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## CONFORMAL RICCI SOLITON IN AN INDEFINITE TRANS-SASAKIAN MANIFOLD

**S. Girish Babu<sup>1</sup>, P. S. K. Reddy<sup>2</sup> and G. Somashekara<sup>2</sup>**

<sup>1</sup> Acharya institute of Technology, Bengaluru 560107, Karnataka, India;

<sup>2</sup> JSS Science and Technology University, Mysuru 570006, Karnataka, India;

<sup>3</sup> Ramaiah University of Applied Sciences, Bengaluru 560054, Karnataka, India

E-mail: sgirishbabu84@gmail.com, pskreddy@jssstuniv.in,  
somashekara.mt.mp@msruas.ac.in

**Abstract.** Conformal Ricci solitons are self similar solutions of the conformal Ricci flow equation. A new class of  $n$ -dimensional almost contact manifold namely trans-Sasakian manifold was introduced by Oubina in 1985 and further study about the local structures of trans-Sasakian manifolds was carried by several authors. As a natural generalization of both Sasakian and Kenmotsu manifolds, the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kahler manifolds introduced by Oubina. This paper deals with the study of conformal Ricci solitons within the framework of indefinite trans-Sasakian manifold. Further, we investigate the certain curvature tensor on indefinite trans-Sasakian manifold. Also, we have proved some important results.

**Key words:** indefinite trans-Sasakian manifold, trans-Sasakian manifold, Ricci flow, conformal Ricci flow.

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### 1. Introduction

In 1982 Hamilton [3] discovered that the Ricci solitons move under the Ricci flow simply by diffeomorphisms of the initial metric; that is, they are stationary points of the Ricci flow given by

$$\frac{\partial g}{\partial t} = -2S. \quad (1.1)$$

In 2004 Fischer [4] introduced the concept of conformal Ricci flow which is a variation of the classical Ricci flow equation. In Ricci flow equation the unit volume constraint plays a important role but in conformal Ricci flow equation scalar curvature  $r$  is considered as constraint.

$$\frac{\partial g}{\partial t} + 2 \left( S + \frac{g}{n} \right) = -pg, \quad (1.2)$$

where  $p$  is a scalar non-dynamical field and  $n$  is the dimension of the manifold.

In the year 2015, Basu and Bhattacharyya [1] introduced the notion of conformal Ricci soliton equation as:

$$L_V g + 2S = \left[ 2\lambda - \left( p + \frac{2}{n} \right) \right] g. \quad (1.3)$$

In 1985 J. A. Oubina [5] introduced a new class of almost contact manifold namely trans-Sasakian manifold.

## 2. Preliminaries

A smooth manifold  $(M^n, g)$  is said to be indefinite almost contact metric manifold, if there exists a a  $(1, 1)$  tensor field  $\varphi$ , structure vector field  $\xi$ , a 1-form  $\eta$  and an indefinite metric  $g$  such that (see [2]):

$$\varphi^2 X_1 = -X_1 + \eta(X_1)\xi, \quad \varphi\xi = 0, \quad \eta(\varphi X_1) = 0, \quad \eta(\xi) = 1, \quad g(\xi, \xi) = \varepsilon, \quad (2.1)$$

$$\eta(X_1) = \varepsilon g(\xi, X_1), \quad g(\varphi(X_1), \varphi(Y_1)) = g(X_1, Y_1) - \varepsilon \eta(X_1)\eta(Y_1), \quad (2.2)$$

$$g(\varphi X_1, Y_1) = -g(X_1, \varphi Y_1), \quad g(\varphi X_1, X_1) = 0, \quad (2.3)$$

for all vector fields  $X_1, Y_1$  on manifold  $M$ , where  $\varepsilon = \pm 1$  accordingly as  $\xi$  is space like vector field and rank  $\varphi$  is  $n - 1$ .

If

$$d\eta(X_1, Y_1) = g(X_1, \varphi Y_1), \quad (2.4)$$

then  $M^n(\varphi, \xi, \eta, g)$  is called an indefinite contact metric manifold.

Indefinite almost contact metric manifold is called an indefinite trans-Sasakian manifold if it is of the form

$$\nabla_{X_1} \varphi Y_1 = \alpha(g(X_1, Y_1)\xi - \varepsilon \eta(Y_1)X_1) + \beta(g(\varphi X_1, Y_1)\xi - \varepsilon \eta(Y_1)\varphi X_1), \quad (2.5)$$

for any  $X_1, Y_1 \in \Gamma(TM)$ , where  $\nabla$  is a metric connection of indefinite metric  $g$ ,  $\alpha$  and  $\beta$  are smooth function on a manifold  $M^n$ .

On using (2.1), (2.2), (2.3), (2.4) and (2.5), we get

$$\nabla_{X_1} \xi = \varepsilon [-\alpha \varphi X_1 + \beta(X_1 - \eta X_1)\xi], \quad (2.6)$$

$$(\nabla_{X_1} \eta) Y_1 = -\alpha g(\varphi X_1, Y_1) + \beta[g(X_1, Y_1) - \varepsilon \eta(X_1)\eta(Y_1)]. \quad (2.7)$$

The indefinite trans-Sasakian manifold  $M^n$ , the following relation holds:

$$\begin{aligned} R(X_1, Y_1)\xi &= (\alpha^2 - \beta^2)(\eta(Y_1)X_1 - \eta(X_1)Y_1) + 2\alpha\beta(\eta(Y_1)\varphi X_1 - \eta(X_1)\varphi(Y_1)) \\ &\quad + \varepsilon((Y_1\alpha)\varphi X_1 - (X_1\alpha)\varphi Y_1 + (Y_1\beta)\varphi^2 X_1 - (X_1\beta)\varphi^2 Y_1), \end{aligned} \quad (2.8)$$

$$\begin{aligned} R(\xi, Y_1)Z_1 &= (\alpha^2 - \beta^2)(\varepsilon g(Y_1, Z_1)\xi - \eta(Z_1)Y_1) + 2\alpha\beta(\varepsilon g(\varphi Y_1, \varphi Z_1)\xi + \eta(Z_1)\varphi Y_1) \\ &\quad + \varepsilon(Z_1\alpha)\varphi Y_1 + \varepsilon g(Y_1, \varphi Z_1)(grad\alpha) - \varepsilon g(\varphi Y_1, \varphi Z_1)(grad\beta) + \varepsilon(Z_1\beta)(Y_1 - \eta(Z_1)\xi), \end{aligned} \quad (2.9)$$

$$S(Z_1, \xi) = ((n-1)(\alpha^2 - \beta^2) - \varepsilon(\xi\beta))\eta(Z_1) - \varepsilon(n-2)(Z_1\beta), \quad (2.10)$$

$$S(\xi, \xi) = (n-1)(\alpha^2 - \beta^2) - \varepsilon(n-1)(\xi\beta), \quad (2.11)$$

$$Q\xi = \varepsilon(n-1)(\alpha^2 - \beta^2)\xi - (\xi\beta)\xi + \varepsilon\varphi(grad\alpha) - \varepsilon(n-2)(grad\beta), \quad (2.12)$$

where  $R$  is the Riemannian curvature tensor,  $S$  is the Ricci tensor and  $Q$  is the Ricci operator. Then we have that  $S(X_1, Y_1) = g(QX_1, Y_1)$  ( $\forall X_1, Y_1 \in \Gamma(TM)$ ).

Now from equation 1.3, we have

$$S(X_1, Y_1) = A_1 g(X_1, Y_1) + A_2 \eta(X_1)\eta(Y_1), \quad (2.13)$$

where  $A_1 = \frac{1}{2}(2\lambda - (p + \frac{2}{n}) - \varepsilon\beta)$ ,  $A_2 = \varepsilon\beta$

$$QX_1 = A_1 X_1 + A_2 \eta(X_1)\xi, \quad (2.14)$$

$$S(X_1, \xi) = A_4 \eta(X_1), \quad (2.15)$$

where  $A_4 = (\varepsilon A_1 + A_2)$

$$Q\xi = A_3 \xi, \quad (2.16)$$

where  $A_3 = A_1 + A_2$ .

### 3. Conformal Ricci Soliton in an Indefinite Trans-Sasakian Manifold Satisfying $R(\xi, X_1) \cdot \tilde{C} = 0$

Let a  $n$ -dimensional conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1) \cdot \tilde{C} = 0$ , where  $\tilde{C}$  is quasi conformal curvature tensor on a manifold  $M$  and is defined by

$$\begin{aligned}\tilde{C}(X_1, Y_1)Z_1 &= aR(X_1, Y_1)Z_1 + b(S(Y_1, Z_1)X_1 - S(X_1, Z_1)Y_1 + g(Y_1, Z_1)QX_1 \\ &\quad - g(X_1, Z_1)QY_1) - \left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)(g(Y_1, Z_1)X_1 - g(X_1, Z_1)Y_1),\end{aligned}\tag{3.1}$$

where  $r$  is scalar curvature.

Substituting  $Z_1 = \xi$ , we get

$$\begin{aligned}\tilde{C}(X_1, Y_1)\xi &= aR(X_1, Y_1)\xi + b(S(Y_1, \xi)X_1 - S(X_1, \xi)Y_1 + g(Y_1, \xi)QX_1 \\ &\quad - g(X_1, \xi)QY_1) - \left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)(g(Y_1, \xi)X_1 - g(X_1, \xi)Y_1).\end{aligned}\tag{3.2}$$

Using equation (2.2), (2.8), (2.13) and (2.14) in (3.2), we get

$$\tilde{C}(X_1, Y_1)\xi = A_5(\eta(Y_1)X_1 - \eta(X_1)Y_1),\tag{3.3}$$

where  $A_5 = \left(a(\alpha^2 - \beta^2) + bA_4 + b\varepsilon A_1 - \varepsilon\left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)\right)$ . Taking inner product with  $Z_1$  equation (3.3) becomes

$$-\eta(\tilde{C}(X_1, Y_1), Z_1) = A_5\varepsilon(\eta(Y_1)g(X_1, Z_1) - \eta(X_1)g(Y_1, Z_1)).\tag{3.4}$$

We assume that  $R(\xi, X_1) \cdot \tilde{C} = 0$ , which implies that

$$\begin{aligned}R(\xi, X_1)(\tilde{C}(Y_1, Z_1)Z_2) - \tilde{C}(R(\xi, X_1)Y_1, Z_1)Z_2 \\ - \tilde{C}(Y_1, R(\xi, X_1)Z_1)Z_2 - \tilde{C}(Y_1, Z_1)R(\xi, X_1)Z_2 = 0,\end{aligned}\tag{3.5}$$

for all vector fields  $X_1, Y_1, Z_1, Z_2$  on a manifold  $M$ .

Putting  $Z_2 = \xi$  and using (2.9), we get

$$\begin{aligned}\varepsilon(\alpha^2 - \beta^2)g(X_1, \tilde{C}(Y_1, Z_1)\xi) - \varepsilon(\alpha^2 - \beta^2)g(X_1, Y_1)\tilde{C}(\xi, Z_1)\xi \\ + (\alpha^2 - \beta^2)\eta(Y_1)\tilde{C}(X_1, Z_1)\xi - \varepsilon(\alpha^2 - \beta^2)g(X_1, Z_1)\tilde{C}(Y_1, \xi)\xi \\ + (\alpha^2 - \beta^2)\eta(Z_1)\tilde{C}(Y_1, Z_1)\xi - (\alpha^2 - \beta^2)\eta(X_1)\tilde{C}(Y_1, Z_1)\xi \\ + (\alpha^2 - \beta^2)\tilde{C}(Y_1, Z_1)X_1 = 0,\end{aligned}\tag{3.6}$$

Taking inner product with  $\xi$  and using (2.2), (3.3), equation (3.6) reduces to

$$g(X_1, \tilde{C}(Y_1, Z_1)\xi) + \eta(\tilde{C}(Y_1, Z_1)X_1) = 0,\tag{3.7}$$

provided  $(\alpha^2 - \beta^2) \neq 0$ .

Substituting  $Z_1 = \xi$  and using (3.3) in (3.7), we obtain

$$A_5g(X_1, Y_1) - A_5\varepsilon\eta(X_1)\eta(Y_1) + \eta(\tilde{C}(Y_1, \xi)X_1) = 0.\tag{3.8}$$

Again substituting  $Y_1 = \xi$  in (3.1), we get

$$\begin{aligned}\tilde{C}(X_1, \xi)Z_1 &= aR(X_1, \xi)Z_1 + b(S(\xi, Z_1)X_1 - S(X_1, Z_1)\xi + g(\xi, Z_1)QX_1 \\ &\quad - g(X_1, Z_1)Q\xi) - \left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)(g(\xi, Z_1)X_1 - g(X_1, Z_1)\xi),\end{aligned}\tag{3.9}$$

Taking inner product with  $\xi$  and using (2.1), (2.2), (2.9), (2.10), (2.11), (2.12), equation (3.9) reduces to

$$\eta(\tilde{C}(X_1, \xi)Z_1) = A_6g(X_1, Z_1) + A_7\eta(X_1)\eta(Z_1) - bS(X_1, Z_1),\tag{3.10}$$

where

$$\begin{aligned}A_6 &= \left(-\varepsilon a(\alpha^2 - \beta^2) - b\varepsilon(A_1 + A_2) + \left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)\varepsilon\right), \\ A_7 &= \left(a(\alpha^2 - \beta^2) + b\varepsilon(A_1 + A_3 + A_4) - \left(\frac{r}{2n+1}\right)\left(\frac{a}{2n} + 2b\right)\right)\end{aligned}$$

replacing  $X_1$  with  $Y_1$  and  $Z_1$  with  $X_1$  in (3.10), we obtain

$$\eta(\tilde{C}(Y_1, \xi)X_1) = A_6g(X_1, Y_1) + A_7\eta(X_1)\eta(Y_1) - bS(X_1, Y_1).\tag{3.11}$$

Substituting (3.11) in (3.8), we get

$$S(X_1, Y_1) = A_8g(X_1, Y_1) + A_9\eta(X_1)\eta(Y_1),\tag{3.12}$$

where  $A_8 = A_5 + A_6$ ,  $A_9 = A_7 - \varepsilon A_5$ .

Hence we can state the following theorem

**Theorem 3.1.** *A conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)\tilde{C} = 0$  is an  $\eta$ -Einstein manifold.*

#### 4. Conformal Ricci Soliton in an Indefinite Trans-Sasakian Manifold Satisfying $R(\xi, X_1).S = 0$

Leta a  $n$ -dimensional conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1).S = 0$ , which implies that

$$S(R(\xi, X_1)Y_1, Z_1) + S(Y_1, R(\xi, X_1)Z_1) = 0.\tag{4.1}$$

Using (2.1), (2.2), (2.9) and (2.13) in (4.1), we get

$$\begin{aligned}&A_1((\alpha^2 - \beta^2)\varepsilon g(X_1, Y_1)\eta(Z_1) - (\alpha^2 - \beta^2)\eta(Y_1)g(X_1, Z_1)) \\ &+ A_1((\alpha^2 - \beta^2)\varepsilon g(X_1, Z_1)\eta(Y_1) - (\alpha^2 - \beta^2)\eta(Z_1)g(X_1, Y_1)) \\ &+ A_2(\alpha^2 - \beta^2)(g(X_1, Y_1)\eta(Z_1) - \varepsilon\eta(X_1)\eta(Y_1)\eta(Z_1)) \\ &+ g(X_1, Z_1)\eta(Y_1) - \varepsilon\eta(X_1)\eta(Y_1)\eta(Z_1)) = 0.\end{aligned}\tag{4.2}$$

Substituting  $Z_1 = \xi$  and using (2.1), (2.2) in (4.2), we get

$$g(X_1, Y_1) = \varepsilon\eta(X_1)\eta(Y_1),\tag{4.3}$$

provided  $A_2(\alpha^2 - \beta^2) \neq 0$ .

Hence, we state the following theorem

**Theorem 4.1.** *A conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)S = 0$ , then  $g(X_1, Y_1) = \varepsilon\eta(X_1)\eta(Y_1)$ .*

### 5. Conformal Ricci Soliton in an Indefinite Trans-Sasakian Manifold Satisfying $R(\xi, X_1)P = 0$

Let a  $n$ -dimensional conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)P = 0$ , where  $P$  is projective curvature tensor on a manifold  $M$  and is defined by

$$P(X_1, Y_1)Z_1 = R(X_1, Y_1)Z_1 - \frac{1}{2n} (S(Y_1, Z_1)X_1 - S(X_1, Z_1)Y_1). \quad (5.1)$$

We assume that  $R(\xi, X_1)P = 0$ , which implies that

$$\begin{aligned} & R(\xi, X_1)(P(Y_1, Z_1)Z_2) - P(R(\xi, X_1)Y_1, Z_1)Z_2 \\ & - P(Y_1, R(\xi, X_1)Z_1)Z_2 - P(Y_1, Z_1)R(\xi, X_1)Z_2 = 0, \end{aligned} \quad (5.2)$$

for all vector fields  $X_1, Y_1, Z_1$  and  $Z_2$  on  $M$ .

Putting  $Z_1 = \xi$  and using equation (2.9) in (5.2), we get

$$\begin{aligned} & \varepsilon g(X_1, P(Y_1, \xi)Z_2)\xi - \eta((P(Y_1, \xi)Z_2)X_1) - \varepsilon g(X_1, Y_1)P(\xi, \xi)Z_2 + \eta(Y_1)P(X_1, \xi)Z_2 \\ & - \varepsilon\eta(X_1)P(Y_1, \xi)Z_2 + P(Y_1, X_1)Z_2 - \varepsilon g(X_1, Z_2)P(Y_1, \xi)\xi + \eta(Z_2)P(Y_1, \xi)X_1 = 0. \end{aligned} \quad (5.3)$$

Substituting  $Y_1 = \xi$  in equation (5.1), we get

$$P(X_1, \xi)Z_1 = R(X_1, \xi)Z_1 - \frac{1}{2n}(S(\xi, Z_1)X_1 - S(X_1, Z_1)\xi), \quad (5.4)$$

using equation (2.9) and (2.15) in (5.4), we get

$$\begin{aligned} & P(X_1, \xi)Z_1 = -\varepsilon(\alpha^2 - \beta^2)g(X_1, Z_1)\xi \\ & + \left( (\alpha^2 - \beta^2) - \frac{A_4}{n-1} \right) \eta(Z_1)X_1 + \frac{1}{n-1}S(X_1, Z_1)\xi. \end{aligned} \quad (5.5)$$

Replacing  $X_1$  with  $Y_1$  and  $Z_1$  with  $Z_2$  in (5.5), we get

$$\begin{aligned} & P(Y_1, \xi)Z_2 = -\varepsilon(\alpha^2 - \beta^2)g(Y_1, Z_2)\xi \\ & + \left( (\alpha^2 - \beta^2) - \frac{A_4}{n-1} \right) \eta(Z_2)Y_1 + \frac{1}{n-1}S(Y_1, Z_2)\xi. \end{aligned} \quad (5.6)$$

Now substituting  $Z_2 = \xi$  and using (5.6) in (5.3), we get

$$\varepsilon \left( \frac{A_4}{n-1} \right) g(X_1, Y_1)\xi + \frac{A_4}{2n}\eta(Y_1)X_1 - \frac{A_4}{2n}\eta(X_1)Y_1 + \frac{1}{n-1}S(X_1, Y_1)\xi = 0. \quad (5.7)$$

Taking inner product with  $\xi$  and using (2.1), (2.2), equation (5.7) becomes

$$S(X_1, Y_1) = -A_{10}g(X_1, Y_1), \quad (5.8)$$

where  $A_{10} = \varepsilon A_4$ . Hence we can state the following theorem

**Theorem 5.1.** *A conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)P = 0$  is an Einstein manifold.*

## 6. Conformal Ricci Soliton in an Indefinite Trans-Sasakian Manifold Satisfying $R(\xi, X_1)\tilde{P} = 0$

Let a  $n$ -dimensional conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)\tilde{P} = 0$ , where  $\tilde{P}$  is pseudo projective curvature tensor on a manifold  $M$  and is defined by

$$\begin{aligned}\tilde{P}(X_1, Y_1)Z_1 &= aR(X_1, Y_1)Z_1 + b(S(Y_1, Z_1)X_1 - S(X_1, Z_1)Y_1) \\ &\quad - \frac{r}{n} \left( \frac{a}{n-1} \right) (g(Y_1, Z_1)X_1 - g(X_1, Z_1)Y_1).\end{aligned}\tag{6.1}$$

We assume that  $R(\xi, X_1)\tilde{P} = 0$ , which implies that

$$\begin{aligned}R(\xi, X_1)(\tilde{P}(Y_1, Z_1)Z_2) - \tilde{P}(R(\xi, X_1)Y_1, Z_1)Z_2 \\ - \tilde{P}(Y_1, R(\xi, X_1)Z_1)Z_2 - \tilde{P}(Y_1, Z_1)R(\xi, X_1)Z_2 = 0,\end{aligned}\tag{6.2}$$

for all vector field  $X_1, Y_1, Z_1$  and  $Z_2$  on  $M$ .

Putting  $Z_2 = \xi$  and using (2.9) in (6.2), we get

$$A_{11}g(X_1, Y_1)Z_1 + A_{12}g(X_1, Z_1)Y_1 + \tilde{P}(Y_1, Z_1)X_1 = 0,\tag{6.3}$$

provided  $(\alpha^2 - \beta^2) \neq 0$  and where

$$A_{11} = \left( a\varepsilon + bA_4\varepsilon - \frac{r}{n} \left( \frac{a}{n-1} + b \right) \right), \quad A_{12} = \left( a\varepsilon - bA_4\varepsilon + \frac{r}{n} \left( \frac{a}{n-1} + b \right) \right).$$

Substituting  $Z_1 = \xi$  in (6.3), we get

$$A_{11}g(X_1, Y_1)\xi + A_{12}g(X_1, \xi)Y_1 + \tilde{P}(Y_1, \xi)X_1 = 0.\tag{6.4}$$

Taking inner product with  $\xi$  and using (2.1), (2.2), equation (6.4) becomes

$$A_{14}g(X_1, Y_1) + A_{13}\eta(X_1)\eta(Y_1) + \eta(\tilde{P}(Y_1, \xi)X_1) = 0,\tag{6.5}$$

where  $A_{13} = \varepsilon A_{12}$ ,  $A_{14} = \varepsilon A_{11}$ . In the view of (6.1) and (6.5) we have

$$S(X_1, Y_1) = A_{15}g(X_1, Y_1) + A_{16}\eta(X_1)\eta(Y_1),\tag{6.6}$$

where

$$\begin{aligned}A_{15} &= \left( \frac{A_{11} - a(\alpha_2 - \beta_2) - \varepsilon \frac{r}{n} \left( \frac{a}{n-1} + b \right)}{b\varepsilon} \right), \\ A_{16} &= \left( \frac{A_{13} + a\varepsilon(\alpha_2 - \beta_2) + bA_4\varepsilon + \frac{r}{n} \left( \frac{a}{n-1} + b \right)}{b\varepsilon} \right).\end{aligned}$$

Hence we can state the following theorem

**Theorem 6.1.** *A conformal Ricci soliton in an indefinite trans-Sasakian manifold satisfying  $R(\xi, X_1)\tilde{P} = 0$  is an  $\eta$ -Einstein manifold.*

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### References

1. Basu, N. and Bhattacharyya, A. Conformal Ricci Soliton in Kenmotsu Manifold, *Glob. J. Adv. Res. Class. Mod. Geom.*, 2015, vol. 4, no. 1, pp. 15–21.

2. Blair, D. E. *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics, Vol. 509, Springer Verlag, 1976.
3. Hamilton, R. S. The Ricci Flow on Surfaces, *Mathematics and General Relativity*, Santa Cruz, CA, 1986, pp. 237–262. *Contemp. Math.*, 71, Amer. Math. Soc., Providence, RI, 1988.
4. Fischer, A. E. An Introduction to Conformal Ricci Flow, *Class. Quantum Grav.*, 2004, vol. 21, pp. S171–S218.
5. Oubina, J. A. New Classes of Almost Contact Metric Structures, *Publ. Math. Debrecen*, 1985, vol. 32, pp. 187–193.

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SHIVANNA GIRISH BABU

Acharya institute of Technology,

Bengaluru 560 107, India

Research Scholar

E-mail: sgirishbabu84@gmail.com,

POLAEPALLI SIVA KOTA REDDY

JSS Science and Technology University,

Mysuru 570 006, Karnataka, India,

Professor

E-mail: pskreddy@jssstuniv.in

GANGANNA SOMASHEKHARA

Ramaiah University of Applied Sciences,

Bengaluru 560 054, India,

Associate Professor

E-mail: somashekara.mt.mp@msruas.ac.in

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## КОНФОРМНЫЕ СОЛИТОНЫ РИЧЧИ НА НЕОПРЕДЕЛЕННОМ ТРАНССАСАКИЕВОМ МНОГООБРАЗИИ

Гириш Бабу С.<sup>1</sup>, Редди П. С. К.<sup>2</sup>, Сомашекхара Г.<sup>3</sup>

<sup>1</sup> Технологический институт Ачары, Бангалор 560107, Карнатака, Индия;

<sup>2</sup> Научно-технический университет JSS, Майсур 570006, Карнатака, Индия;

<sup>3</sup> Университет прикладных наук Рамая, Бангалор 560054, Карнатака, Индия

E-mail: sgirishbabu84@gmail.com, pskreddy@jssstuniv.in,  
somashekara.mt.mp@msruas.ac.in

**Аннотация.** Конформные солитоны Риччи являются автомодельными решениями конформного уравнения потока Риччи. Новый класс  $n$ -мерных почти контактных многообразий, а именно транссасакиевые многообразия, был введен Обиной в 1985 г. Дальнейшее изучение локальной структуры транссасакиевых многообразий было проведено несколькими авторами. Транссасакиевые многообразия, являющиеся естественным обобщением как сасакиевых многообразий, так и многообразий Кенмоцу, тесно связаны с локально конформными келеровыми многообразиями. В настоящей статье изучаются конформные солитоны Риччи в контексте неопределенного транссасакиева многообразия. Исследован тензор кривизны на неопределенном транссасакиевом многообразии и доказаны некоторые важные результаты.

**Ключевые слова:** неопределенное транссасакиево многообразие, поток Риччи, конформный поток Риччи.

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