Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume 9 (2014), 149 – 166

A COVARIANT STINESPRING TYPE THEOREM FOR τ-MAPS

Harsh Trivedi

Abstract. Let τ be a linear map from a unital C^*-algebra $\mathcal{A}$ to a von Neumann algebra $\mathcal{B}$ and let $\mathcal{C}$ be a unital C^*-algebra. A map T from a Hilbert $\mathcal{A}$-module E to a von Neumann $\mathcal{C}$-$\mathcal{B}$ module F is called a τ-map if

$$\langle T(x), T(y)\rangle = \tau(\langle x, y\rangle) \text{ for all } x, y \in E.$$

A Stinespring type theorem for τ-maps and its covariant version are obtained when τ is completely positive. We show that there is a bijective correspondence between the set of all τ-maps from E to F which are (u', u)-covariant with respect to the dynamical system (G, η, E) and the set of all (u', u)-covariant $\widetilde{\tau}$-maps from the crossed product $E \times_\eta G$ to F, where τ and $\widetilde{\tau}$ are completely positive.

Full text

Acknowledgement. The author would like to express thanks of gratitude to Santanu Dey for several discussions.

References

[1] Panchugopal Bikram, Kunal Mukherjee, R. Srinivasan, and V. S. Sunder, *Hilbert von Neumann modules*, Commun. Stoch. Anal. **6** (2012), no. 1, 49–64. MR2890849

[2] B. V. Rajarama Bhat, G. Ramesh, and K. Sumesh, *Stinespring's theorem for maps on Hilbert C^*-modules*, J. Operator Theory **68** (2012), no. 1, 173–178. MR 2966040. Zbl 1265.46091.

[3] B. V. Rajarama Bhat and K. Sumesh, *Bures distance for completely positive maps*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **16** (2013), no. 4, 1350031, 22. MR 3192708. Zbl 1295.46043.

2010 Mathematics Subject Classification: Primary: 46L08, 46L55; Secondary: 46L07, 46L53.
Keywords: Stinespring representation; Completely positive maps; Von Neumann modules; Dynamical systems.
This work was supported by CSIR, India.

[4] Man Duen Choi and Edward G. Effros, *The completely positive lifting problem for C^*-algebras*, Ann. of Math. (2) **104** (1976), no. 3, 585–609. MR 0417795 (54 #5843). Zbl 0361.46067.

[5] Santanu Dey and Harsh Trivedi, *$\mathfrak{K}$-families and CPD-H-extendable families*, arXiv:1409.3655v1 (2014). arXiv:1409.3655v1.

[6] Siegfried Echterhoff, S. Kaliszewski, John Quigg, and Iain Raeburn, *Naturality and induced representations*, Bull. Austral. Math. Soc. **61** (2000), no. 3, 415–438. MR 1762638 (2001j:46101). Zbl 0982.46051.

[7] Heo Jaeseong, *Completely multi-positive linear maps and representations on Hilbert C^*-modules*, J. Operator Theory **41** (1999), no. 1, 3–22. MR 1675235. Zbl 0994.46019.

[8] Heo Jaeseong and Ji Un Cig, *Quantum stochastic processes for maps on Hilbert C^*-modules*, J. Math. Phys. **52** (2011), no. 5, 053501, 16pp. MR 2839082.

[9] Maria Joiţa, *Covariant version of the Stinespring type theorem for Hilbert C^*-modules*, Cent. Eur. J. Math. **9** (2011), no. 4, 803–813. MR 2805314 (2012f:46110). Zbl 1243.46049.

[10] Maria Joiţa, *Covariant representations of Hilbert C^*-modules*, Expo. Math. **30** (2012), no. 2, 209–220. MR 2928201. Zbl 1252.46055.

[11] Alexander Kaplan, *Covariant completely positive maps and liftings*, Rocky Mountain J. Math. **23** (1993), no. 3, 939–946. MR 1245456 (94k:46139). Zbl 0796.46042.

[12] E. C. Lance, *Hilbert C^*-modules*, London Mathematical Society Lecture Note Series, vol. 210, Cambridge University Press, Cambridge, 1995, A toolkit for operator algebraists. MR 1325694 (96k:46100). Zbl 0822.46080.

[13] William L. Paschke, *Inner product modules over B^*-algebras*, Trans. Amer. Math. Soc. **182** (1973), 443–468. MR 0355613 (50 #8087). Zbl 0239.46062.

[14] Vern Paulsen, *A covariant version of Ext*, Michigan Math. J. **29** (1982), no. 2, 131–142. MR 654474 (83f:46076). Zbl 0507.46060.

[15] Marc A. Rieffel, *Induced representations of C^*-algebras*, Advances in Math. **13** (1974), 176–257. MR 0353003 (50 #5489). Zbl 0284.46040.

[16] Iain Raeburn and Shaun J. Thompson, *Countably generated Hilbert modules, the Kasparov stabilisation theorem, and frames with Hilbert modules*, Proc. Amer. Math. Soc. **131** (2003), no. 5, 1557–1564 (electronic). MR 1949886 (2003j:46089). Zbl 1015.46034.

[17] Iain Raeburn and Dana P. Williams, *Morita equivalence and continuous-trace C^*-algebras*, Mathematical Surveys and Monographs, vol. 60, American Mathematical Society, Providence, RI, 1998. MR 1634408 (2000c:46108) Zbl 0922.46050.

[18] Michael Skeide, *Generalised matrix C^*-algebras and representations of Hilbert modules*, Math. Proc. R. Ir. Acad. **100A** (2000), no. 1, 11–38. MR 1882195 (2002k:46155). Zbl 0972.46038.

[19] Michael Skeide, *Hilbert modules and applications in quantum probability*, Habilitationsschrift (2001). Habilitationsschrift.

[20] Michael Skeide, *Hilbert von Neumann modules versus concrete von Neumann modules*, arXiv:1205.6413v1 (2012). arXiv:1205.6413v1.

[21] Skeide Michael and Sumesh K., *CP-H-extendable maps between Hilbert modules and CPH-semigroups*, J. Math. Anal. Appl. **414** (2014), no. 2, 886–913. 3168002.

[22] W. Forrest Stinespring, *Positive functions on C^*-algebras*, Proc. Amer. Math. Soc. **6** (1955), 211–216. 0069403 (16,1033b). Zbl 0064.36703.

[23] Dana P. Williams, *Crossed products of C^*-algebras*, Mathematical Surveys and Monographs, vol. 134, American Mathematical Society, Providence, RI, 2007. 2288954 (2007m:46003). Zbl 1119.46002.

Harsh Trivedi
Department of Mathematics, Indian Institute of Technology Bombay,
Powai, Mumbai-400076,
India.
E-mail: harsh@math.iitb.ac.in