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Kinematic Analysis of Pericyclic Transmission Mechanism

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The article presents a kinematic analysis of the pericyclic mechanism, where the gear is internally engaged with fixed central wheel. Geometric features of the trajectory of the mechanism allows use of this type of mechanisms in machines, which provide such kind of technological processes where slight fluctuations of the driven link sometimes are necessary at the extreme position of the system. Mechanisms of this type can be used in different technological machines such as metal cutting machines as well used in light industry, where the processing of any tissue requires slight fluctuations of the executive body at the temporary position.

Keywords and phrases: Mechanism, cycloid, planetary gears, trajectory, velocity, acceleration.

AMS subject classification: 70B10.

1 Introduction

Planetary gears with a satellite or with satellites, which allow an approximate quasi-stop of the output link, are widely used in textile and light industry machines. Such allowances for minor deviations during the dwell of the working body of the mechanism are due to the flexibility and pliability of the materials processed and driven in this industry.

For example, in special sewing machines, there are mechanisms for transverse or longitudinal vibrations of the needle when stitching the material. This behavior of the tool does not have a damaging effect on the material being processed, in the process of puncturing the material with a needle. In such cases, it is possible to use mechanisms with less kinematic complexity and making it possible to develop as high speeds as possible for high-speed sewing and textile machines.

2 Main part

In this paper, we consider a planetary gear with a planetary gear that is in internal engagement with a stationary central wheel (Figure 1). Our task is to determine the speed and acceleration

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of the hole for the sewing thread of the driven link when the driving link O_1O_2 rotates with a certain angular velocity and angular acceleration. The angular velocities of the carrier and the moving link are related by the relation:

$$\omega_1 \cdot \ |O_1O_2| = \omega_2 \cdot |PO_2|$$

whence the angular velocity of the link 2 will be:

$$\omega_2 = \frac{|O_1O_2|}{|PO_2|} \cdot \omega_1 = \frac{r_2 - r_1}{r_2} \cdot \omega_1$$

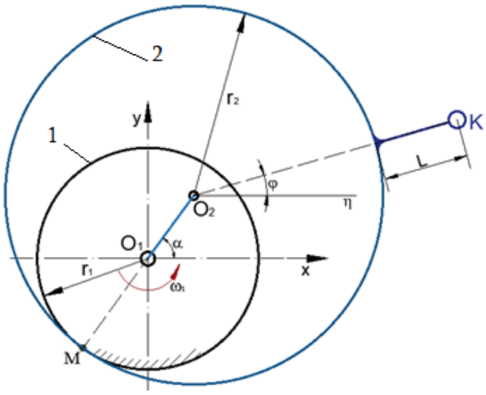


Figure 1

To determine the speeds of any point of the driven link, it is necessary to find the position of the instantaneous center of speed of the gear with internal gearing. The mentioned, instantaneous center of speed is the point P of contact between gear 2 and wheel of gear 1.

The crank O_1O_2 moves with an angular velocity ω_1 and with an angular acceleration ε_1 relative to the point O_1. Based on this, the speed of the point O_2 is determined by the formula:

$$V_{O_2} = \omega_1 \cdot |O_1O_2| = (r_2 - r_1) \cdot \omega_1 .$$

After that, we find the speed of point K by the following formula:

$$V_{KO_2} = \omega_2 \cdot |KO_2| = \frac{(r_2 - r_1) \cdot \dot{\alpha}}{r_2} \cdot (r_2 + L) .$$

Full velocity of the point K, can be written as:

$$\vec{V}_K = \vec{V}_{O_2} + \vec{V}_{KO_2};$$

Projections of the full velocity of the point K on the axes x and y will be written as:

$$V_{Kx} = V_{O_2} \cdot cos\left(90^{\circ} + \alpha\right) + V_{KO_2} \cdot cos\left(90^{\circ} + \varphi\right)$$
$$= -(r_2 - r_1) \cdot \dot{\alpha} \cdot sin\alpha - \frac{(r_2 - r_1) \cdot \dot{\alpha}}{r_2} \cdot (r_2 + L) \cdot sin\left(1 - \frac{r_1}{r_2}\right)\alpha;$$

$$V_{Ky} = V_{O_2} \cdot sin\left(90^{\circ} + \alpha\right) + V_{KO_2} \cdot sin\left(90^{\circ} + \varphi\right)$$
$$= (r_2 - r_1) \cdot \dot{\alpha} \cdot cos\alpha + \frac{(r_2 - r_1) \cdot \dot{\alpha}}{r_2} \cdot (r_2 + L) \cdot cos\left(1 - \frac{r_1}{r_2}\right)\alpha;$$

and hence we will have:

$$V_K = \sqrt{V_{Kx}^2 + V_{Ky}^2}.$$

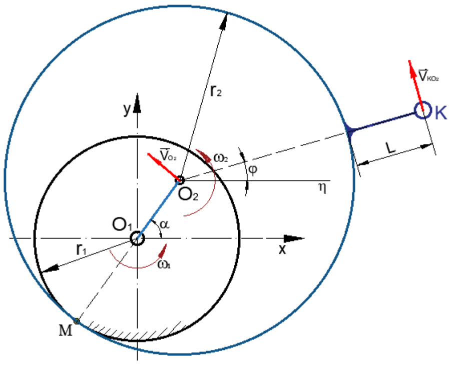


Figure 2

At the same time, if the crank O_1O_2 rotates around the point O_1 with a constant angular velocity ω_1 then the centripetal acceleration $W_{O_2}^n$ will have the form

$$W_{O_2}^n = \omega_1^2 \cdot (r_2 - r_1) = \dot{\alpha}^2 \cdot (r_2 - r_1).$$

Centripetal acceleration of point K of the gear in relative motion can be determined as:

$$W_K^n = \omega_2^2 \cdot (r_2 + L) = \frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2.$$

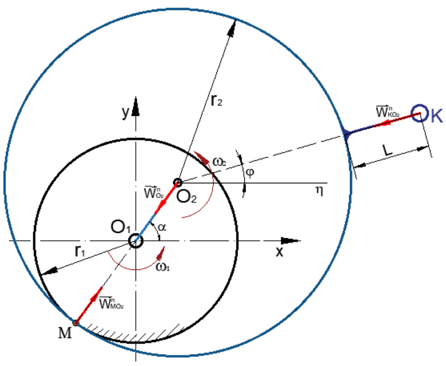


Figure 3

The vector of full acceleration of point K is found as the sum of vectors of translational and relative acceleration

$$\overrightarrow{W}_K = \overrightarrow{W}_{O_2}^n + \overrightarrow{W}_{KO_2}^n.$$

At the same time the meanings of modules of here given accelerations are equal

$$W_{Kx} = -W^n_{O_2} \cdot cos\alpha - W^n_{KO_2} \cdot cos\varphi$$
$$= -\dot{\alpha}^2 \cdot (r_2 - r_1) \cdot cos\alpha - \frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2 \cdot \cos\left(1 - \frac{r_1}{r_2}\right)\alpha;$$

$$W_{Kx} = -W^n_{O_2} \cdot sin\alpha - W^n_{KO_2} \cdot sin\varphi$$
$$= -\dot{\alpha}^2 \cdot (r_2 - r_1) \cdot sin\alpha - \frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2 \cdot \sin\left(1 - \frac{r_1}{r_2}\right)\alpha.$$

The angle among them is:

$$W_K = \sqrt{W^2_{Kx} + W^2_{Ky}}.$$

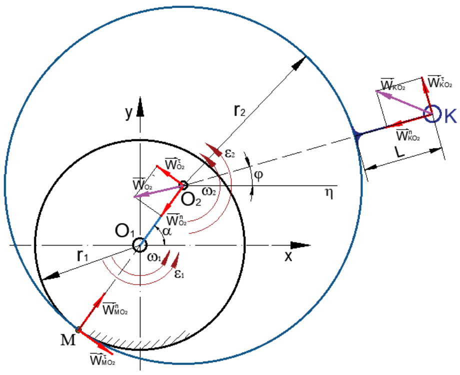


Figure 4

If the crank, in addition to the angular velocity, also has an angular acceleration, then to determine the total acceleration of the point K, it is necessary to use the following analytical dependence

$$\omega_2 \cdot r_2 = (r_2 - r_1)\,\dot{\alpha};$$
$$\omega_2 = \frac{(r_2 - r_1)\,\dot{\alpha}}{r_2}.$$

Differentiating this fraction with respect of time, we find the angular acceleration of the gear:

$$\varepsilon_2 = \frac{d\omega_2}{dt} = \frac{(r_2 - r_1)\,\ddot{\alpha}}{r_2}.$$

The tangential acceleration of the point O_2, as the point of the crank O_1O_2, is directed towards O_1 and is equal in absolute value to:

$$W^n_{O_2} = \dot{\alpha}^2 \cdot (r_2 - r_1)$$

The module of rotation acceleration of the point O_2 is perpendicular to the straight-line O_1O_2can be calculated as:

$$W^\tau_{O_2} = \ddot{\alpha} \cdot (r_2 - r_1)\,.$$

If the point O_2 is taken as the pole, then the total acceleration of the point K can be found by the following equation: based on this we will have:

$$\overrightarrow{W}_K = \overrightarrow{W}_{O_2} + \overrightarrow{W}^n_{KO_2} + \overrightarrow{W}^\tau_{KO_2}.$$

The acceleration of the point K has already been found through the components$W^n_{O_2}$and$W^\tau_{O_2}$. The tangent acceleration of the point K relative to the point O_2 is directed towards $O_2$the module of this acceleration is equal to $\omega_2^2 \cdot (r_2 + L)$. Substituting the value ω_2, we will have:

$$W^n_{KO_2} = \omega_2^2 \cdot (r_2 + L) = \frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2.$$

The rotational acceleration of the point K relative to the point O_2 is directed perpendicular to the straight-lineO_2K and the algebraic value of this acceleration is:

$$W^\tau_{KO_2} = \varepsilon_2 \cdot (r_2 + L) = \frac{(r_2 - r_1) \cdot (r_2 + L) \cdot \ddot{\alpha}}{r_2};$$

Projecting both sides of equality (1) onto the x and y axes

$$\begin{aligned} W_{Kx} &= -W^n_{O_2O_1} \cdot cos\alpha - W^\tau_{O_2O_1} sin\alpha - W^n_{KO_2} \cdot cos\varphi - W^\tau_{KO_2} \cdot sin\varphi; \\ W_{Ky} &= -W^n_{O_2O_1} \cdot sin\alpha + W^\tau_{O_2O_1} cos\alpha - W^n_{KO_2} \cdot sin\varphi + W^\tau_{KO_2} \cdot cos\varphi. \end{aligned} \tag{1}$$

In an expanded form, these equations can be written as:

$$W_{Kx} = -(r_2 - r_1) \cdot \dot{\alpha}^2 \cdot cos\alpha - (r_2 - r_1) \cdot \ddot{\alpha} sin\alpha$$
$$-\frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2 \cdot cos\left(1 - \frac{r_1}{r_2}\right)\alpha - \frac{(r_2 - r_1) \cdot (r_2 + L) \cdot \ddot{\alpha}}{r_2} \cdot sin\left(1 - \frac{r_1}{r_2}\right)\alpha;$$

$$W_{Ky} = -(r_2 - r_1) \cdot \dot{\alpha}^2 \cdot sin\alpha + (r_2 - r_1) \cdot \ddot{\alpha} cos\alpha$$
$$-\frac{(r_2 - r_1)^2 \cdot (r_2 + L)}{r_2^2} \cdot \dot{\alpha}^2 \cdot sin\left(1 - \frac{r_1}{r_2}\right)\alpha + \frac{(r_2 - r_1) \cdot (r_2 + L) \cdot \ddot{\alpha}}{r_2} \cdot cos\left(1 - \frac{r_1}{r_2}\right)\alpha.$$

The full acceleration value is calculated as follows:

$$W_K = \sqrt{W_{Kx}^2 + W_{Ky}^2}.$$

The distance from the point K to the instantaneous center of acceleration Q is calculated by the following formula:

$$|KQ| = \frac{W_K}{\sqrt{\omega_2^4 + \varepsilon_2^2}}.$$

To determine the angle between the acceleration W_K and the segment KQ, we use the formula

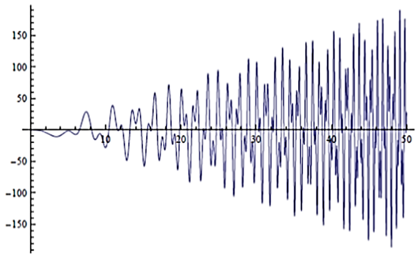
$$tg\theta = \frac{\varepsilon}{\omega^2}$$

Regarding the given equations and using computer program “Wolfram” the numerical examples have been calculated. Based on these examples, changing of the values of speed and acceleration in time in the graphical form are shown below.

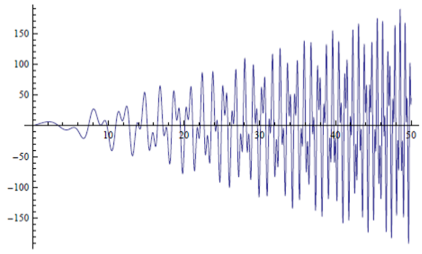
During calculations the angular acceleration ε was taken equal to 0.4 rad/sec; The angular velocity $\omega = 0$ when $t = 0$. The step during calculations have been taken equal to 0.05 and the time was varied from 0 up to 50 sec.

```
V1[t_] := Plot[-1.6 t×Sin[0.2×t^2] - 2.311 t×Sin[0.088×t^2], {t, 0, 50}]
V2[t_] := Plot[1.6 t×Cos[0.2×t^2] + 2.311 t×Cos[0.088*t^2], {t, 0, 50}]
V[t_] := Plot[Sqrt[V1^2 + V2^2], {t, 0, 50}, Axes → {True, True}]
Show[V1, V2]
```

```
V1 = Plot[-1.6 t×Sin[0.2×t^2] - 2.311 t×Sin[0.088×t^2], {t, 0, 50}]
```

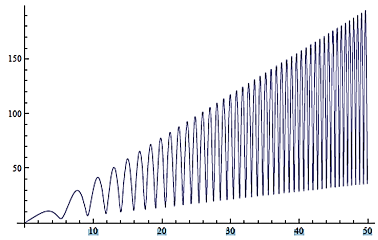


```
V2 = Plot[1.6 t×Cos[0.2×t^2] + 2.311 t×Cos[0.088*t^2], {t, 0, 50}]
```



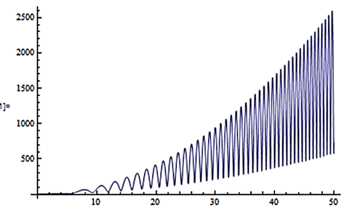
By summing these curves of velocities we obtain the following summarized plot.

```
V = Plot[Sqrt[(1.6 t×Cos[0.2×t^2] + 2.311 t×Cos[0.088*t^2])^2 +
    (-1.6 t×Sin[0.2×t^2] - 2.311 t×Sin[0.088×t^2])^2], {t, 0, 50}]
```



```
In[1]:= W =
  Plot[Sqrt[(-0.64 t^2×Cos[0.2×t^2] - 1.6 t×Sin[0.2*t^2] - 0.41 t^2×Cos[0.088*t^2] - 2.311 Sin[0.088 t^2])^2 +
    (-0.64 t^2×Sin[0.2×t^2] + 1.6 t×Cos[0.2*t^2] - 0.41 t^2×Sin[0.088*t^2] + 2.311 Cos[0.088 t^2])^2], {t, 0, 50}]
```

Out[1]=



3 Summary

The article presents a kinematic solution of a planetary mechanism with a satellite, which is in external engagement with a fixed central wheel. The geometric features of the mechanism path allow the use of this mechanism in machines, providing technological processes for which minor oscillations of the driven link are sometimes required at the stop stage. Mechanisms of this type are used in different fields of industry, like mechanical engineering as well textile industry, where in the treatment of a material, is required that the executive body can make slight fluctuations relative to the temporary position.

Acknowledgements

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