FOURTH-GRADE STUDENTS' PERFORMANCE ON GRAPHICAL LANGUAGES IN MATHEMATICS

Tom Lowrie

Charles Sturt University,

AUSTRALIA

Carmel Diezmann

Queensland University of Technology, AUSTRALIA

This study investigated the performance of 217 fourth-grade students (9 or 10 years) on a 36-item test that comprised items from six distinct graphical languages (e.g., maps) that are commonly used to convey mathematical information. The results of the study revealed that: fourth graders have difficulty decoding a variety of graphics; some graphical languages are more difficult for students to access than others; only one of the graphic languages revealed gender differences; and there were positive correlations in performance between all pairings of graphic languages. The implications of this study include the need to support the development of students' ability to decode graphics beyond activities usually investigated in mathematics curricula.

The importance of representation in teaching, learning and understanding mathematics is widely acknowledged (e.g., Cucuo & Curcio, 2001). However, although our society utilises a vast array of "information graphics" (e.g., graphs, diagrams, charts, tables, maps) for the management, communication, and analysis of information (Harris, 1996), there has been scant attention to the interrelationship between numeracy and representation (Pugalee, 1999). This relationship involves the ability to decode mathematical information from graphics and encode mathematical information into graphics (Baker, Corbett, & Koedinger, 2001). This paper focuses on primary students' ability to decode the embedded mathematical information in graphics. These students need to become "code-breakers" in order to access the mathematical information in graphics that are employed in tasks, texts, tests, software, and other everyday situations.

INFORMATION GRAPHICS

Information graphics convey quantitative, ordinal and nominal information through a range of perceptual elements (Mackinlay, 1999). These elements are position, length, angle, slope, area, volume, density, colour saturation, colour hue, texture, connection, containment, and shape (Cleveland & McGill, 1984). Although there are thousands of graphics in use, they can be categorised into six "graphic languages" that link the perceptual elements via particular encoding techniques (Mackinlay, 1999) (see Table 1). An example item for each of these graphic languages is included in Appendix A.

Students' performance in decoding information graphics is likely to be influenced by their age and the particular types of graphics in use, and their visual-spatial abilities. Each of these influences is briefly discussed.

Graphical Languages	Examples	Encoding Technique
Axis Languages	Horizontal and vertical axes	A single-position encodes information by the placement of a mark on an axis.
Opposed-position Languages	Line chart, bar chart, plot chart	Information is encoded by a marked set that is positioned between two axes.
Retinal-list Languages	Graphics featuring colour, shape, size, saturation, texture, orientation	Retinal properties are used to encode information. These marks are not dependent on position.
Map Languages	Road map, topographic map	Information is encoded through the spatial location of the marks.
Connection Languages	Tree, acyclic graph, network	Information is encoded by a set of node objects with a set of link objects.
Miscellaneous Languages	Pie chart, Venn diagram	Information is encoded with additional graphical techniques (e.g., angle, containment).

Table 1: Overview of six Graphical Languages.

Age and Types of Graphics

Some students find some graphics difficult to decode. For example, on the National Assessment of Educational Progress [NAEP], many fourth graders had difficulty reasoning from a bar graph (National Center for Education Statistics [NCES], n.d., 1992-4M7-03, 1992-8M7-03) and using a scale (NCES, n.d., 2003-4M6-19, 2003-8M6-18 (see Table 2). These students' success on the scale item was no better than chance accuracy (1 out of 4, 25%). Although eighth graders outperformed fourth graders' on both these items, many older students' also had difficulty with graphics (see Table 2). The performance differences between the bar graph item (Opposed-position) and the scale item (Axis) at each grade level indicate variance in the difficulty level of particular graphical languages. Differences in the relative difficulties of some graphics and increased success with age were also reported by Wainer (1980), however, that study investigated limited graphics.

A plausible reason for differences in students' success on particular items and at different grade levels is their knowledge of the embedded mathematics content. However, the particular graphic used to represent information is a major factor in students' success. Baker et al. (2001) reported substantial variance in eighth- and ninth-grade students' ability to interpret informationally equivalent graphics with students' comparative success rates of 95% on a histogram, 56% on a scatterplot, and 17% on a stem-and-leaf plot. They argued that this performance variance was due to students' transfer of knowledge about bar graphs to the other three graphics, and that

although histograms and scatterplots share surface features with bar graphs, stem-and-leaf plot vary at the surface level from bar graphs.

NAEP Item Description	Graphical Languages	Grade 4	Grade 8	Performance Difference
Reason from a bar graph	Opposed-position	49% (n = 1622)	62% (n = 1759)	13%
Use a scale to find a distance between two points	Axis	24% (n = 36764)	39% (n = 30578)	15%

Table 2: Two graphic items from NAEPs Assessment for Grades 4 and 8.

Visual-Spatial Abilities

Decoding information graphics involves the interpretation of information presented in a visual-spatial format. Hence, it draws on spatial ability, which is a composite of abilities rather than a unitary construct and includes mental rotation, spatial perception, and spatial visualisation (Voyer, Voyer, & Bryden, 1995). Some students may be predisposed to high or low performance on decoding tasks. For example, students who have high spatial ability may decode graphics with relative ease due to their enhanced ability to process visual information (e.g., Raven, 1998). By contrast, students who have visual perception or processing problems may experience particular difficulties decoding graphics (e.g., Zangemeister & Steihl, 1995). Although gender and spatial ability has been the subject of much research, a meta-analysis revealed that the only gender difference in under 13-year-old students is limited to performance on mental rotation tests, which favour males (Voyer et al., 1995). Mental rotation tasks focus on orientation, which is included in the Retinal-list languages (see Table 1).

METHODOLOGY

This investigation is part of a 4-year longitudinal study that is designed to enhance our understanding of the development of primary students' ability to decode information graphics that represent mathematical information. Here, we report on the first stage of our study, which was:

1. To document fourth-grade students' knowledge of particular graphical languages in mathematics;

2. To establish whether there are gender differences in students' decoding performance in relation to the six graphical languages; and

3. To determine the relationships between decoding performance across the six graphical languages.

The Instrument

The Graphical Languages in Mathematics [GLIM] Test is a 36-item test that was developed to determine students' decoding performance for each of the six graphical languages (see Appendix A for an example of each of the six graphical languages items.). A bank of 58 items was variously trialled with primary-aged children (N = 796) in order to select items that: (a) varied in complexity; (b) required substantial levels of graphical interpretation; and (c) conformed to reliability and validity measures. The items were selected from state and national year-level mathematics tests that have been administered to students in their final three years of primary school or to similarly aged students (e.g., QSCC, 2000).

The Participants

The participants (n = 217) were randomly selected from six primary schools in a large rural city in Australia. Fourth-grade students (aged 9 or 10) were selected for investigation because other aspects of this study will monitor these students' decoding performance over the last three years of their primary schooling.

RESULTS AND DISCUSSION

The first aim of the investigation was to document the participants' knowledge of particular graphical languages in mathematics. As there were six items in every category, "6" is the maximum score. As shown on Table 3, the fourth-grade students were more successful in completing the Miscellaneous [$\overline{X}$ = 4.24, S.D.= 1.39] and Map [$\overline{X}$ = 4.06, S.D.= 1.29] languages than the other four categories of graphics. Interestingly, these two types of graphical languages are explicitly taught in key learning areas outside the mathematics syllabus. By contrast, the mean score for the Opposed-position category was considerably lower (15% less than the Miscellaneous category) despite the concentration of activities involving line charts, bar graphs and histograms in the curriculum and in state numeracy tests (e.g., QSCC, 2000). These results indicate differences in the difficulty level of various graphical languages for fourth-grade students. Although some of this variance could be attributed to content, the particular graphical languages used are likely to be a major factor influencing students' performance (Baker et al., 2001).

The second aim of this investigation was to establish whether there were gender differences in students' decoding performance in relation to the six graphical languages. The mean scores for the male fourth-grade students were higher than that of the female students in all six categories. T-tests were conducted to determine whether there were statistically significant differences between the performances of males and females across the six graphical language categories. There were no statistically significant differences across the gender variable for five out of the six categories [*Opposed-position* (t = .001, p = .98); *Retinal-list* (t = .84, p = .36); *Map* (t = 2.71, p = .10); *Connection* (t = 1.28, p = .26); and *Miscellaneous* (t = .15, p = .69)]. There was, however, a statistically significant difference between the mean scores of

the male and the female students in relation to the Axis graphical languages [*Axis* (t = 12.2, p ≤ .001)]. A gender difference for Axis languages was unanticipated because no mental rotation was required and the students were under 13 (Voyer et al., 1995). However, these results are consistent with Hannula's (2003) findings of gender differences on a number line task (Axis) in favour of boys for fifth-grade (n = 1154) and seventh-grade Finnish students (n = 1525). Hannula's explanation that gender differences appeared to occur on tasks that were more difficult for students is inadequate for our study because Axis items were the third easiest of the six graphical languages (see Table 3).

Graphical Languages	Total	Male (n = 115)	Female (n = 102)
Miscellaneous	4.24 *(1.39)*	4.27 *(1.39)*	4.20 *(1.38)*
Map	4.06 *(1.29)*	4.20 *(1.28)*	3.91 *(1.29)*
Axis	3.79 *(1.19)*	4.04 *(1.13)*	3.50 *(1.20)*
Opposed-position	3.26 *(1.71)*	3.27 *(1.30)*	3.26 *(1.02)*
Connection	3.13 *(1.28)*	3.32 *(1.32)*	3.03 *(1.23)*
Retinal-list	2.95 *(1.38)*	3.03 *(1.41)*	2.86 *(1.34)*

Table 3: Means (and Standard Deviations) of the six Graphical Languages.

The third aim of the investigation was to determine relationships between students' decoding performance across the six graphical languages. The six graphical languages were all positively correlated with each other (see Table 4). With the exception of the Axis-Opposed-position correlation (r = .15, p ≤ .05), all correlations were statistically significant at a p ≤ .01 level. Nevertheless, even the strongest relationships [Connections-Maps (r = .39, p ≤ .01) and Miscellaneous-Connections (r = .41, p ≤ .01)] were only moderately correlated with each other despite the strong statistical significance. The Connections-Map and Miscellaneous-Connections correlations accounted for approximately 16% of the variance. The Miscellaneous and Connection categories had the strongest correlations with the other graphical language (with all correlations ≤ 0.27).

By contrast, the correlations between the Retinal-list languages category and other language categories were somewhat weaker (most correlations less than 0.30). In most cases, the Retinal-list items required the participants to consider graphical features including shape, size, saturation, texture, and orientation. Thus, decoding these graphics required an understanding of the use of perceptual elements to convey mathematical information. The weakest correlation was between the Axis and Opposed-position languages. Although both of these languages use axes, they differ substantially at a structural level with information encoded in only one dimension in Axis languages and in two dimensions in Opposed-positional languages (Mackinlay, 1999).

Graphical Languages	Axis	Opposed-position	Retinal-list	Map	Connection
Axis					
Opposed-position	.15*				
Retinal-list	.26**	.19**			
Map	.33**	.24**	.29**		
Connection	.32**	.31**	.27**	.39**	
Miscellaneous	.37**	.34**	.32**	.31**	.41**

* p = .05 level   ** p = .01 level

Table 4: Correlations among the six Graphical Languages.

CONCLUSIONS AND IMPLICATIONS

The ability to decode information graphics is fundamental to numeracy. However, the results of this study revealed that many fourth-grade students have difficulty decoding the graphics used in each of the graphical languages and that some languages are more difficult for students than others. It is also noteworthy that boys outperformed girls in each of the graphical languages (with a significant gender difference for Retinal-list languages). There were positive correlations among the six graphical languages although most significant relationships were only moderately correlated.

The results of this study indicate five educational implications.
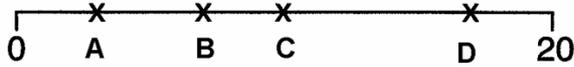
1. Teachers need to be proactive in supporting the development of students' ability to decode information graphics. However, the provision of appropriate support may be challenging for teachers because they have difficulty identifying which types of graphics are easier or harder for students (Baker et al., 2001).

2. Due to gender differences in performances on Retinal-list languages, girls should receive strategic support in activities incorporating these languages.

3. Learning opportunities should be broad and include graphical languages that are typically used outside formal mathematics contexts (i.e., Maps, Connections, Miscellaneous, Retinal-list) in addition to those explicitly incorporated into the mathematics syllabus (Axis, Opposed-position).

4. Where appropriate, explicit links between graphical languages should be made to facilitate cognitive transfer.

5. The informational content of graphics used for instructional purposes should be explicated to ensure that all students have access to embedded information.

References
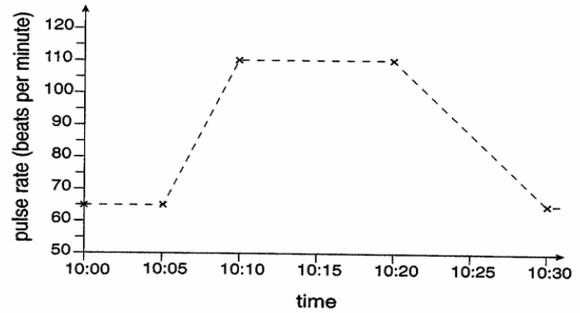
Baker, R. S., Corbett, A. T., Koedinger, K. R. (2001). Toward a model of learning data representations. In *Proceedings of the 23rd Annual Conference of the Cognitive Science Society*, (pp. 45-50). Mahwah, NJ: Erlbaum.

Cleveland, W. S., & McGill, R. (1984). Graphical perception: Theory, experimentation, and application to the development of graphical methods. *Journal of the American Statistical Association, 79*(387), 531-554.

Cucuo, A. A., & Curcio, F. (2001). *The role of representation in school mathematics: National Council of Teachers of Mathematics Yearbook.* Reston, VA: NCTM.

Educational Testing Centre (2001). *Australian Schools Science Competition Year 6.* Sydney: University of New South Wales.

Educational Testing Centre (1999). *Australian Schools Science Competition Year 5.* Sydney: University of New South Wales.

Hannula, M. S. (2003). Locating fractions on a number line. In N A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA* (Vol. 3, pp. 3-24). Honolulu, USA: PME.

Harris, R. L. (1996). *Information graphics: A comprehensive illustrated reference.* Atlanta, GA: Management Graphics.

Mackinlay, J. (1999). Automating the design of graphical presentations of relational information. In S. K. Card, J. D. Mackinlay, & B. Schneiderman (Eds.), *Readings in information visualization: Using vision to think* (pp. 66-81). San Francisco, CA: Morgan Kaufmann.

National Center for Educational Statistics (n.d.). *NAEP questions.* Retrieved January 10, 2003, from http://nces.ed.gov/nationsreportcard/itmrls/search.asp

Pugalee; D. K. (1999). Constructing a model of mathematical literacy. *The Clearing House, 73*(1), 19-22.

QSCC (2000). *2000 Queensland Year 3 test: Aspects of Numeracy.* VIC: ACER.

QSCC (2001a). *2001 Queensland Year 3 test: Aspects of Numeracy.* VIC: ACER.

QSCC (2001b). *2001 Queensland Year 5 test: Aspects of Numeracy.* VIC: ACER.

QSCC (2002). *200 Queensland Year 3 test: Aspects of Numeracy.* VIC: ACER.

Raven, J. C. (1998). *Raven's progressive matrices.* London: Oxford Psychologists.

Voyer D, Voyer S, & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: a meta-analysis and consideration of critical variables. *Psychological Bulletin, 117*(2), 250-270.

Wainer, H. (1980). A test of graphicacy in children. *Applied Psychological Measurement, 4*, 331-340.

Zangemeister, W. H., & Steihl, H. S. (1995). *Visual attention and cognition.* Amsterdam: Elsevier.

APPENDIX A: EXAMPLES OF SIX GRAPHICAL LANGUAGES ITEMS

Estimate where you think 17 should go on this number line.
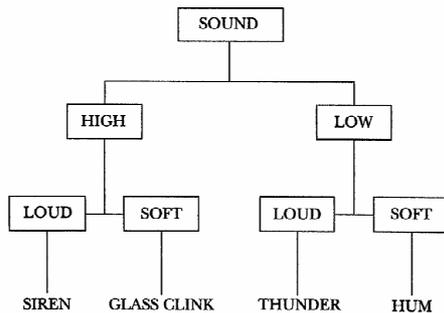
Axis Item (Adapted from QSCC, 2000, p. 11)

This graph shows how Sam's pulse rate changed while she exercised. What is the difference between Sam's lowest and highest pulse rate in beats per minute?

Opposed-position Item (Educational Testing Centre, 1999, p. 8)
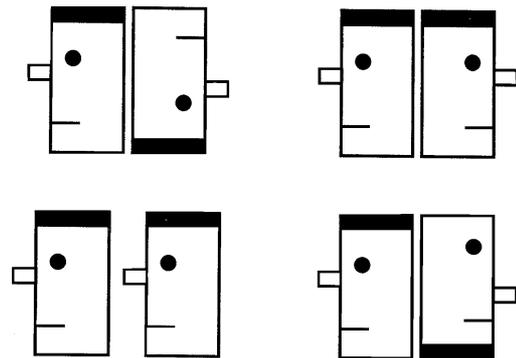
This flowchart shows a way to describe sounds.

Which of the following describes a 'hum'?

Connection Item (Educational Testing Centre, 2001, p. 2)

Which two faces show a flip?

Retinal-list Item (Adapted from QSCC, 2001a, p. 13)

Deb rides her bike along the bike track. What part of the park won't she ride through?

Map Item (QSCC, 2001b, p. 16)

Which date is 3 weeks before 29 May?

MAY 2003

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Miscellaneous Item (QSCC, 2002, p. 9)