

FAMILIES OF QUASI-PSEUDO-METRICS GENERATED BY PROBABILISTIC QUASI-PSEUDO-METRIC SPACES

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Abstract. This paper contains a study of families of quasi-pseudo-metrics (the concept of a quasi-pseudo-metric was introduced by Wilson [22], Albert [1] and Kelly [9]) generated by probabilistic quasi-pseudo-metric-spaces which are generalization of probabilistic metric space (PM-space shortly) [2, 3, 4, 6]. The idea of PM-spaces was introduced by Menger [11, 12], Schweizer and Sklar [18] and Serstnev [19]. Families of pseudo-metrics generated by PM-spaces and those generalizing PM-spaces have been described by Stevens [20] and Nishiure [14].

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