STABILITY AND STABILIZATION OF A ACTIVE SURFACE.

Vladimir V. Alexandrov FCFM - BUAP
and University M.V. Lomonosov, Moscú.
Humberto Salazar Ibargüen FCFM - BUAP.
Francisco Guerra Vázquez - INAOE.
Filadelfo Aguilar Andrade FCFM - BUAP.

The construction of the Large Millimeter Telescope (LMT) has phrased the necessity for solving some theorical and practical problems in Physics and Mathematics, such as the stability and stabilization of the active surface. The dimensions of the LMT and its 126 segments, makes very difficult to control the movement of all the segments at once. For this reason we start studing the movement of just one piece of the surface, then we have that to propouse mathematics models to solve some problems, such as:

1.- The stability of the ideal position of one segment.

2.- The stability of the ideal position of one ring with six segments.

3.- The stability of the ideal position of a part of the active surface with seven segments.

In this talk we will show the following progress related with the first task.

a) The absolute motion and complete controllability of one segment in its proper vertical plane.

b) The absolute motion and complete controllability of one segment in the space.

MOTION OF ONE SEGMENT AS CONTROLABLE DYNAMICS SYSTEM

We make the followings assumptions:

a) The segment is a rigid body, with a aproximately mass of 275 Kgs.

b) The actuators have three spherical charnelas and three cilyndrical charnelas.

c)The mass of the actuators is neglected.

The motion equations for the segment are in terms of two systems of coordinates (x,y,z) and $(\tilde{x}, \tilde{y}, \tilde{z})$.

The relation betwen this two systems is given by:

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \Theta \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Where Θ is the rotation matrix:

$$\begin{pmatrix} Cos\phi Cos\psi - Sin\phi Sin\psi Sin\gamma & Sin\phi Cos\psi + Cos\phi Sin\psi Sin\gamma & -Cos\gamma Sin\psi \\ -Sin\phi Cos\gamma & Cos\phi Cos\gamma & Sin\gamma \\ Cos\phi Sin\psi + Sin\phi Sin\gamma Cos\psi & Sin\phi Sin\psi - Cos\phi Sin\gamma Cos\psi & Cos\gamma Cos\psi \end{pmatrix}$$

We take the generalized coordinates:

$$(x_c, y_c, z_c, \phi, \gamma, \psi)$$

The motion equations of the mass center have the following form:

$$m\ddot{x}_c = \sum_{i=1}^{3} F_i \frac{\Delta x_i}{l_i}$$

$$m\ddot{y}_c = \sum_{i=1}^{3} F_i \frac{\Delta y_i}{l_i} - mg \tag{1}$$

$$m\ddot{z}_c = \sum_{i=1}^{3} F_i \frac{\Delta z_i}{l_i}$$

Where $l_i^2 = \Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2$ and the relation betwen the distinct coordinate is:

$$\begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} + \Theta^{\top} P_i - \begin{pmatrix} x_{m_i} \\ y_{m_i} \\ z_{m_i} \end{pmatrix}$$

$$P_i = \begin{pmatrix} \tilde{x_{p_i}} \\ \tilde{y_{p_i}} \\ \tilde{z_{p_i}} \end{pmatrix}$$

In the above relation, we fixed the values of the $x_{m_i}, y_{m_i}, z_{m_i}, \tilde{x_{p_i}}, \tilde{y_{p_i}}, \tilde{z_{p_i}}$ coordinates and the F_i are the control forces of the actuators.

The relative motion equations for a rigid body with respect to the center mass have the form:

$$\frac{d}{dt}\begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} + \Omega \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} M_{\tilde{x}} \\ M_{\tilde{y}} \\ M_{\tilde{z}} \end{pmatrix} \qquad (2)$$

where:

$$K_1 = I_{11}p - I_{12}q - I_{13}r$$

$$K_2 = -I_{21}p + I_{22}q - I_{23}r$$

$$K_3 = -I_{31}p - I_{32}q - I_{33}r$$

Where I_{ij} are the inertial moments.

$\omega = (p, q, r)$ is the angular velocity with its own proyections over the axes $(\tilde{x}, \tilde{y}, \tilde{z})$.

The angular velocity components are given by:

$$p = \dot{\gamma} Cos\psi - \dot{\phi} Cos\gamma Sin\psi$$

$$q = \dot{\psi} + \dot{\phi} Sin\gamma \qquad (3)$$

$$r = \dot{\gamma} Sin\psi + \dot{\phi} Cos\gamma Cos\psi$$

The matrices $\Omega and M$ are defined as:

$$\Omega = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}$$

$$\begin{pmatrix} M_{\tilde{x}} \\ M_{\tilde{y}} \\ M_{\tilde{z}} \end{pmatrix} = \sum_{i=1}^{3} \begin{pmatrix} 0 & -\tilde{z_{p_i}} & \tilde{y_{p_i}} \\ \tilde{z_{p_i}} & 0 & -\tilde{x_{p_i}} \\ -\tilde{y_{p_i}} & \tilde{x_{p_i}} & 0 \end{pmatrix} \frac{F_i}{l_i}\Theta \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix}$$

For the particular case when the motion is in the vertical plane (x, y) the equations $(1), (2)$ and (3) take the form:

$$m\ddot{x}_c = F_1 \frac{\Delta x_1}{l_1} + 2F_{2(3)} \frac{\Delta x_2}{l_2}$$

$$m\ddot{y}_c = F_1 \frac{\Delta y_1}{l_1} + 2F_{1(3)} \frac{\Delta y_2}{l_2} - mg$$

$$I_3\ddot{\phi} = \frac{F_1}{l_1}\tilde{a}(-Sin\phi\Delta x_1 + Cos\phi\Delta y_1) + 2\frac{F_2}{l_2}\tilde{c}(Sin\phi\Delta x_2 - Cos\phi\Delta y_2)$$

The last equation is obtained from a Lagrangian:

$$L = T - V$$

where T is the kinetic energy and V is the potential energy, and the Lagranges equations are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

where the Q_i are generalized forces.

The positions are given now by:

$$\Delta x_1 = x_c + Cos\phi\tilde{a} - a$$

$$\Delta x_2 = x_c - Cos\phi\tilde{c} + c$$

$$\Delta y_1 = y_c + Sin\phi\tilde{a}$$

$$\Delta y_2 = y_c + Sin\phi\tilde{c}$$

We define the ideal position of the segment as:

$$x_c \equiv 0$$

$$y_c \equiv h > 0$$

$$\phi \equiv 0$$

So we find the solution represented by the following system equations:

$$0 = f_1(F_1, F_2, h)$$

$$0 = f_2(F_1, F_2, h)$$

$$0 = f_3(F_1, F_2, h)$$

We proved that the solution of this system equations, is independent of the value of the parameter h.

$$h_{min} \leq h \leq h_{max}.$$

LINEAR AND STATIONARY MODEL OF THE ESTABILIZATION FOR THE IDEAL POSITION OF A SEGMENT.

Now, we are going to use the motion equations of the segment; to make an algorithm for the stability and stabilization, such that we like that the motion is around of a desirable position, then we write the equations in varations of the controllable dynamic system:

$$\ddot{q} - Cq = B\Delta p$$

The equation of the actuators, $j = 1, 2, 3$ is:

$$\tau\Delta\dot{p}_j + a_0\Delta p_j = r_j^\top q + s_j^\top \dot{q} + \Delta u_j$$

We use a system of linear Feedback Control.

$$\Delta u = K\tilde{x}$$

And the sensors are given by:

$$z = Hq + \eta(t)$$

The desviation of the positions is represented by:

$$q^\top = (\Delta x, \Delta y, \Delta z, \Delta\phi, \Delta\gamma, \Delta\psi)$$

And in the ideal position they are reduced to:

$$\Delta x = x_c - 0, \Delta y = y_c - h, \Delta z = z_c - 0, \Delta\phi = \phi - 0, \Delta\gamma = \gamma - 0, \Delta\psi = \psi - 0$$

Now the aditional pressures of the actuators is given by:

$$\Delta p^\top = (\Delta p_1, \Delta p_2, \Delta p_3)^\top$$

and the aditional controls represented by:

$$\Delta u^\top = (\Delta u_1, \Delta u_2, \Delta u_3)$$

The main objetive in this part, is to find out an algorithm, which could describe the motions of segments around of a desirable position. With this in mind we rewrite the motion equations in variations of the controlable dynamic system.

The stimations for the desviations for $\tau = 0$, and $a_0 = 1$ are represented by:

$$\tilde{x}^\top = (\tilde{q}^\top, \tilde{\dot{q}}^\top)$$

and the sensors signals of a segment:

$$z^\top = (z_1, z_2, z_3, z_4, z_5, z_6)$$

and the constant matrices:

$$R_{3*6} = \begin{pmatrix} r_1^\top \\ r_2^\top \\ r_3^\top \end{pmatrix}$$

$$S_{3*6} = \begin{pmatrix} s_1^\top \\ s_2^\top \\ s_3^\top \end{pmatrix}$$

$$C_{6*6}, B_{6*3}, O^1_{6*6}, O^2_{6*3}, H_{6*6}, K_{3*12}, \tilde{K}_{12*6}$$

and the error in the sensors are represented by:

$$\eta^\top = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6)$$

To make out the stabilization, we have to solve three asociated problems:

1.-Complete controlability of two matrices:

$$\begin{pmatrix} 0 & E \\ C+BR & BS \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ B \end{pmatrix}$$

2.- Complete observability of two matrices

$$\begin{pmatrix} 0 & E \\ C+BR & BS \end{pmatrix}$$

and

$$H$$

3.- Stabilization:

a) ASYMPTOTIC: $(Re(\lambda_i) < 0, i = 1, ..., 24) \Rightarrow$ we have to find the values for K and $\tilde{K}$

b) OPTIMAL: $(max_{|q(0)| \leq 1} \int_0^\infty q^\top E q dt \rightarrow min K, \tilde{K} \in (Re(\lambda_i < -\alpha_0))$

The complete linear model is described by:

$$\ddot{q} - Cq = B\Delta p$$

$$q(0) \neq 0; \dot{q}(0) \neq 0;$$

$$\tau\Delta\dot{p}_j + a_0\Delta p_j = r_j^\top q + s_j^\top \dot{q} + \Delta u_j$$

$$\Delta p(0) \neq 0$$

$$j = 1, 2, 3$$

$$\Delta u = K\tilde{x}$$

$$\dot{\tilde{x}} = \begin{pmatrix} 0 & E \\ C + BR & BC \end{pmatrix} \tilde{x} + \begin{pmatrix} 0 \\ B \end{pmatrix} \Delta u + \tilde{k}(z - H\tilde{q})$$

$$\tilde{x}(0) = 0$$

$$z = Hq + \eta(t)$$

References

Lee, K. and Shad, D. : Kinematic Analysis of a three Degrees of Freedom In Parallel Actuated Manipulator, IEEE Journal of Robotics and Automatization, 4,3, pp. 354-60, 1988
David R. Smith: General Report on LMT, August 27, 1992
Mast, T.S. and Nelson, J.E.: Figure Control for Fully Segmented Telescope Mirror, Applied Optics, 21, 14, pp. 2631-41, 1982.