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ECONOMETRIC MODELING AT MIXED FREQUENCIES

Abstract. A helpful first step in building an adequate structural model of the present Russian economy, is to build a high-frequency system based on monthly statistics for very short run projections of overall economic performance. The period of adequate statistical documentation is very short; so three steps are proposed: 1) a very short run monthly model for projecting major aggregative indicators for periods of 6-months-to- one-year ahead; 2) construction of a more detailed structural model of the whole economy patterned after those that have been prepared for most major economies of the world, as enough components of an adequate database have been collected, with an aim of making quarterly or annual projections; 3) a mathematical linkage of the two systems, so that the high frequency system provides immediate short-run projections and the medium-term structural system of lower frequency generates projections of three-to-five years ahead that are "as close as possible," in the sense of a minimized loss function, to the high-frequency projections of major variables.

Statement of the problem

During the celebration of St Petersburg's tercentenary, I presented a research paper to the Academy of Sciences (host organization), together with the collaboration of Vladimir Eskin and Andrei Rudoi, on the concept of "Empirical Regularities in the Russian Economy". There do seem to be such regularities in the dynamic properties of Russia's economy now, and for the past decade or so, that can be used for building a model of the system for short-term forecasting. In that paper our concern was with monthly data, which define implicitly a high frequency, in economic terms, but it might even draw upon higher frequency information, especially from financial markets and also from commodity markets that report relevant information daily, or even more frequently. We can use weekly, daily, or even real time data, in principle, for model building, and much of this kind of information refers to world commodity and financial markets.

Reaction to our findings in the St Petersburg anniversary paper has frequently been, "How can this approach be used for longer term application?" The motivation for the present paper is to indicate how high-frequency modeling may be extended to forecast or simulation horizons of 5 to 10 years.

The approach taken, up to the present time, has been to develop a working system, not a *structural* econometric model, of Russia, but to make as much use as possible of a database with a very limited time span. The focus will be on a monthly model that can be used for short run quarterly forecasts, up to two quarters ahead. The starting point will be near 1995, allowing the sample to reach more than 120 observations. We shall exploit the serial dependence in the statistical observations by assuming that the serial effects last for a few months. A sample starting in 1995 would include the 1998 financial crisis, but that one event will not destroy the usefulness of the whole available sample. In the work done to date, we have been able to review and update forecasts every month, and perceive a reasonable amount of statistical stability. All the main features of the new IT facilities in hardware, software, and internet will be used as fully as possible in order to compensate for the paucity of historical information.

Eventually, the statistical record of the performance of the Russian economy will provide ample information for the building of a *structural* model of the Russian economy on the basis of a data sample that covers about 15-20 annual observations. That is approximately the data span that led Jan Tinbergen to construct the first *structural* models of his native Holland and later, for the League of Nations, the United States. Of course one would want to have even a longer sample of annual statistical data, in the framework of an integrated national accounting system, but now is an excellent time to lay foundations for statistical modeling of the Russian economy as soon as possible. The purpose of the present paper is to indicate how the short run models are now being adapted for immediate analysis of Russia, at a monthly or even shorter frequency, and have them ready to be combined with an annual model that will be suitable for extrapolation into longer time horizons. The high-frequency monthly model is already operational, but still being enhanced, along the lines of the paper presented to the meetings in St Petersburg during summer, 2003. The rest of this presentation will deal with its efficient combination with the longer term, lower frequency model of Russian economic structure. Those are, in short, the objectives of this paper.

Abstract formulations of an annual Russian economic model

The aim in a structural model of Russia will be based on two basic kinds of variables, y_{it}, the ith endogenous variable for year t, $i = 1, 2, \ldots, n$, and x_{it}, $i = 1, 2, \ldots, m$; the ith exogenous variable, chronological time, is denoted by t. At the macroeconomic level of analysis, y_{it} will denote such magnitudes as total production (GDP = gross domestic product), consumer spending, fixed capital formation, exports, imports, wage and salary payments, capital payments such as interest, rent, and profits, public spending, public receipts, prices, wage rates, interest rates, foreign exchange rates, money supply, and other standard macroeconomic measures. The exogenous variables are formed outside the economic system as such, but they have significant effect on the economy. Prototypes are weather and climate variables. A linear specification of an economic model is

$$A(L)y_t + Bx_t = e_t$$

Models are not generally linear, but it is easy and clear to express the system in closed form if the specification is linear. Implicit forms accompanied by iterative algorithms enable us to solve nonlinear systems quite easily in practice, but the pedagogy of linear systems shows our motivations for analysis quite plainly.

Here L is a lag operator

$$Ly_t = y_{t-1},$$

y_t, x_t, and e_t are column vectors

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ M \\ y_{nt} \end{pmatrix}, \begin{pmatrix} x_{1t} \\ x_{2t} \\ M \\ x_{mt} \end{pmatrix}, \begin{pmatrix} e_{1t} \\ e_{2t} \\ M \\ e_{nt} \end{pmatrix},$$

$A(L)$ is a square matrix $(n \times n)$, while B is rectangular $(n \times m)$ and e_t is a vector of random errors.

A closed-form expression for the solution of the dynamic system is

$$||\Delta(L)||y_t = -a(L)Bx_t + a(L)e_t,$$

where $||\Delta(L)||$ is a diagonal matrix with $\Delta(L)$ on the main diagonal, $\Delta(L)$ is the determinantal polynomial of $A(L)$, and $a(L)$ is the adjoint matrix of $A(L)$. If the characteristic roots (eigenvalues) are all distinct, the solution may be written as

$$y_{it} = \sum_{j=1}^{np} k_{ij}\lambda_j^t + \sum_{s=0}^{\infty}\sum_{j=1}^{np} \frac{\lambda_j^{s+np-1}}{\prod_{j\neq k}(\lambda_j - \lambda_k)}\varphi(t-s),$$

$$\varphi_i(t-s) = \delta_0 y_{i,t} + \delta_1 y_{i,t-1} + \ldots + \delta_{np} y_{i,t-np},$$

where $p =$ longest lag length in the system.

This solution is quite interesting. It states that the time-path solution of each variable is a function of powers of the eigenvalues. If some are complex, they introduce sinusoidal cycles in the solution, which can display business-cycle movements of the economy. If the eigenvalues are all less than unity (absolute value) that implies that cycle movements will tend to die out; that would contradict the measurements by students of economic cycles, who claim that business cycles are sustained over time and do not fade away. Eugen Slutsky (and others, too) have shown that the stochastic component of the solution in

$$a(L)e_t$$

is essentially a form of moving average (or moving totals) of past errors and may very well tend to a cyclical limit.[1]

From the expression for the solution to a general linear dynamic model

$$y_t = -[A(L)]^{-1}BX_t + [A(L)]^{-1}e_t$$

which appears above explicitly in terms of eigenvalues and distributed lags of error terms, we can determine what econometricians call multiplier values. These are sensitivities to external (i.e. exogenous) variables and to changes in parameter values or random error terms. These are all adjustments that are used to "fine-tune" a solution to make it agree more closely with new information about latest values of exogenous variables, or disturbances. Essentially they enable us to estimate approximate numerical values for $\frac{\partial y_{it}}{\partial x_{jt}}$, or $\frac{\partial y_{it}}{\partial e_{jt}}$. It is therefore possible to change a solution to the equation system when new information changes from the world over.

This means that the model operator (or proprietor) can change the solution path of any variable in known ways. In the process of altering

[1] See E. Slutsky, *The summation of random causes as the source of cyclical processes*, Econometrica **5** (1937), 105–46.

the solution, the model operator could be guided by personal judgment - of himself or a senior executive who has close interests in outcomes from the economic system.

To keep the solution presentation from being judgmental and to retain *objectivity*, one can rely on an a priori loss function, to show how discrepancies between

$$\begin{pmatrix} \widehat{y}_{1t_0} \\ \widehat{y}_{2t_0} \\ M \\ \widehat{y}_{n_1 t_0} \\ n_1 < n \end{pmatrix}$$

and any other solution to the same equation system at $t = t_0$ that does not use any information from the companion higher frequency system. It is the operator's preferred solution, given the knowledge that the model operator has about the inputs and outputs (solution value) corresponding to the *lower* frequency system, without making any particular use of latest information. A preferred procedure should be that the model operator makes objective adjustments to bring the solution of the model "as close as possible" to the solution of the high frequency model over the normal forecast horizon of the latter system. This is done by adjusting the values of x_{jt} or e_{jt}, when these are not known in advance. In other words, the operator steers the system towards a solution that has minimum loss.

Let the loss function be some function of the specified subset of $y_{1t_0} \ldots y_{nt_0}$. Call this subset $y_{1t_0} y_{2t_0} \ldots y_{n_1 t_0}$. In a formal sense the model operator follows the rule

$$\min L \left[\widehat{y}_{1t_0} - y_{1t_0}, \widehat{y}_{2t_0} - y_{2t_0} \, \Lambda \, \widehat{y}_{n_1 t_0} - y_{n_1 t_0} \right]$$

subject to

$$y_t = -[A(L)]^{-1} X_t - [A(L)] e_t .$$

If the loss function is specified as a weighted sum of discrepancies between the two models, it may be written as $L = \sum_{t=1}^{h} \sum_{i=1}^{n_1} w_i (\widehat{y}_{it} - y_{it})^2$, where h is the high-frequency forecast horizon; n_1 is the number of high-frequency target variables. Co-variation terms could be used in order to generalize the concept "as close as possible to ..." So, without personal judgment, once the loss function has been specified and the two models estimated – one at low frequency and one at high frequency the operator can adjust the low frequency model to come close to the results from the high frequency model. The latter takes precedence and the low frequency system is set out to get ready for following its solution path, but first must come "as close as possible" to the main variables of the high frequency system on their solution path, over a finite period in which its path is influenced by a path that is restricted to be as close as possible to that of the high frequency solution in the specified variables.

After registering estimated values that are kept close to the high-frequency target path in selected variables, the relationships to the latter are dropped from consideration and the low frequency model is allowed to follow its own paths. For the short-run start-up path the model solution is restricted, but after that period is past, the low frequency model is allowed to go on its own course for as long as the model operator wants to compute values.

The noise-to-signal ratio in quantitative economics

Lest it be thought that there is a large judgmental element in economic data that renders mathematical treatment unsound, it is important to understand the role of uncertainty and error mechanisms in econometric models. While Slutsky showed how successive averaging (or smoothing) can generate cyclical dynamics, the range of uncertainty can be narrowed by careful treatment of underlying data and frequent replication of extrapolative forecasting exercises. That is the point at which high-frequency transmission in this information age can be used advantageously.

The higher the frequency, the greater is the likelihood of significant data revision. As one moves from annual to quarterly, to monthly, to weekly, to daily observation of economic variables, the more likely are there to be data revisions. Samples and crude estimates must be made for international trade statistics, for profits, for inventory change, for savings, for non-merchandise (i.e. "invisible") transactions, for labor market statistics, and for partial time-series values of some observations. It is nevertheless, important for the investigator to keep re-making forecasts as the collectors of primary information supply better and better estimates of these hard-to-measure items as frequently as possible.

Data revision is common and frequent – even more so at recent high-frequency intervals; therefore the model operator in making forecasts must adapt and learn the way markets work by adjusting individual equations. These adjustments are noted above as $\frac{\partial y_{it}}{\partial x_{jt}}$ or $\frac{\partial y_{it}}{\partial e_{jt}}$. The latter are closely related to changes in the constant-term parameters of linear equations. These adjustments are referred to in the literature of econometrics as "add factors". There is much criticism of model builders and operators for changing the *level* of equations, i.e. by changing the constant-term parameter values in order to achieve a preordained subjective result, but the combination of low and high frequency models are introduced, primarily in order to remove subjectivity of personal choice.

The high frequency models are built so that they rely only on observed data, but are subject to revision or change as frequently as the high frequency data become available, i.e. every day or every week. Those data streams and revised high frequency forecasts are not subjective; they change because underlying data or policy interventions change frequently. Solutions of high frequency models can be re-calculated, without *subjective choice*, by the model operator. After their fresh values are obtained every week or day, the *low frequency* models are re-calibrated, especially by changing constant terms or other obvious parameters of low frequency models so as to minimize loss functions according to the control adaptations described in the previous section. The low frequency models will be projected for a few (or several) years ahead, but they will follow rules of change so that other operators could, in principle, obtain the same values. Since there is generally high serial correlation in modern economies, big deviations from observed values will not last more than one month, but usually even less frequently than that, since models can be resolved every week or even every few days.

Not only do combinations of mixed frequencies enable the forecaster to improve accuracy; they also lead to elimination of personal judgment. Replicability of forecasts by others, at least in principle, is an important aim to be achieved.

Also, economists generally aim to reduce *risk* in making analyses of situations that are not fully known in advance. They do this often by diversifying their tools of analysis. The combination of models, among frequencies, provides a form of diversification. The high frequency models can potentially be estimated and projected ahead at least every week, but also more frequently, if desired, or if necessitated by data changes or new information flows; thus a method of analysis that combines results of many model environments over time is one way of trying to reduce risk by forecasting often.

Postscript

The primary objective of the memorial program of January 2004, in St.Petersburg was to honor and remember the fundamental contributions of Acad Leonid Kantorovich. The paper that was actually presented in the meetings was one that I have most recently been examining. I presented it because it dealt with information technology that I felt was important for Russian colleagues. It drew more upon the work of the late Wassily Leontief than of Leonid Kantorovich. Consequently, for the written record, I submitted this different paper. It was put together in response to inquiries from Russian colleagues how models that are based on monthly (high-frequency) data can be used for medium or longer-term analysis. My answer has been that it is possible to combine models of both low and high frequencies in order to study trends of intermediate length in the Russian economy. It turns out that there is a small and straightforward application of control theory, requiring *optimization* in order to do this. Since optimization was a major contribution of Leonid Kantorovich, I felt that a short analysis of an optimization problem would be eminently suitable for the memorial issue to honor Kantorovich.

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