\, T^{|} + \left[1 - \varepsilon\, T^{|}\right]\left(2W + W^2/R\right) \tag{11}$$

$$\frac{DW}{D\tau} = -\frac{\partial P^{||}}{\partial \theta} + \frac{1}{Re}\nabla^2 W - \left[1 - \varepsilon\, T^{|}\right]\left(2V + VW/R\right) \tag{12}$$

$$\frac{DT}{D\tau} = \frac{1}{RePr}\nabla^2 T \tag{13}$$

In the above equations the operator ∇^2 indicates diffusion in the corresponding direction, P is the pressure, the dimensionless temperature T' is defined as $T' = (T - T_o)/\Delta T$, and u, v, w are the respective velocity components for the axial, radial and circumferential (z, r, θ) directions. In Equations (11) and (12): R^2 is the centrifugal force term due to the rotating frame of reference; 2V and 2W are the components of Coriolis force, also due to the rotating frame of reference; W^2/R and VW/R are centrifugal and Coriolis forces, respectively, due to the base vector variations of the cylindrical system used to describe the flow. These non-linear terms couple the radial and circumferential components of motion. Equations. (10-12) result after linearisation of the density as a function of the temperature in all those source terms involving external or inertial forces. Note that the more familiar conservation equations for isothermal flow would be obtained for the limiting case $\varepsilon \longrightarrow 0$.

Calculations have been performed using the CUTEFLOWS numerical algorithm for unsteady incompressible flows. The algorithm is fully documented in Schuler et al. (1991). It has been extensively tested and applied to a variety of isothermal and non-isothermal flow configurations including rotating disks, bluff bodies and two backward-facing step benchmark problems. Briefly, the algorithm is based on a staggered-grid, control volume discretisation approach to derive finite difference forms of the conservation equations in terms of primitive variables. Central differencing is used to approximate the pressure and diffusion terms in the momentum equations. The second order accurate QUICK scheme is used for the convective terms. Continuity yields a discrete Poisson equation which is solved for pressure using the conjugate gradient method. The algorithm is explicit in time and the use of a second order Runge-Kutta scheme renders it second order accurate in time as well as in space.

The finite difference approximations to the conservation equations are solved numerically subject to the following boundary conditions for velocity: u = 0, v = 0 and w = 0 along all rotating surfaces and $w = -\Omega R_2$ at the enclosure wall. In the circumferential direction, a periodic flow boundary condition is imposed. In non-isothermal calculations a uniform temperature distribution is prescribed for both disk surfaces while the cylindrical walls are set as adiabatic. All boundary conditions are implemented with second order accuracy. Depending on the objective of a run, calculations are started either by interpolation of previous results or from an initial state which is motionless with respect to a fixed frame of reference. In the latter case, all moving walls are impulsively set to their corresponding angular velocity at t = 0. The dimensionless integration time-step was set to $\Delta(\Omega t) \leq 0.006$ (see Table I). Several 2D test-runs show that this time-step is small enough to guarantee both stable behaviour and accuracy in the course of calculation. Results presented below for isothermal and non-isothermal flow have been obtained on computational grids featuring 40x74 and 50x90 nodes in the (z-r) plane, respectively, and 52 nodes in the θ direction. The latter figure corresponds to 12 calculations nodes per wave-cycle plus the extra nodes required to implement the periodicity boundary conditions.

2.3 Results and discussion

The isothermal unsteady 3D ECDP flow at Re = 20,565 develops a periodic structure with a wave-number *m*=4. An example of the evolution of the axial velocity component u at $r' = (r-R_1)/(R_2-R_1) \approx 0.8$ on the midplane is shown in Fig. 5a for this Reynolds number. Unsteadiness sets in at Ωt about 60, after the whole system has already undergone up to 9-10 complete revolutions and the typical structure of 2D steady ECDP flow is almost fully developed. Then a transient is required before reaching the final characteristic shape of the cross-stream 3D structure for ECDP flow illustrated in Fig. 5b, which is periodic both in θ and time. In this figure the structure is shown in terms of the stream function. The sense of rotation of these vortices is dictated by the outflow in both disk Ekman layers, which is the result of the imbalance between the radial pressure gradient and the centrifugal force, first and third terms respectively in the RHS. of Eq. (11).

The oscillation present in the u velocity component in Fig. 5a corresponds to alternating axial displacements of the midplane jet that is inwards directed between the cross-stream vortices portrayed in Fig. 5b. Maxima or minima in u at z/H = 0.5 imply that fluid with low (more negative) circumferential velocity is displaced above or below the midplane. As a consequence, the Coriolis force component in Eq. (11) induces a larger radial motion in the jet between the two cross-stream vortices. Because of the mass conservation constraint, Eq. (9), these radial and circumferential accelerations above/below the midplane are associated with alternating positive and negative axial motions. The resulting oscillation produces significant deformations in the cross-stream vortices, as illustrated by the velocity vectors plotted in Figs 6a and 6b.

Figures 6a and 6b also include the distribution of the axial vorticity component at the cross-stream plane. Regions with high (positive) ω_z at large radial positions in the disk Ekman layers are typical of ECDP flow. As the cross-stream motion circulates fluid from the Ekman layers into the midplane, positive axial vorticity is also advected. Maximum values of positive axial vorticity arise about the midplane at circumferential locations corresponding to maxima or minima in u. Note how the axial departures of the midplane jet are associated with intrusions of positive axial vorticity which is even transported inwards from the enclosure wall to lower radial positions. As Humphrey et al. (1995) pointed out, the distribution of the circumferential velocity component is to a large extent dictated by the cross-stream flow, while the relation between w and the axial vorticity component is obvious from the definition of ω_z itself. Figs. 6a and 6b also portray the great complexity acquired by the vorticity field, with different regions with relatively low and relatively high ω_z at a given line of constant z above or below the midplane. In our opinion, and in agreement with Humphrey et al. (1995), the radial and circumferential alternation of minima and maxima of ω_z accounts for the observations of axially-aligned vortical structures reported in the earlier literature.

When the flow is *isothermal* anomalous steady solutions are detected for Re>45,000 in 2D calculations only. Figure 7a shows an example of one of these solutions at Re = 82,380. Although the flow portrayed in Fig. 7a is steady, the streamline pattern in this plot is quite different from that typical of steady ECDP flow at low Reynolds numbers (Fig. 5b). Note that 3D calculations of the same flow yield almost chaotic velocity fields. A completely different behaviour has been detected in a 3D non-isothermal flow calculation at Re = 82,380, Pr = 0.7 and $\varepsilon = 0.1$. The evolution of the axial and circumferential velocity components at z/H = 0.5 and r'=0.8 is plotted in Fig. 7b. This 3D non-isothermal calculation begins at a dimensionless time $\Omega t \approx 315$, after the corresponding isothermal ECDP calculation has already developed an irregular pattern. At this time $\Omega t \approx 315$ the preselected temperature difference between both disks is suddenly set up. Figure 7b shows clearly that at about $\Omega t = 360$ the flow has evolved from highly irregular to a quasi-periodic behaviour. The cross-stream structure that has developed after this time at a given θ-plane is depicted in Fig. 7c in terms of the velocity vectors. Note the similarities that exist with the streamline pattern of the anomalous steady, isothermal ECDP solution given in Fig. 7a for the same Reynolds number. The presence of buoyancy alters the bifurcation map for ECDP flow, as demonstrated by the anomalous solution reported in Fig. 7 for Re = 82,380.

3. STRUCTURES IN A TURBULENT OPEN CHANNEL FLOW

3.1 Problem definition and governing equations

The problem considered is the incompressible flow in an open channel driven by gravity as shown in Fig. 8. A no-slip condition is imposed at the channel wall and free-slip condition, i.e., velocity fluctuations parallel to the surface are not restrained and have zero gradients, at the gas-liquid interface. The coordinates x_1, x_2 and x_3 represent the streamwise, spanwise and normal directions, respectively. The flow is assumed to be fully developed and homogeneous in the streamwise and spanwise directions. The direct numerical simulation results analysed here were obtained by Lam (1989) and reported by Lam and Banerjee (1992). At the gas-liquid interface velocity fluctuations parallel to the interface are unimpeded due to the much lower density (inertia) of the gas and there are no waves.

The governing equations are the mass and momentum conservation equations (2) and (3) for an incompressible Newtonian fluid. The pressure gradient may be eliminated from these conservation equations to yield

$$\frac{\partial \omega_3^*}{\partial t^*} - \frac{\partial C_2^*}{\partial x_1^*} + \frac{\partial C_1^*}{\partial x_2^*} = \frac{1}{Re}\nabla^{*2}\omega_3^* \tag{14}$$

$$\frac{\partial}{\partial t^*}(\nabla^{*2}u_3^*) = \nabla^{*2}C_3^* - \frac{\partial}{\partial x_3^*}(\frac{\partial C_j}{\partial x_j^*}) + \frac{1}{Re}\nabla^{*4}u_3^* \tag{15}$$

for the normal velocity and vorticity components, u_3 and ω_3. In these equations C_i are the non-linear convective terms minus the mean kinematic pressure gradient, and variable are made dimensionless with respect to the interface velocity and fluid film height. With u_3 and ω_3 known it is possible to calculate u_1 and u_2 by solving simultaneously the continuity and vorticity equations,

$$\frac{\partial u_1^*}{\partial x_1^*} + \frac{\partial u_2^*}{\partial x_2^*} = - \frac{\partial u_3^*}{\partial x_3^*} \tag{16}$$

$$\frac{\partial u_2^*}{\partial x_1^*} - \frac{\partial u_1^*}{\partial x_2^*} = \omega_3^* \tag{17}$$

A "pseudospectral" method was used by Lam and Banerjee (1992) to solve efficiently the non-linear terms in Eqns. (14) and (15) following the procedure proposed by Kim et al. (1987). Time advancement was done by the two-level explicit Adams-Bashforth scheme for the convective terms and by the implicit Crank-Nicolson method for the diffusion terms. Finite Fourier expansions were used to represent solutions in the homogeneous x_1 and x_2 directions, while Chebyshev polynomials were applied in the cross-stream or normal direction x_3. The turbulence data analysed below are the result of the calculations reported in Lam (1989).

3.2 Pattern recogniton of turbulent velocity fields

The case analysed here is one with no shear at the interface. The Reynolds number Re based on the half depth h and the effective shear velocity is 60.4. The corresponding Reynolds number based on average velocity and equivalent diameter is Re_D = 11,000. This relatively low Reynolds number is a practical limitation of direct numerical simulation of turbulence. More details are given elsewhere (Lam, 1989).

The pattern recognition analysis of the 3D data base has been carried out with self-organising maps (Kohonen, 1982a,b). These artificial neural nets consist of an unsupervised, competitive learning scheme. In the present study the turbulent velocity data were first pre-processed to extract the features of the different motions present. This pre-processing consisted in filtering the raw signals according to a predetermined phase spatial distribution of the velocity vectors. The extracted features of the different classes were then used to train the net. Finally, the raw velocity data were analysed at all planes x_1-x_2, x_1-x_3 and x_2-x_3 of the domain to detect and classify statistically significant events. Figure 8 shows a typical velocity vector plot at the x_2-x_3 plane where fluid exits the domain.

Of all possible coherent structures that occur in wall-bounded turbulent flows (Kline, 1988; Ottino, 1990; Glendinning, 1994) Fig. 9 depicts four of the classes identified in the present work. They correspond to two of the three characteristic flow zones identified by Hunt et al. (1988) in turbulent flows. The four classes identified in the turbulent open channel flow by the Kohonen map are divergence and convergence zones (Fig. 9 a & b), and two eddies with a counter-clockwise and clockwise rotation (Fig. 9c & d). When the divergence and convergence zones occur at the free interface (x_3^* = 0) they correspond to down-wellings and up-wellings from and to this (x_1-x_2) surface. Also, most of the vortices classified in cross-stream planes, such as the one depicted in Fig. 8, are the result of projecting into these surfaces the spiralling motion of 3D funnel-like vortices with ω_1 vorticity. These eddies were first visualised by Kaftori (1993). They are the dominant coherent wall structures and are the high speed regions observed in wall layers. The regions between funnel vortices correspond to low speed streaks.

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Figure Captions.

Figure 1. Physical model of the cubical cavity heated from below.

Figure 2. Schematic of the ECDP geometry investigated. Dimensional values are H = 9.53 mm, R_1 = 56.4 mm, R_2 = 105 mm, $\Gamma = H/(R_2-R_1) = 0.196$ and $\eta = R_2/R_1 = 1.862$.

Figure 3. Isosurfaces of the second invariant velocity gradient tensor at Ra=9000 for the structures S1 (a) Π^*=-3000; S2 (b) Π^*=-4000; S3 (c) Π^*=-3000; S4 (d) Π^*=-500.

Figure 4. Distribution of local Nusselt number at the bottom plate (a, b, c & d) and top plate (e, f, g, & h) at Ra=9000 for the structures S1 (a & b); S2 (c & d); S3 (e & f); S4 (g & h).

Figure 5. Three-dimensional calculations at Re = 20,565. (a) Evolution of the axial velocity component at the location z/H = 0.5 and r' ≈ 0.8. (b) Streamtube illustrating the 3D structure of the periodic flow.

Figure 6. Combined plot of velocity vectors and contours of the dimensionless axial vorticity component, ω_z, at two cross-stream planes (a) $\theta = \pi/8$ and (b) $\theta = 3\pi/8$.

Figure 7. (a) Streamlines for the anomalous isothermal 2D solution at Re = 82,380. (b) Time evolutions of the dimensionless axial and circumferential velocity components at z/H = 0.5 and r' = 0.8 for a non-isothermal 3D calculation with Pr = 0.7 and $\varepsilon = 0.1$. (c) Velocity vectors at the cross-stream plane for the 3D calculation in (b).

Figure 8. Flow system showing the coordinate system and the cross-stream plane x_2-x_3 at the exit of the domain. A free-slip condition is imposed at the interface ($x_3^* = 0$) and a no-slip condition at the wall ($x_3^* = -1$). A coherent structure with ω_1^* vorticity exiting the domain is also shown.

Figure 9. Classes of coherent structures detected by a Kohonen map. (a) divergence zone (up-welling at $x_3^* = 0$); (b) convergence zone (down-welling at $x_3^* = 0$); (c) counter-clockwise vortex; (d) clockwise vortex.