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ASPECTS OF SPONTANEOUS $N = 2 \to N = 1$ BREAKING IN SUPERGRAVITY

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ABSTRACT. We discuss some issues related to spontaneous $N = 2 \to N = 1$ supersymmetry breaking. In particular, we state a set of geometrical conditions which are necessary for such a breaking to occur. Furthermore, we discuss the low energy $N = 1$ effective Lagrangian and show that it satisfies non-trivial consistency conditions which can also be viewed as conditions on the geometry of the scalar manifold.

1. Introduction

The possibility of spontaneous $N = 2 \to N = 1$ supersymmetry breaking in four space-time dimensions ($D = 4$) was first discussed in string theory [1] and then further developed in global supersymmetry and supergravity (see, for example, refs. [2–11]).[1] In this talk we focus on the situation in supergravity where so far only a few models with spontaneous $N = 2 \to N = 1$ breaking are known [4, 5]. Thus it is of interest to uncover the general story of spontaneous $N = 2 \to N = 1$ breaking or in other words ask the questions

(i) Under what (geometrical) conditions does the $N = 2$ theory have ground states that are $N = 1$ supersymmetric.
(ii) What is the low energy effective $N = 1$ action which describes the interactions below the scale of supersymmetry breaking.

As we will see the first question (i) can be rephrased as geometrical conditions on the scalar manifold which is spanned by the scalar fields in vector- and hypermultiplets. The second question (ii) imposes a set of consistency conditions on the couplings of both the original $N = 2$ theory and the low energy effective $N = 1$

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[1]The truncation of $N = 2 \to N = 1$ theories has been worked out in [12] and aspects about partial supersymmetry breaking for $N > 2$ have been discussed in [13].

theory. Again these conditions can be stated as geometrical properties of the scalar manifold. In this talk we address both issues and give some partial results but leave a more detailed and complete analysis to a separate publication [14].

2. The starting point: gauged $N = 2$ supergravity

Let us first briefly recall the starting point of our analysis. In $D = 4$ the spectrum of a generic $N = 2$ theory consists of a gravitational multiplet, n_v vector multiplets and n_h hypermultiplets [15]. The gravitational multiplet $(g_{\mu\nu}, \Psi_{\mu A}, A_\mu^0)$ features the space-time metric $g_{\mu\nu}, \mu, \nu = 0, \ldots, 3$, two gravitini $\Psi_{\mu A}, A = 1, 2$ and the graviphoton A_μ^0. A vector multiplet (A_μ, λ^A, z) contains a vector A_μ, two gaugini λ^A and a complex scalar z. Finally, a hypermultiplet (ζ_α, q^u) contains two hyperini ζ_α and 4 real scalars q^u. For an arbitrary number of vector- and hypermultiplets there is a total of $2n_v + 4n_h$ real scalar fields in the spectrum with σ-model type interactions of the form

$$\mathcal{L} \;=\; -g_{i\bar{j}}(z, \bar{z})\, D_\mu z^i D_\mu \bar{z}^{\bar{j}} - h_{uv}(q)\, D_\mu q^u D_\mu q^v + \ldots \;, \tag{1}$$

where the range of the indices is $i, \bar{j} = 1, \ldots, n_v$ and $u, v = 1, \ldots, 4n_h$. The scalars (z^i, q^u) can be viewed as coordinates of the manifold

$$\mathcal{M} = \mathcal{M}_v \times \mathcal{M}_h \;, \tag{2}$$

where $g_{i\bar{j}}$ is the metric of the $2n_v$-dimensional space $\mathcal{M}_v$ while h_{uv} is the metric on the $4n_h$-dimensional space $\mathcal{M}_h$. $N = 2$ supersymmetry imposes that $\mathcal{M}_v$ is a special Kähler manifold [16,17] while $\mathcal{M}_h$ has to be a quaternionic-Kähler manifold [18].

Both sets of scalar fields can be charged under the isometries of $\mathcal{M}$

$$\delta q^u \;=\; \Lambda^I k_I^u(q) \;, \qquad \delta z^i \;=\; \Theta^I k_I^i(z) \;, \qquad I = 0, \ldots, n_v \;, \tag{3}$$

where $k_I^u(q), k_I^i(z)$ are Killing vectors of $\mathcal{M}_h$ and $\mathcal{M}_v$, respectively, and Λ^I, Θ^I are the respective gauge parameters. This in turn fixes the covariant derivatives to be

$$D_\mu q^u = \partial_\mu q^u + k_I^u A_\mu^I \;, \qquad D_\mu z^i = \partial_\mu z^i + k_I^u A_\mu^I \;. \tag{4}$$

In the following we are mainly interested in the case where $k_I^i = 0$ and for simplicity we focus on this situation henceforth.

On a quaternionic manifold the Killing equation $\nabla_u k_v + \nabla_v k_u = 0$ determines the Killing vectors in terms of a triplet of Killing prepotentials $P^x(q), x = 1, 2, 3$ [19]

$$k_I^u\, K_{uv}^x \;=\; D_v P_I^x \;, \tag{5}$$

where K^x_{uv} is the triplet of covariantly constant hyper-Kähler two-forms which exist on a quaternionic-Kähler manifold. They are related to the triplet of complex structures J^x (which satisfy the quaternionic algebra $J^x J^y = -\delta^{xy}\mathbf{1} + \epsilon^{xyz} J^z$) via

$$K^x_{uv} = h_{uw}(J^x)^w_v \ . \tag{6}$$

D_v in (5) is a covariant derivative with respect to the $Sp(1)$ connection of the holonomy group $Sp(1) \times Sp(2n_h)$.

$N = 2$ supersymmetry determines the Lagrangian and thus the interaction of the various multiplets [15–19]. Here we do not recall all the couplings but only focus on those terms which are relevant for our analysis. Apart from the σ-model terms (1) there is a set of mass-like terms for the fermions and the scalar fields interact via the potential (the conventions follow [15])

$$V = -12 S_{AB}\bar{S}^{BA} + g_{i\bar{j}} W^{iAB}\bar{W}^{\bar{j}}_{AB} + 2N^A_\alpha \bar{N}^\alpha_A \ , \tag{7}$$

where $S_{AB}, W^{iAB}, N^A_\alpha$ are scalar field dependent quantities given by

$$\begin{aligned} S_{AB} &= \frac{1}{2} e^{\frac{K}{2}} X^I P^x_I (\sigma^x \epsilon)_{AB} \ , \\ W^{iAB} &= i(\sigma^x \epsilon)^{AB} P^x_I g^{i\bar{j}} \bar{D}_{\bar{j}} \bar{X}^I \ , \\ N^A_\alpha &= 2 e^{\frac{K}{2}} U^A_{\alpha u} k^u_I \bar{X}^I \ . \end{aligned} \tag{8}$$

The $X^I(z)$ are holomorphic functions of the z^i and their covariant derivatives are defined as $D_i X^I = \partial_i X^I + (\partial_i K) X^I$ where K is the Kähler potential of $\mathcal{M}_v$, i.e. $g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K$. The $U^A_{\alpha u}$ is the 'vielbein' of the quaternionic metric $h_{uv} = U^A_{\alpha u} U^B_{\beta v} \epsilon_{AB} C^{\alpha\beta}$ with $C^{\alpha\beta}$ being the invariant $Sp(2n_h)$ matrix.

S_{AB} is also the mass matrix of the two gravitini and W^{iAB}, N^A_α are related to the mass matrix of the spin-1/2 fermions. Furthermore, these quantities appear in the supersymmetry transformations of the fermions

$$\begin{aligned} \delta\Psi_{\mu A} &= S_{AB}\gamma_\mu \epsilon^B + \dots \ , \\ \delta\lambda^{iA} &= W^{iAB}\epsilon_B + \dots \ , \\ \delta\zeta_\alpha &= N^A_\alpha \epsilon_A + \dots \ , \end{aligned} \tag{9}$$

where γ_μ are Dirac matrices and ϵ^A are the parameters of the two supersymmetry transformations.

3. Spontaneous $N = 2 \to N = 1$: the necessary conditions

We just sketched a generic $N = 2$ supersymmetric theory. It is of interest to understand under what conditions the potential V can have minima which preserve only $N = 1$ supersymmetry but not the full $N = 2$. So far there are only a few examples known where this situation is realized [4, 5].

The presence of an unbroken $N = 1$ supersymmetry amounts to the requirement that the supersymmetry transformations (9) of the fermions evaluated in the ground states vanish for one of the two supersymmetry transformations, say ϵ_2

$$\langle \delta\Psi_{\mu A}\rangle = \langle \delta\lambda^{iA}\rangle = \langle \delta\zeta_\alpha\rangle = 0 \; . \tag{10}$$

Here the bracket $\langle \; \rangle$ indicates that the fermion transformtions have to be evaluated in the ground states and for $\epsilon_1 = 0, \epsilon_2 \neq 0$. So the answer to the first question (i) of the introduction amounts to determining the properties that the couplings $S_{AB}, W^{iAB}, N_\alpha^A$ have to obey in order to yield a solution of (10). From (8) we see immediately that this is equivalent to geometrical conditions on the scalar manifold $\mathcal{M}$ and its (gauged) isometries.

The ground states can have various space-time properties, they can be flat Minkowski spaces, anti-de Sitter spaces or extended domain walls or p-brane solutions. For simplicity we impose in the following the additional requirement that the ground states spontaneously break $N = 2 \to N = 1$ in flat Minkowski space leaving the more general cases to a separate publication [14]. However, we do allow for the possibilty that there is a continuous family of ground states which have $N = 1$ supersymmetry. This means that there can be solutions of (10) which do depend on (some of) the scalar fields.

Solving (10) directly from the definitions (8) is not straightforward. Instead we can gain a little more insight by further using the fact that S_{AB} is also the mass matrix for the two gravitinos. A necessary condition for the existence of $N = 1$ ground states is that the two eigenvalues m_{Ψ^1}, m_{Ψ^2} of S_{AB} are non-degenerate, i.e. $m_{\Psi^1} \neq m_{\Psi^2}$. In Minkowski ground states one also needs $m_{\Psi^2} = 0$ or in other words one of the two gravitini has to become massive while the second one stays massless. Furthermore, the unbroken $N = 1$ supersymmetry implies that the massive gravitino has to be a member of an entire $N = 1$ massive spin-3/2 multiplet which has the spin content $s = (3/2, 1, 1, 1/2)$. This in turn requires that also two vectors, say A_μ^0, A_μ^1 and a spin-1/2 fermion χ have to become massive or equivalently there have to be two massless gauge bosons together with two Goldstone bosons, a Goldstone fermion and a massive fermion [5]. The Goldstone bosons have to be 'recruited' out of a hypermultiplet while the two gauge bosons require at least one vector multiplet. Thus, the minimal $N = 2$ spectrum which allows the possibility of a spontaneous breaking to $N = 1$ consists of the $N = 2$ supergravity multiplet, one hypermultiplet and one vector multiplet.

Since our analysis assumes Minkowskian ground states the unbroken $N=1$ supersymmtry implies that the spin-3/2 multiplet has to be degenerate in mass, i.e.

$$m_{\Psi^1}=m_{A^0}=m_{A^1}=m_\chi\equiv m\ ,\qquad m_{\Psi^2}=0\ . \tag{11}$$

As we already said the fermionic mass matrices are directly determined by the $S_{AB}, W^{iAB}, N_\alpha^A$ defined in (8). On the other hand the gauge bosons obtain their mass via a Higgs mechanism. This means that among the hypermultiplet scalars there have to be two Goldstone bosons η^1,η^2 with couplings

$$D_\mu\eta^1=\partial_\mu\eta^1+e_0A_\mu^0\ ,\qquad D_\mu\eta^2=\partial_\mu\eta^2+e_1A_\mu^1\ , \tag{12}$$

where e_0,e_1 are constant charges and we have arbitrarily chosen $I=0,1$ as the massive gauge bosons. In geometrical terms this means that $\mathcal{M}_h$ has to admit two commuting, translational $\mathbb{R}^2$-isometries and these isometries have to be gauged [5]. In this case the σ-model interactions (1) imply a set of mass terms

$$\mathcal{L}=-\frac{1}{2}m_{IJ}^2A_\mu^IA_\mu^J+\dots \tag{13}$$

where

$$\frac{1}{2}m_{IJ}^2=k_I^uh_{uv}k_J^v\ ,\qquad k_I^u=e_0\,\delta_{I0}\delta^{u1}+e_1\,\delta_{I1}\delta^{u2}\ . \tag{14}$$

The constraint (11) implies that the two Killing vectors k_0^u,k_1^u have to be orthonormal, i.e. satisfy

$$k_0^uh_{uv}k_0^v=k_1^uh_{uv}k_1^v\equiv\frac{1}{2}m^2\ ,\qquad k_0^uh_{uv}k_1^v=0\ . \tag{15}$$

We just established the fact that we need two orthonormal Killing vector and thus via equation (5) we have two Killing prepotentials P_0^x,P_1^x which generically span a plane in $SU(2)$. Thus, without loss of generality we can always choose an SU(2) basis where $P_0^3=P_1^3=0$. This choice fixes an $SU(2)$ gauge and leaves a $U(1)$ rotation intact. In this basis S_{AB} is diagonal and given by

$$S_{AB}=\frac{1}{2}\begin{pmatrix} m_{\Psi^1} & 0\\ 0 & m_{\Psi^2}\end{pmatrix}\ , \tag{16}$$

where

$$m_{\Psi^1}=e^{\frac{K}{2}}X^I(P_I^1-iP_I^2)\ ,\qquad m_{\Psi^2}=-e^{\frac{K}{2}}X^I(P_I^1+iP_I^2)\ . \tag{17}$$

Since the P_I are real we see immediately that $m_{\Psi^2}=0$ cannot be satisfied when the X^I are linearly independent [4,10]. This implies that on $\mathcal{M}_v$ one has to choose a particular basis of gauge fields where the couplings (12) are realized. Leaving the detailed analysis to ref. [14] let us just state that indeed such a basis exists for a large class of special Kähler manifolds $\mathcal{M}_v$ [4, 5, 10, 20] and that

$$\mathcal{M}_v = \frac{SU(1,1)}{U(1)} \ , \qquad \text{with} \qquad X^0 = -\frac{1}{2} \ , \quad X^1 = \frac{i}{2} \ , \tag{18}$$

is a representive choice [4]. (Other manifolds can be found in [5, 10].) For (18) the constraint $m_{\Psi^2} = 0$ is solved by

$$P_0^2 = P_1^1 \ , \qquad P_1^2 = -P_0^1 \ . \tag{19}$$

This is a strong constraint on the scalar manifold $\mathcal{M}_h$ and its gauged isometries and generically defines a subspace of $\mathcal{M}_h$. On this subspace one shows that (11) implies

$$V| \equiv 0 \ , \tag{20}$$

where the $|$ indicates that the potential is evaluated on the subspace where (19) holds.

Before we turn our attention to question (ii) let us summarize the necessary conditions found so far. The possibilty of $N = 1$ ground states is equivalent to the existence of solutions of $\langle \delta\Psi_{\mu A}\rangle = \langle \delta\lambda^{iA}\rangle = \langle \delta\zeta_\alpha\rangle = 0$. By relating it to properties of a massive $N = 1$ spin-3/2 multiplets we were able to rephrase this as conditions on the scalar manifold $\mathcal{M}$. In particular on $\mathcal{M}_v$ one has to choose a basis where the X^I are linearly dependent while $\mathcal{M}_h$ has to admit two commuting, orthonormal, translational $\mathbb{R}^2$-isometries which additionally obey (19) and (11). It would be interesting to rephrase the constraints (19), (11) in a more geometrically language and to determine the manifolds $\mathcal{M}$ which satisfy all these constraints.

4. The low energy effective $N = 1$ theory

Let us now turn to question (ii) of the introduction and focus on the properties of the low energy effective $N = 1$ theory which is valid well below the scale of the supersymmetry breaking set by m_{Ψ^1}. The Lagrangian of this effective theory can be derived by 'integrating out' the massive gravitino multiplet together with other ($N = 1$) multiplets which might have acquired a mass of the same order. At the two derivative level this is achieved by using the equation of motions of the massive fields to first non-trivial order in p/m_{Ψ^1} where $p \ll m_{\Psi^1}$ is a characteristic momentum. For the fermions and scalars this is a straightforward procedure in that they are simply set to zero in the $N = 2$ Lagrangian. This in turn truncates the scalar manifold to a subspace spanned by the left over massless states. For the spin-1 gauge bosons the situation is slightly more complicated. Due to their couplings to the Goldstone bosons (12) eliminating $A_\mu^{0,1}$ also elimates the two Goldstone bosons and furthermore changes the σ-model interactions of the remaining scalar fields.

This amounts to taking the quotient of $\mathcal{M}_h$ with respect to the two translational $\mathbb{R}^2$-isometries [21].[2]

The effective theory contains the left over massless $N=1$ multiplets which include a gravity multiplet, n_v' vector multiplets and n_c chiral multiplets. In addition, the effective theory has to be manifestly $N=1$ supersymmetric. This implies in particular that the scalar manifold is Kähler, the gauge coupling functions $f(z)$ are holomorphic and the potential is expressed in terms of a holomorphic superpotential W. This imposes a set of conditions implied by the consistency of the integrating out procedure and the following (non-trivial) facts have to hold

(a) Quaternionic-Kähler manifolds which admit $\mathbb{R}^2$-isometries of the type specified in section 3 have a quotient $\mathcal{M}_h/\mathbb{R}^2$ which is Kähler (with Kähler potential K_h).[3]

(b) The inverse gauge couplings g of the gauge bosons are harmonic
$$g^{-2} = f(z) + \bar{f}(\bar{z}) \ .$$

(c) The $N=1$ potential obeys
$$V^{N=1} = e^{K_v+K_h}\left(g^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} + g^{u\bar{v}} D_u W \bar{D}_{\bar{v}} \bar{W} - 3|W|^2\right) \ ,$$
where W is holomorphic and
$$D_i W = \partial_i W + (\partial_i K_v) W \ , \qquad D_u W = \partial_u W + (\partial_u K_h) W \ .$$
($g_{u\bar{v}}$ denotes the Kähler metric on the quotient $\mathcal{M}_h/\mathbb{R}^2$.)

A generic $N=2$ theory does not satisfy (a)–(c) but supersymmetry imposes these conditions on the low energy effective $N=1$ theory. The fact that we have chosen to consider an $N=2$ spectrum with only one vector multiplet immediately implies that the low energy $N=1$ theory contains no vector multiplets and hence (b) is trivially satisfied. Furthermore for Minkowskian ground states the $N=1$ gravitino Ψ_μ^2 is exactly massless which implies $W = V^{N=1} \equiv 0$ and hence also (c) is satisfied. Thus, for the case at hand the only non-trivial constraint left to check is condition (a).

Eliminating the two massive gauge bosons via their equations of motions results in σ-model type couplings in the effective $N=1$ Lagrangian which are as in eq. (1) but with h_{uv} replaced by the metric $\hat{h}_{uv}$ on the quotient given by
$$\hat{h}_{uv} = h_{uv} - \frac{2}{m^2}\left(k_{0u}k_{0v} + k_{1u}k_{1v}\right) \ , \qquad k_{Iu} \equiv k_I^w h_{wu} \ . \tag{21}$$

[2] We thank G. Horowitz for reminding us of this fact.

[3] The scalar manifold for the vector multiplets is already Kähler so that no new constraint arises for $\mathcal{M}_v$.

$\hat{h}_{uv}$ satisfies

$$\hat{h}_{uv} k_I^v = 0 \ , \qquad \hat{h}_{uv} h^{vw} \hat{h}_{wr} = \hat{h}_{ur} \ , \tag{22}$$

where $h^{vw} h_{wu} = \delta_u^v$. Thus $\hat{h}_{uv}$ has two null directions and h^{vw} is the inverse metric.

Among the three hyper-Kähler two-forms K_{uv}^3 plays a preferred role in that it points in the direction (in $SU(2)$-space) normal to the plane spanned by P_0^x, P_1^x. By using a two-dimensional σ-model one can compute the two-form which decends from K_{wu}^3 to the quotient to be[4]

$$\hat{K}_{uv} = K_{uv}^3 - \frac{1}{k}\left(k_0^w K_{wu}^3 k_1^w K_{wv}^3 - k_1^w K_{wu}^3 k_0^w K_{wv}^3\right) \ , \tag{23}$$

where $k \equiv k_0^w K_{wu}^3 k_1^u$. From (10) one derives

$$k_0^w K_{wu}^3 = -k_{1u} \ , \qquad k_1^w K_{wu}^3 = k_{0u} \ . \tag{24}$$

which in turn can be used to show[5]

$$d\hat{K} = 0 \ , \qquad \hat{J}^2 = -\mathbf{1} \ , \tag{25}$$

where $\hat{K}_{uv} = \hat{h}_{uw} \hat{J}_v^w$. This proves that the quotient is indeed a Kähler manifold with Kähler form $\hat{K}$ and complex structure $\hat{J}$.[6] Hence the consistency condition (a) is satisfied.

Let us close by summarizing the properties of the Kähler manifold just constructed. We started from a quaternionic manifold $\mathcal{M}_h$ which admits two orthonormal Killing vectors of an $\mathbb{R}^2$-isometry. We showed that if in addition (24) holds the quotient manifold $\mathcal{M}_h/\mathbb{R}^2$ is Kähler. It would be interesting to determine the quaternionic geometries which do satisfy (24) and thus (25).

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[4]We thank E. Zaslow for suggesting this procedure.

[5]The details of the computation will be presented in [14]. The proof of (25) does not require the ground state to be Minkowskian.

[6]It is tempting to conjecture that this Kähler manifold is somehow related to the twistor space [21, 22]. We thank P. Aspinwall, S. Vandoren and B. de Wit for discussions related to this conjecture.

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