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THE MATHEMATICIAN HUGO RIBEIRO

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Being always concerned with the world around him, Hugo Ribeiro was actively involved in student organizations during his student years at the Faculdade de Ciências de Lisboa. Under a political regime that allowed no dissenting voices or criticism, this granted him some persecution which led to suspension of his student status and even arrest. While he was a student, he also acquired some teaching experience at private secondary schools.

As a student of Mathematics, Hugo Ribeiro always kept a critical view before what was being offered to him. He attended very few lectures, as he preferred to figure out things by himself. At that time, most teachers at the Faculdade de Ciências came to the university just to give their lectures, they usually did no research, and they directed their teaching mostly towards the applications of science they were really concerned with. To the student Hugo Ribeiro who was eager to learn real Mathematics and not just technical manipulations, this situation was most unsatisfactory. But he had enough initiative and motivation to follow his own ways, and he did so brilliantly.

In 1939, Hugo Ribeiro completed his degree (licenciatura) in the Mathematical Sciences at the Faculdade de Ciências de Lisboa. That same year, he started what proved to be a very fruitful collaboration with António Monteiro. Having obtained a doctoral degree under the supervision of M. Fréchet in Paris in 1936, António Monteiro brought to Portugal knowledge of a new field of Mathematics (Abstract Topology) along with his contagious will and capacity to work. Outside the framework of the official academia, he fostered so much activity around him that it is perhaps appropriate to say that he started a revolution in Mathematics in Portugal. He organized seminars, founded the "Sociedade Portuguesa de Matemática", a research journal, "Portugaliae Mathematica", and a bulletin of the portuguese mathematical community, "Gazeta de Matemática". He promoted cooperation with mathematicians abroad. He insisted before the "Instituto de Alta Cultura" to give scholarships to young graduates to allow them to go to the

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best schools in Europe to work on doctoral dissertations.

Naturally, Hugo Ribeiro was willingly caught up in the wave of activity created by António Monteiro. He collaborated with Monteiro on all his initiatives and became his brilliant disciple.

In the seminar of "General Analysis", founded by Monteiro, Hugo Ribeiro participated actively in the study of different axiomatizations of topological spaces, trying to clarify the role played by various topological operators. This area, which we might call today the "Algebra of Topology", had recently been developed by, among others, people such as Fréchet, Kuratowski, Riesz, and Sierpiński. There were at the time many definitions of topological space ranging from E.H. Moore's [16] set 1 endowed with a transformation Φ of its subsets (formally: a pair $[1,\Phi]$, where $\Phi : \mathcal{P}(1) \to \mathcal{P}(1)$ is a function) to spaces endowed with distance functions. Fréchet [6] proposed two levels of abstraction: one — Fréchet's topological spaces — was formulated in terms of the derived set A' (set of "accumulation points") of a subset A of the universe 1; the other considered only some features of neighbourhoods and the corresponding spaces were then known as "(V) spaces".

According to Fréchet's definition, a *topological space* is a Moore space $[1,']$ such that, for any $x \in 1$ and $A \subseteq 1$,

$$x \in A' \iff \left(A\backslash\{x\} \neq \emptyset \text{ and } x \in (A\backslash\{x\})'\right) .$$

In his first research paper on the subject [17], published in the first volume of Portugaliae Mathematica in 1940, Hugo Ribeiro reported his results on equivalent formulations of this notion in terms of other topological operators, such as closure, interior, border. For instance, in terms of the closure operator, a Fréchet topological space is a Moore space $[1,\overline{}]$ such that

$$\overline{\emptyset} = \emptyset \quad \text{and} \quad A \subseteq \overline{A}$$

for any $A \subseteq 1$. In a joint paper with Monteiro [13], published in the same issue of Portugaliae Mathematica, he worked on the same question for (V) *spaces*. Fréchet defined these spaces as pairs $[1,\Phi]$ where $\Phi\colon 1 \to \mathcal{P}\mathcal{P}(1)$ is a function associating to each point in the space a set of "neighbourhoods" of the point such that, for any $x \in 1$ and any $V \in \Phi(x)$, $x \in V$. He considered only equivalence classes of such pairs, where $[1,\Phi] \sim [1,\Psi]$ if, for any $x \in 1$ and $V \in \Phi(x)$ [resp. $\Psi(x)$] there is $W \in \Psi(x)$ [resp. $\Phi(x)$] such that $W \subseteq V$. Fréchet had already shown that this definition is equivalent to the following in terms of the operator of derivation: a (V) space is a (Fréchet) topological space $[1,']$ such that $A' \cup B' \subseteq (A \cup B)'$ for any $A, B \subseteq 1$. Ribeiro and Monteiro gave equivalent characterizations of (V) spaces in terms of other topological operators. For example: a (V) space is a Fréchet space $[1,\overline{}]$ such that $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$ for any $A, B \subseteq 1$ (equivalently, $A \subseteq B$

$\Rightarrow \overline{A} \subseteq \overline{B}$). In [19], Hugo Ribeiro considered these questions for the coherence operator c: $c(X) = X \cap X'$.

The modern notion of topological space was also formulated by Kuratowski in terms of the operator $\overline{}$: it is a Moore space $[1, \overline{}]$ such that

$$\overline{\emptyset} = \emptyset, \quad A \subseteq \overline{A}, \quad \overline{A \cup B} = \overline{A} \cup \overline{B} \quad \text{and} \quad \overline{\overline{A}} = \overline{A}$$

for any $A, B \subseteq 1$. Adding the separation axiom which is nowadays known as T_1 (singletons are closed) one obtains the so-called "accessible spaces", which, in terms of the operator $'$ may be described as a (V) space $[1, ']$ such that

$$(A \cup B)' = A' \cup B', \quad A'' \subseteq A' \quad \text{and} \quad \{x\}' = \emptyset$$

for any $A, B \subseteq 1$ and $x \in 1$. Monteiro had determined in [12] when such a space is Hausdorff (T_2). In [18], Hugo Ribeiro did the same for regularity (T_3), normality (T_4) and complete normality. For instance, an accessible space $[1, ']$ is normal if and only if, for any nonempty $B_1, B_2 \subseteq A'$ such that each of the sets B_1 and B_1' is disjoint from each of the sets B_2 and B_2', there is a partition $A = A_1 \cup A_2$ into two nonempty classes such that $A_1' \cap B_2 = A_2' \cap B_1 = \emptyset$.

In a joint paper with Armando Gibert [8], Hugo Ribeiro considered, from the same point of view of axiomatization, the "spaces of finite character" (or "(Cf) spaces") introduced by Destouches [5]: a (V) space is a *(Cf) space* if, for any point a, there is a smallest neighbourhood of a. In [8], the authors prove, among other things, that a (V) space $[1, ']$ is a (Cf) space if and only if $'$ behaves well with respect to arbitrary unions. They also consider the relation ρ on a (V) space $[1, \overline{}]$ defined by $x \,\rho\, y$ if $x \neq y$ and $x \in \overline{y}$. They show that the association of the pair $[1, \rho]$ to the pair $[1, \overline{}]$ establishes a bijective correspondence between (Cf) spaces and pairs consisting of a set together with a binary irreflexive operation on it. They also identify various other kinds of spaces in terms of properties of ρ.

In a recent paper, Garrett Birkhoff [1] had given a characterization of topological spaces in terms of convergence of nets (Moore–Smith sequences). In [20], Hugo Ribeiro extends the notion of net by considering mappings defined on a set endowed with a transitive relation with values in a space 1. He then defines convergence for such a function in a (V) space in a natural way and establishes the properties of this convergence which characterize (V) spaces. He further shows that, in a (V) space $[1, ']$, the topology is determined by convergence of nets if and only if $(A \cup B)' = A' \cup B'$ for any $A, B \subseteq 1$.

In another joint paper with António Monteiro [14], Hugo Ribeiro considered an abstraction of the topological closure operator. They define a closure operator on a partially ordered set $(\mathcal{P}, \subseteq)$ to be a function $\varphi\colon \mathcal{P} \to \mathcal{P}$ such that $X \subseteq \varphi(X)$ for all $X \in \mathcal{P}$. They say that $I \in \mathcal{P}$ is *φ-invariant* if $\varphi(I) = I$ and they prove that φ is completely determined by its invariants if and only if

$$X \subseteq Y \implies \varphi(X) \subseteq \varphi(Y) \ \text{ and } \ \varphi(\varphi(X)) = \varphi(X)$$

for any $X, Y \in \mathcal{P}$. They call a pair $[\mathcal{P}, \varphi]$ with these properties a *topological partially ordered system*. The rest of the paper is concerned with various properties of such structures as well as the study of the set Φ of all topological partially ordered systems on a partially ordered set $(\mathcal{P}, \subseteq)$ ordered by: $[\mathcal{P}, \alpha] \leq [\mathcal{P}, \beta]$ if $\alpha(X) \subseteq \beta(X)$ for all $X \in \mathcal{P}$. At the end of the paper, they propose the problem of determining whether Φ is a lattice in case $\mathcal{P}$ is inductive (every chain has an upper bound). In his review for Zentralblatt für Mathematik, Dieudonné answers this question in the affirmative by using the axiom of choice.

The generalization of concepts and results from the theory of topological spaces to partially ordered sets with closure operators is also considered in another paper [15] written in collaboration with António Monteiro. Specifically, the authors examine the extent to which elementary results concerning the notion of continuity for a function $f\colon 1 \to 1$ on a (V) space $[1, \overline{\ }]$ depend on the properties defining these spaces and the formulation of the definition of continuity.

Hugo Ribeiro carried his interest in Abstract Topology to Zürich when he went there in 1942 to prepare a doctoral dissertation with a scholarship from the Instituto de Alta Cultura. In [22, 23], he moved further up in the hierarchy of definitions of topological spaces by considering a generalization of a metric motivated by a course on topological spaces given by H. Hopf. He calls a function $\rho\colon 1 \times 1 \to [0, +\infty[$ a *weak metric* on 1 if

$$x = y \implies \rho(x,y) = 0 \ \text{ and } \ \rho(x,z) \leq \rho(x,y) + \rho(y,z)$$

for all $x, y \in 1$. The objective of the paper is to give a topological characterization of the spaces which admit a weak metric. The main theorem states that a (V) space is weakly metrizable if and only if each point x admits a countable basis of neighbourhoods $(V_{n,x})_n$ such that

$$y \in V_{n,x} \implies V_{m,y} \subseteq V_{k,x} \ \text{ where } k = \min\{m,n\} - 1 \ .$$

As an open question, Hugo Ribeiro asks whether every normal weakly metrizable space is metrizable, while he proves this is the case for compact Hausdorff spaces.

But, in Zürich, Hugo Ribeiro became interested in Lattice Theory and he must have read avidly Birkhoff's recent monograph [2] on this subject.

In 1943, he received the Artur Malheiros award of the Lisbon Academy of Sciences for which he submitted a monograph [21] on the foundation of probability theory. In this work, Hugo Ribeiro first introduces the elements of Lattice Theory, which in fact occupies half the manuscript. In this section, he includes questions about complementation in lattices and, in particular, he proposes what was then an open question: to determine whether every lattice in which each element has a unique complement is distributive (answered, in the negative, by Dilworth [4] a couple of years later). He then proceeds to study the Boolean algebra of binary operations on a set 1 and the interaction of the Boolean operations with other natural operations on binary relations. In the final section, Hugo Ribeiro studies "positive modular" functionals on a lattice L, namely functions $p\colon L \to \mathbb{R}$ which are order-preserving and satisfy the equation

$$p(x \cup y) + p(x \cap y) = p(x) + p(y) \ .$$

To such a functional, one associates a quasi-metric on L given by $\rho(x, y) = p(x \cup y) - p(x \cap y)$. Finally, the notion of probability field is identified with a Boolean ring endowed with a quasi-metric ρ such that $\rho(0, 1) = 1$ and the order topology is finer than the quasi-metric topology, where the quasi-metric is obtained by taking $\rho(x, y) = p(x + y)$ and the probability measure is recovered from $p(x) = \rho(0, x)$. This constitutes just a brief summary of the monograph, which clearly reveals how quickly Hugo Ribeiro came to master a variety of contemporary research topics and how he was able to give a global perspective of them, motivated by a need to clarify the foundations of probability theory.

Under the supervision of P. Bernays, Hugo Ribeiro worked on the abstract characterization of the lattices of subgroups of finite abelian groups. His doctoral dissertation, published in [26], solved this problem as follows.

For a group G, denote by $L(G)$ the lattice of all its subgroups (under inclusion). For an abelian group G, this coincides with the lattice of all normal subgroups of G which is known to be modular. For a cyclic group G, $L(G)$ is a distributive lattice. Since the structure of finite abelian groups was already quite well-known, the thesis is really about the theory of finite modular lattices.

The thesis starts by reducing the problem to the study of the primary components of the group G, by observing that, if $G = G_1 \times \cdots \times G_n$ with $|G_i|$ and $|G_j|$ relatively prime for $i \neq j$, then $L(G) = L(G_1) \times \cdots \times L(G_n)$. Moreover, for a primary component G_i, $L(G_i)$ is indecomposable as a direct product. For the case when all these components are cyclic groups, one then obtains a product of chains for $L(G)$ and Hugo Ribeiro characterizes abstractly these lattices as the (finite) distributive lattices in which each element is covered by, at most, the number of atoms in the Boolean algebra of its complemented elements. In [25], he included a proof of the particular case of the product of two chains. From the introduction of this short note, we quote the following which portrays some of his pedagogical concerns.

> If it is possible that the formal development of the theory [of lattices] is easily accessible to anyone who has a certain habit of fol-

> lowing formal developments, what is certain is that — and not just for lattice theory — without a perfect understanding of examples of applications (outside that same formal development) it is not possible to comprehend the meaning of the results and problems. (A mere reading under these circumstances can not bring much knowledge and it carries the risk of contributing to vitiate a formation in Mathematics.) In lattice theory, it happens frequently that the models, the examples at a first level are still abstract concepts and even more so: concepts coming from a great variety of areas of Mathematics, from what may be called (and is actually called among us) "modern Mathematics".

Hugo Ribeiro calls a finite modular lattice *exceptional* if it has a convex sublattice which is a projective geometry not over a field (i.e., non-arguesian). A finite non-exceptional modular lattice is then said to be *u-uniform* if its convex sublattices of height 2 are either chains or have $u+3$ elements. Hugo Ribeiro shows that the 1-uniform lattices are just the distributive lattices. He calls an element of a lattice a *cycle* if the elements below it form a chain.

The main result of Hugo Ribeiro's thesis states that a lattice L is isomorphic to the lattice of subgroups of some finite abelian group if and only if L is the product of ideals of lattices satisfying the following conditions, where the values of u are all distinct:

(i) Every join irreducible element is a cycle;

(ii) The maximal cycles have the same height;

(iii) The lattice is u-uniform;

(iv) The number of covers for each element does not exceed the number of atoms.

This representation as a product of lattices is unique, the component, if any, with $u=1$ is $L(C)$ for a cyclic group C. In particular, for a finite abelian group G, $L(G)$ determines G up to isomorphism if and only if there is no factor with $u=1$ (i.e., there is no distributive factor). In terms of the structure of the group, this means that no primary component is cyclic. As a consequence of these results, he points out in [27] that a lattice satisfying the above conditions (i)–(iv) is self-dual.

In 1946, Hugo Ribeiro returns to Portugal at a particularly bad time for the history of portuguese universities. The government, with the help of the political police, had identified all dissidents or critics of the regime and started to get rid of the undesirables. António Monteiro never managed to find a position at any university in Portugal and had moved to Brasil in 1945. Hugo Ribeiro never even had his doctoral degree recognized in Portugal. In fact, the only support both had from official agencies came from the Instituto de Alta Cultura. When Hugo Ribeiro presented himself at the Instituto after returning from Zürich, the president asked him what he was doing there. This was to say that he was glad he was successful in his work; but forget it, he would never be allowed to work in any university in Portugal. Most of the best mathematicians in portuguese universities were fired at this time and some were even arrested. Many of them ended up immigrating in search for places where they would be allowed to do their work. In 1947, Hugo Ribeiro moved to Berkeley where he worked with A. Tarski for three years with a position of Lecturer in Mathematics at the University of California.

Participating in Tarski's seminar, he worked on a paper by Jónsson and Tarski [9] on the theory of Boolean algebras. In [28], he improves a key lemma in that paper. Jónsson and Tarski consider complete atomistic extensions $\mathcal{U} = \langle A; +, \cdot, 0, 1\rangle$ of a Boolean algebra $\mathcal{B} = \langle B; +, \cdot, 0, 1\rangle$ in which $\mathcal{B}$ satisfies certain technical conditions. Under those conditions, for an operator $f : B^m \to B$ on $\mathcal{B}$, one may define a unique extension $f^+ : A^m \to A$. Such an operator is said to be *additive* if it respects $+$ when restricted to each component. The set of all operators on $\mathcal{B}$ obtained by composing additive operators is denoted by Φ. Jónsson and Tarski had proved that any equation satisfied by specific additive functions is also satisfied by their extensions. This was done by showing that, if $f, g_1, ..., g_m$ are additive operators on $\mathcal{B}$, then

$$\Big(f[g_1, ..., g_m]\Big)^+ = f^+[g_1^+, ..., g_m^+] \ .$$

Hugo Ribeiro shows that this relation is still valid in case the operators $g_1, ..., g_m$ are monotone and $f \in \Phi$.

In 1950, Hugo Ribeiro moved to Lincoln, where he became an Associate Professor (1950–52) and later Professor (1952–61) of Mathematics at the University of Nebraska.

As a proposed "theme of study", he had written in 1943 a note [24] for Gazeta de Matemática on the notion of topological group. The idea was to study this notion from an algebraic point of view. Ten years later, he came back to this theme in his paper [29]. He associates with a topological group G the "complex algebra" of its subsets

$$\Big\langle \mathcal{P}(G); \cup, \emptyset, \cap, G, {}^{-}(\text{complement}), \cdot(\text{product}), e, {}^{-1}, {}^{\sim}(\text{topological closure})\Big\rangle \ .$$

He then writes 15 first order sentences which are satisfied by this algebra exactly when G is a topological group. He also points out that the well-known fact that the closure of a subgroup of a topological group is again a subgroup is an immediate consequence of his axioms.

In Tarski's seminar, Hugo Ribeiro also became interested in Model Theory, which was soon going to occupy him in all his research work. In one of his last papers [33], he communicates some of the beauty of this field to a general readership and tries to attract students to it.

Kalicki and Scott [10] introduced the notion of *equational completeness* for a class of algebras to mean that the class contains nontrivial algebras and, for any identity ϵ, either ϵ holds for all algebras in the class or it is valid only in the trivial algebras of the class. (Of course, for a variety of algebras, equational completeness means that the variety is an atom in the lattice of varieties.) They show, for example, that the class of all distributive lattices is equationally complete.

Hugo Ribeiro wrote four papers exploring the idea behind this concept. In [31], he recalls this notion and the specific example of distributive lattices. Since the class of all distributive lattices is equationally complete, the subclass of all chains is not equational. However, it may be axiomatized by adding to the axioms of distributive lattices the universal sentence

$$\forall x \;\; \forall y \quad \Big(x \cap y = x \quad \vee \quad x \cap y = y\Big)\,.$$

So he asks whether there are any further nontrivial specializations of the theory of chains obtained by adding axioms that are universal sentences. The argument he indicates is simple enough to be reproduced here. If φ is a universal sentence which is not valid in all chains, then its negation $\neg\varphi$ is an existential sentence satisfied by some (finite) chain C and, therefore, by all chains with at least $|C|$ elements. If $n+1$ is the number of elements of the smallest chain satisfying $\neg\varphi$, then, for a chain, φ is equivalent to the sentence $\exists_n!$ which states that there are at most n elements.

This remark (which he found to be already known to Langford [11] after announcing it in [30]) led Hugo Ribeiro to formulate the following analog of the definition of equational completeness. A consistent set Σ of sentences of a first order language with equality is said to be *universally complete* if, for any universal sentence φ which is not a consequence of Σ, there is a natural number n such that the sentence $\varphi \leftrightarrow \exists_n!$ follows from Σ. This notion is stronger than equational completeness for sets of identities.

In [32], Hugo Ribeiro further develops the theory of universally complete theories. The main objective of the paper is to obtain structural properties of the class of all models of a theory that constitute necessary and sufficient conditions for universal completeness. In fact, he shows that a set Σ of sentences is universally complete if and only if the class $\mathbf{K}$ of all its models is nonempty and, for any finite substructure $\mathcal{U}'$ of a member of $\mathbf{K}$, there is a finite substructure

$\mathcal{C}$ of a member of $\mathbf{K}$ such that, for every $\mathcal{B} \in \mathbf{K}$, $\mathcal{U}'$ is embeddable in $\mathcal{B}$ if and only if $|\mathcal{C}| \leq |\mathcal{B}|$. Thus, Hugo Ribeiro takes this property as the definition of *universal completeness* of the class $\mathbf{K}$. (The reviewer of this paper for Math. Reviews apparently completely missed the point of the paper since he writes: "... it would seem simpler to define universal completeness of $\mathbf{K}$ directly as universal completeness of Σ".) Hugo Ribeiro proceeds to specialize this result to the cases when $\mathbf{K}$ is axiomatized by $\forall\,\exists$- sentences or by universal sentences. For instance, in the latter case, he proves that, if $\mathbf{K}$ is nonempty, then Σ is universally complete if and only if, for any $\mathcal{U}, \mathcal{B} \in \mathbf{K}$ with $\mathcal{U}$ finite, either $\mathcal{U}$ is embeddable in $\mathcal{B}$ or $|\mathcal{U}| > |\mathcal{B}|$.

These results are applied to the theory of (linearly) ordered abelian groups in a joint paper with Diana Brignole [3]. They prove that the class of such structures is universally complete, thus obtaining a simpler proof of an equivalent result of Gurevič and Kokorin [7] stating that if a universal sentence holds in some nontrivial ordered abelian group then it holds in all of them. The paper also includes a proof that there is a largest class of infinite structures with one binary operation and one binary relation containing all ordered abelian groups which is universally complete, namely, the class of all linearly ordered commutative cancellative semigroups which are extensions of nontrivial ordered groups.

The notion of equational completeness is reconsidered in a joint note with R. Schwabauer [34]. The reviewer of this paper for Math. Reviews also failed to read it very carefully, as he indicates that the main theorem is stated without proof. The theorem states that the class of all direct products [resp. subdirect products] of algebras of an equationally complete class [resp. closed under the formation of subalgebras] is also equationally complete. The proof is quite easy and it occupies only a few lines. The objective of the paper is to point out that this simple observation yields the equational completeness of several classes of algebras. For instance, since the class of all chains with at most 2 elements is clearly equationally complete and the class of all distributive lattices is the class of all subdirect products of such chains, distributive lattices form an equationally complete class.

In 1961, Hugo Ribeiro moved to the Pennsylvania State University (nicknamed Penn State) where he was Associate Professor for a year, then Professor until 1975, and afterwards Professor Emeritus. Upon his retirement, he moved back to Portugal, where a revolution had restored some basic human rights, including freedom of speech. He and his wife Pilar offered their services to the University of Porto where they taught respectively for four and five years.

Throughout his life, Hugo Ribeiro was concerned with offering the best opportunities to students of Mathematics. While in Porto, he directed two seminars for students finishing their degrees in Mathematics. In 1976–77, one of those students was Carlos Alves. In 1977–78, I was one of them. Hugo Ribeiro encouraged us to pursue graduate studies and offered his recommendations, in particular (but by no means exclusively) if we wanted to attend Penn State. We both ended up going there. Carlos Alves did a Ph.D. in Set Theory under T. Jech. I worked on rational languages with G. Lallement but eventually moved to the border of Finite Semigroup Theory and Universal Algebra. Hugo Ribeiro had introduced us to the latter topic in his seminar, which was dedicated to the Algebra of Logic and Model Theory. After he definitely retired and moved to his residence in Bicesse, he was always available to young assistants that came to him for advice. One of them, Fernando Ferreira, has recently finished a Ph.D. in Logic under S. Simpson, also in Penn State. But, many other portuguese mathematicians have profited from his advice, namely most of our other logicians (including Narciso Garcia, Isabel Loureiro and Franco de Oliveira) as Hugo Ribeiro recommended and incentivated them to do post-graduate work.

I met Hugo Ribeiro in 1976 when I was assigned, as a monitor (student-teacher), tutorials for his third year Logic course, which I again taught the following year. In Penn State, I would meet the Ribeiros during their Summer vacation there. This gave me many opportunities to talk with them and I greatly benefited from their generous friendship.

Over the years, I had many interesting conversations with the Ribeiros. The non-mathematical content of this article reflects, for the most part, recollections of these conversations. As to the mathematical content, it is based on my own recent reading of Hugo Ribeiro's papers. He was always very modest about his achievements and so, while he made me read many papers, he never even suggested that I should read his own.

I did not intend to be exhaustive in relating the mathematical contributions of Hugo Ribeiro, nor did I wish to portray all sides of his captivating personality. However, I hope I contributed to situating properly the beginning of his career and displaying his main mathematical interests and contributions.

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