CARLOS B. DE LYRA[†]

Carlos Benjamin de Lyra was born on November 23, 1927, in Recife, Pernambuco, the son of Carlos Lyra Filho, owner and director of the daily "Diário de Pernambuco", and of Da. Elizabeth Lau de Lyra. After his father's death he moved to the U.S.. From 1939 until 1945, he attended Iona High School in New Rochelle, New York, where he graduated 'summa cum laude'.

In his teens he was already uncommonly learned. When he was about 15 years old, influenced by Courant, the celebrated founder of the Mathematics Institute of New York University, he chose Mathematics as his field of study. Lyra used to recall his frequent conversations with Courant on his train rides to and from New York. He considered these talks very important in the choice of his career. On completion of high school, he came back to serve the Brazilian army, thus choosing his nationality.

In 1946 he started attending lectures at the Faculdade de Filosofia, Ciências e Letras of the University of São Paulo, at the Departament of Mathematics, where he met the French mathematicians André Weil and Jean Dieudonné. With the former he used to take long walks on week-ends in a then sparsely inhabited quarter of town. The recollection of these walks and of the talks with Weil always remained vivid in Lyra's mind. In 1947 he enrolled in the undergraduate mathematics program of the Faculdade de Filosofia Ciências e Letras (U.S.P.). He took, among others, Prof. Candido da Silva Dias, courses on algebraic topology. Prof. Dias himself had recently returned from the U.S. where he had studied the subject with Cartan and Spanier. The deep impression left by these courses on him, plus his earlier and profound interest in point set topology led Lyra to choose algebraic topology as his special field. He was a brilliant student throughout his university years and graduated in 1950.

In June 1951 he went to France on his own but later he was granted a scholarship by the Conselho Nacional de Pesquisas. There he participated in the Seminaire Cartan about fiber spaces and also followed Hurewicz'lectures on homotopy (at the Collège de France, in 1953). His contact with

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Hurewicz led him to the study of Borsuk's theory of ANR spaces, which became the subject of his Ph.D. thesis. At the end of 1953 Lyra returned to Brazil. Right after his homecoming he married Leda Lacerda. They had three children Jorge, Silvia and Eduardo(*). In the same year he joined the University of São Paulo, institution he would never leave and would honour to the end of his life. The first post he held at the University was that of Assistant Professor at the Departament of Mathematics of the F.F.C.L..

During the following years he was influenced by Alexander Grothendieck, then visiting Professor at the U.S.P., who was instrumental not only in stimulating Lyra to go on with his work on fiber spaces but also in introducing him to the theory of cohomology with values in sheaves. Some other remarkable influences were those of M. Goto's lectures (1957) and J.L. Koszul's course on fiber spaces and characteristic classes in which was presented the then recent work of Milnor on the construction of universal fiber spaces of arbitrary topological groups. In 1956 Lyra had taken part in the International Symposium on Algebraic Topology, in Mexico City, where he became interested in cohomological operations and Postnikov systems. The research wich resulted in his Ph.D. was done during 1957-1958, and by the end of 1958 he presented his thesis. In the academic year of 1960-1961 he was a visiting fellow of the Institute for Advanced Studies, Princeton, N.J., on a scholarship of the Rockfeller Foundation. He continued his research on Homotopy Theory and also took advantage of this oportunity to enlarge his knowledge of cohomological operations in the seminar of N. Steenrod and to begin, under J. W. Milnor, studies on Differential Topology. In 1968 he obtained the title of "livre-docente" in the F.F.C.L. and, in 1974 he became Associate Professor of the Instituto de Matemática e Estatística of the U.S.P..

A list of Lyra's publications is given below. A short commentary on them follows.

1) "A note on Zorn's lemma", Bol. Soc. Mat. de São Paulo, 4(1951), 63-65;
2) "Minimal complexes and maps", Bol. Soc. Mat. de São Paulo, 7(1954), 85-98;

(*) Eduardo died in 1971.

3) "Introdução à Topologia Algébrica", 1.º Colóquio Brasileiro de Matemática, 1957;
4) "Espaços de Recobrimento", Capítulo II de Superfícies de Riemann — Dep. de Mat. da F.F.C.L. da U.S.P., 1957-58. New edition: Notas de Matemática n.º 26, IMPA, 1961.
5) "On the homotopy type of a factor space", Anais Acad. Bras. Cien. 30(1958), 37-41.
6) "On circle handles over complex projective spaces", Anais Acad. Bras. Cienc. 31(1959), 17-24.
7) "Sobre os espaços de mesmo tipo de homotopia que os poliedros", tese de doutoramento, U.S.P., 1958.
8) "On spaces of the same homotopy type as polyedra", Bol. Soc. Mat. de São Paulo 12(1960), 43-62;
9) "Sobre os métodos de Whitehead na teoria de homotopia", Esc. de Eng. de São Carlos, U.S.P., 1963.
10) "On a conjecture in homotopy theory", Anais Acad. Bras. Cienc. 37(1965), 137-164.
11) "Sobre uma conjetura na teoria da homotopia" Atas do 5.º Colóquio Brasileiro de Matemática, 1965.
12) "H-equivalencia de Grupos Topológicos", tese de livre docência, U.S.P., 1968.
13) "Grupo Fundamental e Revestimentos", 7.º Colóquio Brasileiro de Matemática, 1967.
14) "Introdução à Topologia Algébrica", I.M.E., 1972.
15) "Quasi-homomorphism of CW-groups", pre-print, 1973.
16) "An Obstruction to Quasi-homomorphism", pre-print, 1973.

Note [1] was the result of some complementary studies on set theory, carried on his last year as undergraduate. Starting from Zorn's lemma Lyra proves Zermelo's theorem on well-ordering.

Theorem 11 of [2] was proved during his work in the Seminaire Cartan 1952/53. He applies the then recent technique of minimal subcomplexes of Eilenberg-Zilber, adapted to the cubic singular complex, to the study of the relationship between n-homotopy equivalence and the isomorphism of the n-skeletons of minimal subcomplexes.

[3] is the result of his first graduate courses on the subject of algebraic topology. It deals with simplicial homology and its applications. These

mimeographed notes were the basis for a course taught in the 1.º Colóquio Brasileiro de Matemática in 1957.

The lecture notes [4] are from lectures in a seminar on Riemann Surfaces, organized by Prof. Alexandre Rodrigues in 1957/58 in the Dept. of Mathematics of the FFCL — University of São Paulo.

In 1954/56 he pursued very actively the atudies on fiber spaces he had begun in Paris. Notes [5] and [6] are the result of this activity. In [5] he proves that for any locally trivial fibration of a S^{2n+1} sphere with fiber S^1, the base M has the same homotopy type as the complex projective space $P_n(C)$. The following theorem is proved, among others in [6]: If $S^1 \longrightarrow S^3 \xrightarrow{p} B$ is a locally trivial fibration with fiber S^1, then the base B is homeomorph to S^2 and the fibration is equivalent to the classical Hopf fibration.

In his Ph.D. thesis (cf. [7], [8]) he gave a characterization of the simply connected spaces which have the same homotopy type as a finite polyhedron. He gave a (partial) answer to a problem on compact ANR posed by K. Borsuk. Paper [8] is mentioned in K. Borsuk's 'Theory of Retracts', Monogr. Matem. n.º 44, Warsaw (1967), p. 218.

[9] are mimeographed lecture notes of a course taught in the Dept. of Mathematics of the Escola de Engenharia de São Carlos, in 1963. Its subject is the Whitehead's methods in homotopy theory, and it presuposes a knowledge of singular homology.

In [10] he extends to non simply connected spaces the research of [8] in the simply connected case. In [10] he introduces, for the category of spaces X dominated by finite complex such that $\pi_1(X)$ is finite, an obstruction $\lambda(X)$ with values in the group $\Gamma(\pi)$ which is the Grothendieck group on the projective finitely generated modules on the group ring $\Lambda = Z(\pi)$. The following theorems are proved: 1) X has the same homotopy type as a finite polyhedron if and only is $\lambda(X) = 0$; 2) For all $u \in \Gamma(\pi)$, there exists a complex X, dominated by a finite polyhedron, such that $\lambda(X) = u$. From 2) one gets a negative answer to a classical problem formulated by J.H.C. Whitehead in the late 40's (Does a space, dominated by a finite complex, have the same homotopy type as a finite complex?). This result was obtained independently by C.T. Wall, 'Annals of Mathematics', 81(1965), with precedence of publication. Wall developed a more complete theory of obstruction to finiteness, without the restriction of $\pi_1(X)$ being

finite, as in 10. J. F. Adams, reviewing in 1970 the development of algebraic topology in the 60's, mentions the problem cited above as 'an outstanding problem raised by J. H. C. Whitehead', solved by the application of Wall's obstruction (cf. 'Algebraic Topology', Proceedings of Symposia in Pure Mathematics, vol XXII, AMS (1971), p. 3). The obstruction introduced in [10], even though defined for a more restricted category of spaces, provided also a negative answer to the problem (see [10], Theorem 9.1). In a second paper "Finiteness condition for CW-complexes II' (proc. Royal Soc., A, vol. 295(1966), p. 139), Wall mentions [10].

Publication [13] resulted from a graduate course given at the University of São Paulo in 1969. It was also used for a course in the 7.º Colóquio Brasileiro de Matemática, in 1969.

The mimeographed notes [14] were written for an introductory course in algebraic topology, given several times in the 60's. The edition of 1972 is out of print, and will be published in more permanent form by the IME.

Notes [12] and [15] study topological groups under the viewpoint of homotopy theory. A more satisfactory and complete theory was achieved in [15]. Roughly speaking, topological groups are considered as Hopf spaces, enriched with aditional structure. This leads to the definition of an apropriate class of morphisms (quasi-homomorphisms) which are studied from the point of view of homotopy. It is proved that the morphisms thus defined are invariant under homotopy and an inversion theorem for these homomorphisms is proved. By comparing these with the more restricted theory of topological groups with continuous homomorphisms, a characterization of the new morphisms is given.

Pre print [16] contains some initial results obtained by application of obstruction theory to quasi-homomorphisms. A theory of this type makes it possible to give criteria for deciding whether a map $\phi: G \longrightarrow G'$ of topological groups is a quasi-homomorphism. For example, it is proved in [16] that every homotopy class of maps $\phi: S^3 \longrightarrow U$, where $U = \lim U(n)$ is the 'large' unitary group and S^3 the Lie group of quaternions of norm 1, is made up of quasi-homomorphisms. The importance of the theory comes from the possibility of applying it to the classification of the maps $B_G \longrightarrow B_G$ of classifying spaces of Lie groups, being studied at present by J. F. Adams, J. R. Hubbuck and others.

This last paper [16] was published post mortem, and it was his friend Prof. Peter Hilton who using Lyra's manuscript, wrote its definitive version after Lyra's death. It contains results obtained by applying obstruction theory to quasi-homomorphisms.

One other aspect of Lyra's activity on the subject of algebraic topology deserves special mention. From 1956 on he directed a weekly seminar on the subject covering the most important topics of the field as for instance: simple homotopy type, topological localization, homology of fiber spaces, cohomological operations, applications of cohomological operations, Postnikov systems, localization and applications. His role in supervising more than a dozen advanced students and a number of beginners was also remarkable. Among the students taking part in this seminar we should mention Prof. Gilberto F. Loibel, Renzo Piccinini, Nelo Allan, who are now professors at several universities in Brazil and abroad.

Lyra's initiative in the important question of obtaining the cooperation of many visiting professors is also worth mentioning. In 1962/63 under his invitation Prof. Benson Brown came to S. Paulo to take part in Lyra's seminar, and lectured on the stable theory of homotopy. In 1968 Prof. Peter Hilton, then from Cornell, visited S. Paulo; he conducted seminars and lectured on General Theories of Cohomology and K-theory. Prof. Hilton since then returned to S. Paulo several times on 69, 72, 73 and 74. Lyra and Prof. Piccinini invited Prof. Liulevicius in 1970 and Prof. Robert Wells in 71, both from the University of Chicago; they also invited Prof. Paul Ledergerber who has remained as a teacher in S. Paulo. In 1972 Lyra invited Prof. John R. Hubbuck, of Oxford University, who lectured on the application of K-theory to Hopf spaces. In 1973, Prof. U. Sutter was a visiting professor at the U.S.P. Lyra's influence was not restricted to the field of algebraic topology: it was thanks to his initiative that Prof. Alfredo J. Rodrigues came to teach Algebra in S. Paulo.

Lyra also left his mark as a leader both in the reform of the University and the organization of an Association of Instructors of the University: as a representative of this Association he had an active participation in the work that led to the creation of the Fundação de Amparo à Pesquisa do Estado de S. Paulo. Later he was an enthusiastic and undefatigable promoter of the university reform. In this role not only his specific knowledge of mathematics but also his humanistic spirit as a defender of culture

and intelligence had a great importance; he was member of the Comittee which presented the proposed text for the reform. One result of this reform was the creating of the Instituto de Matemática e Estatística of the U.S.P.

Other activities of his deserve mention: participation in the organizing Committees of the 1^{st}, 2^{nd} (Chairman) and 6^{th} Colóquios Brasileiros de Matemática, member of the Board and president of the Sociedade de Matemática de S. Paulo; advisor to the Fapesp; member of the Committee which defined the scientific character and organization of the Latin American School of Mathematics, and Brazilian representative in the Per-Permanent Committee of this School; founding member of the Sociedade Brasileira de Matemática; member of the scientific board of advisors of the Instituto de Matemática Pura e Aplicada (Rio de Janeiro) and of the Instituto de Física Teórica (S. Paulo); member of the Editorial Board of 'Ciência e Cultura', published by the Sociedade Brasileira para o Progresso da Ciência. He took part in several federal committees (organized by FUNTEC, CNPq and FINEP) to make an evaluation of the situation of mathematics in Brazil, in 1971, 1972 and 1974.

Besides all this activity he had an important role in the teaching of mathematics: from 1963 with Prof. Elza Gomide he worked in the reorganization of courses on Calculus, and in the planning of honor courses in Advanced Calculus. He also taught courses in the History of Mathematics, and in Calculus, Analysis, Algebra, Topology, as well as advanced courses in Algebraic Topology, Group Theory, etc.

The manifold qualities of Lyra's as well as his scientific work were recognized. This is the appreciation written by the board of examinors when he applied for the title of Associate Professor in 1974.

> "On his scientific work: the seriousness and coherence of the candidate in a recognizedly difficult subject — Algebraic Topology — must be mentioned; special mention must be given to his Ph.D. thesis and his thesis for the title of *livre docente*. These works got international notice and served as a starting point for new research by the candidate, some of which he is concluding now.
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> "On his influence in the development of mathematics in Brazil: special emphasis must be given to the dedication of the candidate

to all problems which arise in the development of mathematics in Brazil. This dedication has had remarkable results not only within the Institute of which he is a member but also in the whole of the University and of the country, through his activities in the Colóquios de Matemática, in the Sociedade Brasileira de Matemática, IMPA, etc.

"On his teaching activity: the Committee gives special emphasis to the dedication of the candidate to his work on advising graduate students, keeping up the interest on Algebraic Topology, a subject in which he is the most active lecturer in Brazil. This has never affected his activity as lecturer on the undergraduate level, as is made evident both by the variety of the courses he has taught as by his work on planning curricula and program reforms.

"Taking all this into account, the Committee unanimously recomends Prof. Carlos Benjamin de Lyra for Professor of the Department of Mathematics of this Institute.

The fact that he was elected to the Academia Brasileira de Ciências with an unprecedented unanimity showed how his colleagues of all scientific branches appreciated his many splendid qualities and his enormous activity on behalf of the scientific community in Brazil.