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ON A THEOREM OF GROTHENDIECK

**С глубокой благодарностью
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ABSTRACT. It is considered a smooth projective morphism $p : T \to S$ to a smooth variety S. It is proved, in particular, the following result. The total direct image $Rp_*(\mathbb{Z}/n\mathbb{Z})$ of the constant étale sheaf $\mathbb{Z}/n\mathbb{Z}$ is locally for Zariski topology quasi-isomorphic to a bounded complex $\mathcal{L}$ on S consisting of locally constant constructible étale $\mathbb{Z}/n\mathbb{Z}$-module sheaves.

1. Introduction

To state the main theorem of this paper consider a field k, an affine k-smooth variety X and a finite family $x = \{x_1, \dots, x_n\}$ of closed points of X. Let $\Lambda = \mathbb{Z}/n\mathbb{Z}$ and let $(n, \operatorname{char}(k)) = 1$.

Main Theorem. *Let $\mathcal{U} = \operatorname{Spec} \mathcal{O}_{X,x}$ and let $p : T \to \mathcal{U}$ be a smooth proper morphism. Let $\mathcal{F}$ be a locally constant constructible sheaf of Λ-modules on T_{et}.*

Then there exists a finite complex $0 \to \mathcal{L}^0 \to \cdots \to \mathcal{L}^n \to 0$ of locally constant constructible sheaves on $\mathcal{U}$ and a functor isomorphism

$$H^q_{\mathrm{et}}(T \times_{\mathcal{U}} \mathcal{U}', \mathcal{F}') \cong H^q_{\mathrm{et}}(\mathcal{U}', \mathcal{L}') \quad (q \geqslant 0)$$

on the category of $\mathcal{U}$-schemes (here for a $\mathcal{U}$-scheme $\mathcal{U}'$ the complex $\mathcal{L}'$ is the pullback of the complex $\mathcal{L}$ under the structural morphism $\mathcal{U}' \to \mathcal{U}$, and $\mathcal{F}'$ is the pullback of $\mathcal{F}$ under the canonical projection $T \times_{\mathcal{U}} \mathcal{U}' \to \mathcal{U}'$).

This theorem is analogous of the well-known result of A. Grothendieck about cohomology of a coherent sheaf (see [2], §5).

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§1. Theorem A

Let $p : T \to \mathcal{U}$ be the morphism from Main Theorem and let I^0 be an injective resolution of the sheaf $\mathcal{F}$ in the category of étale Λ-module sheaves Sh_T (Λ-mod) on T. Let $\mathcal{K} = p_*(I^0)$ be the pushforward of I^0 on $\mathcal{U}$. Clearly one has a canonical isomorphism of functors

$$H^q_{\mathrm{et}}(T, \mathcal{F}) \cong H^q_{\mathrm{et}}(\mathcal{U}, \mathcal{K}) \quad (q \geqslant 0).$$

Since the morphism p has a finite dimension one may replace the complex $\mathcal{K}$ by a quasi-isomorphic subcomplex $\tau_{\leqslant m}\mathcal{K}$, where $(\tau_{\leqslant m}\mathcal{K})_i = \mathcal{K}_i$ for $i < m$, $(\tau_{\leqslant m}\mathcal{K})_i = 0$ for $i > m$ and $(\tau_{\leqslant m}\mathcal{K})_i = \mathrm{Ker}\,(\partial_i)$ for $i = m$, such that for $m \gg 0$ one has the isomorphism of functors

$$H^q_{\mathrm{et}}(\mathcal{U}, \mathcal{K}) \cong H^q_{\mathrm{et}}(\mathcal{U}, \tau_{\leqslant m}\mathcal{K})$$

induced by the embedding $\tau_{\leqslant m}\mathcal{K} \hookrightarrow \mathcal{K}$.

Since the morphism p is smooth and proper and the sheaf $\mathcal{F}$ is locally constant constructible, the cohomology sheaves of the complex $\tau_{\leqslant m}\mathcal{K}$ are locally constant constructible sheaves. Therefore, to prove the main theorem it suffices to prove Theorem A.

Theorem A. *Let $\mathcal{K}$ be a bounded complex of étale Λ-module sheaves on the variety X. Assume the cohomology sheaves of $\mathcal{K}$ are locally constant constructible sheaves. Then there exists an affine open neighborhood U of the family $x = \{x_1, \dots, x_n\}$ and there exists a bounded complex $\mathcal{L}^0$ of locally constant constructible sheaves of Λ-modules on U such that the complexes*

$$\mathcal{L}^0 \quad \text{and} \quad \mathcal{K}\big|_U$$

are isomorphic in the derived category $D^b(\mathrm{Sh}_U$ (Λ-mod)) of bounded complexes of étale Λ-module sheaves.

§2. Proof of Theorem A

To prove Theorem A we need several auxiliary results. To state them recall that X is a smooth affine k-variety over a field k and $x = \{x_1, \dots, x_n\}$ is a finite family of closed points of X and n is an integer such that $(n, \mathrm{char}\,(k)) = 1$, and $\Lambda = \mathbb{Z}/n\mathbb{Z}$.

2.1. Theorem. *Let F be a $\Lambda[\pi_1(X)]$-module finite as the Λ-module. And let $\widetilde{F}$ be the respecting sheaf on X_{et}. Then the canonical map*

$$\varinjlim_{x \in U} H^*(\pi_1(U), F) \to \varinjlim_{x \in U} H^*_{\mathrm{et}}(U, \widetilde{F})$$

is an isomorphism.

2.2. Corollary. *Let L_1, L_2 be two bounded complexes of discrete $\Lambda[\pi_1(X)]$-modules and let the cohomology groups $H^*(L_1)$, $H^*(L_2)$ are finite as the Λ-modules. Then the canonical map*

$$\varinjlim_{x\in U} \operatorname{Ext}^*_{\Lambda[\pi_1(U)]}(L_1, L_2) \to \varinjlim_{x\in U} \operatorname{Ext}^*_{\mathrm{Sh}\,(U)}(\widetilde{L}_1, \widetilde{L}_2)$$

is an isomorphism, where $\mathrm{Sh}\,(U)$ *is the category of étale Λ-module sheaves on U.*

2.3. Corollary. *Let $\mathcal{K}$ be a bounded complex in the category of étale Λ-module sheaves. Assume the cohomology sheaves of $\mathcal{K}$ are locally constant constructible sheaves.*

Then there exists an affine open neighborhood U of the family $x = \{x_1, \dots, x_n\}$ and there exists a bounded complex K of discrete $\Lambda[\pi_1(U)]$-modules such that the complexes

$$\widetilde{K} \quad \text{and} \quad \mathcal{K}\big|_U$$

are isomorphic in the derived category $D^b(\mathrm{Sh}_U\ (\Lambda\text{-mod}))$ of bounded complexes of étale Λ-module sheaves on U.

2.4. Theorem. *Let π be a pro-finite group and let K be a bounded complex of discrete $\Lambda[\pi]$-modules. Assume that the cohomology groups of K are finite Λ-modules. Then there exist a bounded complex L of discrete $\Lambda[\pi]$-modules, which are finite Λ-modules, and a quasi-isomorphism*

$$\varphi : L \to K$$

of $\Lambda[\pi]$-module complexes.

2.5. Remark. Clearly Theorem A follows from Corollary 2.3 and Theorem 2.4.

2.6. Remark. Theorem 2.4 is proved by arguments given in [2] (§5, Lemma 1).

2.7. Reduction of Corollary 2.3 to Corollary 2.2. We shall write $D^b(X)$ for the derived category of bounded complexes of étale Λ-module sheaves on X. And for an object M of the category $D^b(X)$ and a neigborhood X'' of the family x we shall write M'' for the image of M under the canonical functor $D^b(X) \to D^b(X'')$. For a morphism $\varphi : M_1 \to M_2$

in the category $D^b(M)$ we shall write $\varphi'' : M_1'' \to M_2''$ for the respecting morphism in the category $D^b(X'')$.

We shall write $\mathrm{Ext}^j_{X''}(N,P)$ for the Ext-group in the category $D^b(X'')$ and we shall write $\mathrm{Ext}^j_{\pi_1(X'')}(A,B)$ for the Ext-group in the derived category of discrete $\Lambda[\pi_1(X'')]$-modules.

Now let $\mathcal{K}$ be the complex from the hypotheses of 2.3. Set $\sigma_{\leqslant p}(\mathcal{K})^n = \mathcal{K}^n$ for $n < p$, $\sigma_{\leqslant p}(\mathcal{K})^n = 0$ for $n > p$ and $\sigma_{\leqslant p}(\mathcal{K})^n = \mathrm{Ker}\,(\partial^n)$ for $n = p$. One may assume without loss of generality that $\sigma_{\leqslant -1}(\mathcal{K}) = 0$ and $\sigma_{\leqslant N}(\mathcal{K}) = \mathcal{K}$ for N sufficiently large. Consider the canonical exact triangle in the derived category $D^b(X)$

$$\mathcal{H}^r(\mathcal{K})[r-1] \xrightarrow{\mathfrak{f}} \sigma_{\leqslant(r-1)}(\mathcal{K}) \xrightarrow{\mathfrak{g}} \sigma_{\leqslant r}(\mathcal{K}) \xrightarrow{\mathfrak{h}} \mathcal{H}^r(\mathcal{K})[r].$$

Using the inductive hypotheses one may assume that the conclusion of Corollary 2.3 holds for complexes $\mathcal{H}^r(\mathcal{K})$ and $\sigma_{\leqslant(r-1)}(\mathcal{K})$.

Let H^r be a discrete $\Lambda[\pi_1(X)]$-module (finite as the Λ-module) such that the sheaf $\widetilde{H}^r$ is isomorphic to the sheaf $\mathcal{H}^r(\mathcal{K})$. Let L_{r-1} be a bounded complex of discrete $\Lambda[\pi_1(X')]$-modules (X' is an appropriate neighborhood of the family x in X) such that one has an isomorphism $\widetilde{L}_{r-1} \to \sigma_{\leqslant(r-1)}(\mathcal{K})'$ in the derived category $D^b(X')$ (see the agreement about notation).

Denote by $\alpha : \widetilde{H}^r \to \mathcal{H}^r(\mathcal{K})$ and $\beta : \widetilde{L}_{r-1} \to \sigma_{\leqslant(r-1)}(\mathcal{K})'$ the choosen isomorphisms in the category $D^b(X')$. Consider the following diagram in the category $D^b(X'')$ for an appropriate neighborhood X' of the family x (see the agreement about notation)

$$(*)\qquad \begin{array}{ccccccc}
\mathcal{H}^r[r-1]'' & \xrightarrow{\mathfrak{f}''} & \sigma_{\leqslant(r-1)}(\mathcal{K})'' & \xrightarrow{\mathfrak{g}''} & \sigma_{\leqslant r}(\mathcal{K}) & \xrightarrow{\mathfrak{h}''} & \mathcal{H}^r[r]'' \\
\uparrow{\scriptstyle \alpha[r-1]''} & & \uparrow{\scriptstyle \beta''} & & \uparrow{\scriptstyle \gamma} & & \uparrow{\scriptstyle \alpha[r]''} \\
\widetilde{H}^r[r-1]'' & \xrightarrow{\widetilde{f}} & (\widetilde{L}_{r-1})'' & \xrightarrow{\widetilde{g}} & \widetilde{\mathrm{Cone}(f)} & \xrightarrow{\widetilde{h}} & \widetilde{H}^r[r]'',
\end{array}$$

where $f : H^r[r-1]'' \to L''_{r-1}$, $g : L''_{r-1} \to \mathrm{Cone}(f)$ and $h : \mathrm{Cone}(f) \to H^r[r]$ and γ are morphisms (constructed below) in the category $D^b(\Lambda[\pi_1(X'')])$ of bounded complexes of discrete $\Lambda[\pi_1(X'')]$-modules.

The morphism $(\beta')^{-1} \circ \mathfrak{f}' \circ \alpha[r-1]'$ is an element of the group $\mathrm{Hom}_{D^b(X')}(\widetilde{H}^r[r-1]', (\widetilde{L}_{r-1})')$. This group is canonically isomorphic to the group $\mathrm{Ext}^{1-r}_{X'}((\widetilde{H}^r)', (\widetilde{L}_{r-1})')$ (see the agreement about notation). Corollary 2.2 shows that there exists a neighborhood X'' of the family

x and there exists a morphism $f : H^r[r-1]'' \to L''_{r-1}$ in the category $D^b(\Lambda[\pi_1(X'')]$ such that the elements $\widetilde{f}$ and $(\beta'')^{-1} \circ f'' \circ \alpha[r-1]''$ coincide in the group $\mathrm{Hom}_{D^b(X'')}(\widetilde{H}^r[r-1]'', (\widetilde{L}_{r-1})'')$.

Let $g : L''_{r-1} \to \mathrm{Cone}(f)$ be a cone of the morphism f in the category $D^b(\Lambda[\pi_1(X'')])$ and let $\widetilde{g} : (\widetilde{L}_{r-1})'' \to \widetilde{\mathrm{Cone}(f)}$ be the respective cone in the category $D^b(X'')$. Since the category $D^b(X'')$ is triangulated there exists a morphism $\gamma : \widetilde{\mathrm{Cone}(f)} \to \sigma_{\leqslant r}(\mathcal{K})''$ in this category making the diagram $(*)$ commutative.

Since morphisms α'' and β'' are isomorphisms the morphism γ is an isomorphism as well. Thus we constructed the neighborhood X'' of the family x and the bounded complex of discrete $\Lambda[\pi_1(X'')]$-modules $\mathrm{Cone}(f)$ and an isomorphism

$$\gamma : \widetilde{\mathrm{Cone}(f)} \to \sigma_{\leqslant r}(\mathcal{K})''$$

in the category $D^b(X'')$. This completes the reduction of Corollary 2.3 to Corollary 2.2. □

2.8. Reduction of Corollary 2.2 to Theorem 2.1. An induction by the length of complexes L_1 and L_2 reduces the general case to the case of pure complexes. In this case L_1 and L_2 are discrete $\Lambda[\pi_1(X)]$-modules finite as the Λ-modules.

Consider two spectral sequences

$$H^p\left(\pi_1(U), \mathrm{Ext}^q_\Lambda(L_1, L_2)\right) \Longrightarrow \mathrm{Ext}^{p+q}_{\Lambda[\pi_1(U)]}(L_1, L_2),$$

$$H^p_{\mathrm{et}}\left(U, \mathcal{E}\mathrm{xt}^q_\Lambda(\widetilde{L}_1, \widetilde{L}_2)\right) \Longrightarrow \mathrm{Ext}^{p+q}_{\mathrm{Sh}_U(\Lambda\text{-}mod)}(\widetilde{L}_1, \widetilde{L}_2),$$

and consider the canonical morphism $E^{**}_*(\pi_1(U)) \to E^{**}_*(U)$ of these spectral sequences. Taking the direct limit one obtains a morphism

$$\varinjlim_{x \in U} E^{**}_*(\pi_1(U)) \longrightarrow \varinjlim_{x \in U} E^{**}_*(U)$$

of the spectral sequences.

The sheaf $\widetilde{\mathrm{Ext}^q_\Lambda}(L_1, L_2)$ respecting to the discrete $\Lambda[\pi_1(U)]$-module $\mathrm{Ext}^q_\Lambda(L_1, L_2)$ is isomorphic canonically to the sheaf $\mathcal{E}\mathrm{xt}^q_\Lambda(\widetilde{L}_1, \widetilde{L}_2)$. Thus Theorem 2.1 shows that the canonical morphism

$$\varinjlim_{x \in U} \mathrm{Ext}^*_{\Lambda[\pi_1(U)]}(L_1, L_2) \longrightarrow \varinjlim_{x \in U} \mathrm{Ext}^*_{\mathrm{Sh}_U(\Lambda\text{-}mod)}(\widetilde{L}_1, \widetilde{L}_2)$$

is an isomorphism. The reduction of Corollary 2.2 to Theorem 2.1 is completed. □

The end of this section is devoted to the proof of Theorem 2.1. We need in couple lemmas.

2.9. Lemma. *Let U be an open neighborhood of the family x and X and let $\mathcal{F}$ be a locally constant constructible sheaf of Λ-modules on X_{et}. Let $\pi : \widetilde{U} \to U$ be a finite étale morphism and let $\alpha \in H^q_{\text{et}}(\widetilde{U}, \mathcal{F}|_{\widetilde{U}})$ with $q > 0$. Then there exists a Zariski open neighborhood V of the family x and a finite étale morphism $\tilde{\tilde{V}} \to \widetilde{V}$, where $\widetilde{V} = V \times_U \widetilde{U}$, such that $\alpha|_{\tilde{\tilde{V}}} = 0$ in $H^q_{\text{et}}(\tilde{\tilde{V}}, \mathcal{F}|_{\tilde{\tilde{V}}})$.*

2.10. Sublemma. *Let Y be a smooth variety over k and let $\{y_1, \dots, y_m\}$ be a finite family of closed points in Y. Let G be a locally constant constructible sheaf on Y_{et} and let $\beta \in H^q_{\text{et}}(Y, G)$ with $q > 0$. Then there exists a Zariski open W in Y ($y \in W$) and a finite étale morphism $\tilde{\tilde{W}} \to W$ such that $\beta|_{\tilde{\tilde{W}}} = 0$ in $H^q_{\text{ét}}(\tilde{\tilde{W}}, G|_{\tilde{\tilde{W}}})$.*

2.11. Reduction of Theorem 2.1 to Lemma 2.9. Denote the sheaf $\widetilde{F}$ by $\mathcal{F}$. Consider the Hochshild–Serre spectral sequence with the second term $H^*(\pi_1(U), H^*_{\text{et}}(U^{\text{univ}}, \mathcal{F}))$ converging to the group $H^*_{\text{et}}(U, \mathcal{F})$. Taking the direct limit of these spectral sequences one obtains a spectral sequence with the second term $\varinjlim_{x \in U} H^*(\pi_1(U), H^*_{\text{et}}(U^{\text{univ}}, \mathcal{F}))$ converging to the group $\varinjlim H^*_{\text{et}}(U, \mathcal{F})$.

Lemma 2.9 shows that $\varinjlim_{x \in U} H^p(\pi_1(U), H^q_{\text{ét}}(U^{\text{univ}}, \mathcal{F})) = 0$ for $q > 0$. Thus the boundary effect of the last spectral sequence

$$\varinjlim_{x \in U} H^*(\pi_1(U), F) \longrightarrow \varinjlim_{x \in U} H^*_{\text{et}}(U, \mathcal{F})$$

is an isomorphism. The reduction is completed.

2.12. Reduction of Lemma 2.9 to Sublemma 2.10. Take $Y = \widetilde{U}$, $G = \mathcal{F}|_{\widetilde{U}}$ and $\beta = \alpha$. By Sublemma 2.10 there exists a Zariski open W in Y containing the finite set $\pi^{-1}(x)$ and there exists a finite étale morphism $\widetilde{W} \to W$ such that $\beta|_{\widetilde{W}} = 0$ in $H^q(\widetilde{W}, G|_{\widetilde{W}})$. Since W contains $\pi^{-1}(x)$ there exists a Zariski open neighborhood V of the set x in U such that $\widetilde{V}$ is contained in W, where $\widetilde{V} = V \times_U \widetilde{U}$. Set $\tilde{\tilde{V}} = \widetilde{V} \times_W \widetilde{W}$. The couple $(\tilde{\tilde{V}}, V)$ is the desired couple. In fact, $\tilde{\tilde{V}}$ is finite étale over V and $\alpha|_{\tilde{\tilde{V}}} = 0$.

The last holds because already $\alpha = \beta\big|_{\widetilde{W}} = \beta\big|_{\widetilde{W}} = 0$. This completes the reduction. $\square$

2.13. Proof of Sublemma 2.10. We will use the induction by q and $\dim(Y)$. Let $\widetilde{Y} \xrightarrow{\rho} Y$ be a finite étale morphism such that the sheaf $\mathcal{F}\big|_{\widetilde{Y}}$ is a constant sheaf. Choose an Artin's neighborhood $V \subseteq \widetilde{Y} \circ f$ the finite set $\rho^{-1}(y)$ and consider a commutative diagram (see [1], Def. 3.1, Prop. 3.3)

$$\begin{array}{ccccc} V & \xrightarrow{j} & \overline{V} & \xleftarrow{i} & V_\infty \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle \overline{g}} & & \downarrow{\scriptstyle h} \\ B & \xrightarrow{id} & B & \xrightarrow{id} & B \end{array}$$

subjecting to the following conditions

(a) j is an open imbedding which is dense in the each fibre over B;

(b) $V = \overline{V} \setminus V_\infty$;

(c) $\overline{g}$ is a smooth projective morphism and fibres of $\overline{g}$ are geometrically irreducible of dimension 1;

(d) h is finite étale morphism which has no empty fibre.

Since $\mathcal{F}\big|_V$ is a constant sheaf the spectral sequence

$$R^p\,\overline{g}_*\left(R^q\,j_*\left(\mathcal{F}\big|_V\right)\right) \Longrightarrow R^{p+q}\,g_*\left(\mathcal{F}\big|_V\right)$$

shows that $R^l\,g_*\left(\mathcal{F}\big|_V\right) = 0$ for $p > 2$ and $R^1\,g_*\left(\mathcal{F}\big|_V\right)$, $R^0\,g_*\left(\mathcal{F}\big|_V\right) = \mathcal{F}$ are locally constant constructible sheaves on B (see arguments in [1], Proof of 4.5).

This calculation of $R^l\,g_*\left(\mathcal{F}\big|_V\right)$ shows that the spectral sequence

$$H^p_{\text{ét}}\left(B, R^q\,g_*\left(\mathcal{F}\big|_V\right)\right) \Longrightarrow H^{p+q}_{\text{ét}}\left(V, \mathcal{F}\big|_V\right)$$

give rises to a long exact sequence

$$\ldots \longrightarrow H^q_{\text{ét}}(B,\mathcal{F}) \longrightarrow H^q_{\text{ét}}\left(V,\mathcal{F}\big|_V\right) \xrightarrow{\partial} H^{q-1}_{\text{et}}\left(B, R^1\,g_*\left(\mathcal{F}\big|_V\right)\right) \longrightarrow \ldots.$$

Consider the element $\partial(\alpha\big|_V)$ in $H^{q-1}_{\text{ét}}(B, R^1\,g_*(\mathcal{F}\big|_V))$. By the inductive hypotheses there exists a Zariski open heighborhood B_0 of the set $g(\rho^{-1}(y))$ (g was choosen such that $g(\rho^{-1}(y))$ consists of exactly one point) and there exists a finite étale morphism $\widetilde{B}_0 \xrightarrow{\eta} B_0$ such that $\partial((\alpha\big|_V)\big|_{\widetilde{B}_0} = 0$. Then the element $\alpha\big|_{\widetilde{V}_0}$ (here $\widetilde{V}_0 = \widetilde{B}_0 \times_B V$) comes from an element $\gamma \in H^q_{\text{ét}}(\widetilde{B}_0, \mathcal{F})$. By the inductive hypotheses there exists a Zariski open neigborhood $\widetilde{C}_0$ of the set $\eta^{-1}(g(\rho^{-1}(y)))$ and a finite étale morphism $\tilde{\widetilde{C}}_0 \xrightarrow{\varepsilon} \widetilde{C}_0$ such that $\gamma\big|_{\tilde{\widetilde{C}}_0} = 0$ in $H^q_{\text{ét}}(\tilde{\widetilde{C}}_0, \mathcal{F})$.

Further one can find a Zariski open neighborhood $B_{00} \hookrightarrow B_0$ of the $g(\rho^{-1}(y))$ such that $\widetilde{B}_{00}$ is contained in $\widetilde{C}_0$. Set $\tilde{\widetilde{C}}_0 = \varepsilon^{-1}(\widetilde{B}_{00})$ and set $\tilde{\widetilde{V}}_0 = \tilde{\widetilde{C}}_{00} \times_B V$ and set $V_{00} = B_{00} \times_B V$. Then V_{00} is a Zariski open neighborhood of $\rho^{-1}(y)$ and the morphism $\tilde{\widetilde{V}}_{00} \to V_{00}$ is a finite étale morphism such that $(\alpha\big|_V)\big|_{\tilde{\widetilde{V}}_{00}} = 0$ in $H^q_{\text{et}}(\tilde{\widetilde{V}}_{00}, \mathcal{F})$.

Since $\rho^{-1}(y) \subseteq V_{00}$ one can find a Zariski open W in Y containing the finite set y and such that $\widetilde{W} = W \times_Y \widetilde{Y}$ is contained in V_{00}. Set $\tilde{\widetilde{W}} = \tilde{\widetilde{V}}_{00} \times_{V_0} V$. Then clearly the couple $(\tilde{\widetilde{W}}, W)$ is the desired couple. The proof of Sublemma 2.10 is completed. Thus Theorem 2.1 is proved. $\square$

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