GIUSEPPE MASTROIANNI – A MASTER IN INTERPOLATION PROCESSES

Gradimir V. Milovanović, Donatella Occorsio, and Maria Grazia Russo

The present issue of *Publications de l'Institut Mathématique* is dedicated to Professor Giuseppe Mastroianni in honor of his outstanding work in Approximation Theory and Numerical Analysis, especially in Interpolation Processes. Some among the 17 papers published in this issue were presented during the *International Workshop on Approximation Theory and Applications* (IWATA), which was held at Rifreddo (Potenza, Italy), on September 12–13, 2013, to celebrate Professor Mastroianni's scientific activity on the occasion of his retirement as Professor of the Department of Mathematics, Computer Science and Economics of the University of Basilicata, Potenza.

Giuseppe Mastroianni was born in Lagonegro on November 11, 1939. He commenced his university and scientific career at the University of Naples "Federico II" as a Scholarship Holder (1974–1976), Assistant Professor (1976–1983), and Associated Professor (1983–1987). In 1987, he joined the Department of Mathematics of the University of Basilicata, Potenza, as a Full Professor of Numerical Analysis. He served as Head of this Department during the period 1993–1998, a member of the Academic Senate (1998–2001), and Head of the Center for Advanced Studies on Functional Analysis and Approximation Theory (1991–2010). During the period 1989–2009, this Center organized the International Conference on Functional Analysis and Approximation Theory (FAAT) six times, in collaboration with the Department of Mathematics of the University of Bari and the Department of Mathematics of the University of Salento, which took place in Maratea. It was an event of crucial importance for the scientific community and for further studies on Approximation Theory and Functional Analysis. Furthermore, Mastroianni collaborated with the Institute for Applications of Mathematics of the National Council of Researches in Naples, organizing nineteen different editions of the Summer School in Computational Mathematics, which provided a relevant spur to the scientific activity of young researchers.

Mastroianni's research interests lie in approximation theory and numerical analysis (interpolation processes, positive operators, orthogonal polynomials, quadrature rules, numerical treatment of integral equations, weighted polynomial inequalities, Fourier series, etc.). He has published over two hundred remarkable papers in several prestigious mathematical journals[1] and has been invited to deliver lectures to a number of international conferences as well as to give colloquium and seminar

[1]See http://oldwww.unibas.it/utenti/mastroianni/listOfPublications.html

talks in several research centers around the world. He has conducted research in collaboration with several mathematicians, and with some of them he has forged a deep and lasting friendship.

The research on *positive operators* represents the starting point of Giuseppe's scientific activity. Along with Mario Rosario Occorsio, who was his advisor and coauthor, he constructed new classes of positive operators, proving their convergence and the preservation of some properties of the functions. This remained as a topic of interest to him throughout the years of his mathematical activity, by virtue of a long collaboration with Biancamaria Della Vecchia, Jószef Szabados and Péter Vértesi. Classical positive operators were extended to new spaces of functions defined over bounded or unbounded domains, and having singular points inside or at the endpoints of the interval. He also proved direct and converse theorems. In the same context the Shepard rational approximation has been considered, establishing error estimates involving the knots distribution, which are not achievable by means of polynomial approximation. The Shepard operator has been extended to functions on unbounded domain and for different distribution of knots.

In the subject of *polynomial approximation* Mastroianni has obtained several results. In some joint papers with Vilmos Totik, new polynomial inequalities with doubling weights were established (Markov-Bernstein, Nikolskii, Remez and Schur inequalities). The authors have proved that the zeros of the orthonormal polynomials with respect to doubling weights have an arccos-distribution and, moreover, that adjacent Christoffel numbers are equivalent. These properties are equivalent to the doubling property. Furthermore, they estimate the error of best approximation with the previous weights, introduce new moduli of smoothness and related K-functionals for weights having inner zeros. Later, in collaboration with Szabados, these results were extended to generalized Freud and Laguerre weights (on unbounded intervals).

Regarding the polynomial approximation with exponential weights, Mastroianni observed that, as a consequence of the "finite-infinity range inequality", it is sufficient to approximate the function in an interval that is strictly included in the Mhaskar–Rahmanov–Saff interval. By this simplification and by using the one-sided approximation, he proved Jackson and Stechkin theorems for different spaces of functions defined on bounded or unbounded domains. For *polynomial approximation* we could mention a number of other results, but we limit ourselves to mention the Fourier sums with respect to exponential weights. Indeed, these processes in weighted L^p spaces converge for a restricted range of the parameter p. Nevertheless, in collaboration with Péter Vértesi, Incoronata Notarangelo, and Donatella Occorsio, Mastroianni considered Fourier sums of finite-section of functions, proving that they converge with the order of the best approximation.

Lagrange's interpolation is a central topic in Numerical Analysis since it appears in approximation, numerical differentiation and integration. The initial goal has been to enlarge the number of the systems of knots having the Lebesgue constant with a logarithmic behavior. Such a result has been achieved by using the method of the "additional nodes" by Szabados, and the so-called "extended interpolation". There are many papers of Giuseppe relevant to these topics, with various co-authors. Among them we recall Giuliana Criscuolo, with whom he collaborated for a long period and published more than 30 papers. A complete exposition can be found in his survey [Trans. Numer. Anal. **14** (2002), 125–134] and in the monograph "Interpolation Processes – Basic Theory and Applications" (Springer, 2008), written jointly with Gradimir Milovanović.

The weighted interpolation has been another research topic of Giuseppe. One of his main results in this area can be roughly interpreted as follows: "for suitable systems of knots, the weighted L^p norm of the Lagrange polynomial of a function f is equivalent to the discrete weighted norm of the function".

As a consequence, these interpolating polynomials converge with the order of the best approximation for wide classes of functions. These results have been extended to the case of exponential weights on bounded and unbounded intervals, overcoming some well-known difficulties. Relevant to this argument, several papers appeared in collaboration with Péter Vértesi, Jószef Szabados, Incoronata Notarangelo, Maria Grazia Russo, and Donatella Occorsio.

Numerical integration is yet another topic of the research activity of Mastroianni including the computation of the Hilbert transform and the numerical approximation of weakly singular integrals having special behaviors. His main co-authors on this topic are Gradimir Milovanović and Giovanni Monegato, with which he developed a long-lasting collaboration and friendship. In the case of Gaussian rule with respect to Laguerre weights, Mastroianni, in joint work with Monegato, showed that the formula is not optimal in weighted L^1-Sobolev spaces. To overcome this problem they suggested a truncated Gaussian rule, proving that this last formula converges with the order of the best weighted polynomial approximation. Thereafter, this idea has been extended to all the polynomial approximation processes with exponential weights on bounded and unbounded intervals.

In the 90's, Sigmund Prössdorf and Giovanni Monegato encouraged Giuseppe to face the numerical approximation of the solutions of *integral equations*. In the treatment of the Fredholm's integral equation, the main ideas have been to associate to every class of equations a suitable separable space of functions and to use the "truncation" in the case of exponential weights. A number of papers in this subject have been appeared, in collaboration with different co-authors. Among them are Giovanni Monegato, Gradimir Milovanović, Maria Carmela De Bonis, Concetta Laurita, and Maria Grazia Russo.

Besides the above, since the beginning of his career, Mastroianni has served as an advisor, encouraging and initiating young students to the realm of mathematical research. For this purpose, he has created the group of Numerical Analysis at the University of Basilicata. The youngest pupils of the group were his students during his teaching career at the University of Basilicata and therefore they were fully trained under his supervision. He has passionately guided the research activity of the scientific group in Potenza, investing also a lot of time and energy in order to create an atmosphere of sincere collaboration and friendship among them.

The present issue of *Publications de l'Institut Mathématique* is devoted to recent results in Approximation Theory, Numerical Analysis, and various applications of an interdisciplinary character. The papers appear in alphabetical order with respect to the last name of the first-named author. Aimi, Desiderio, Diligenti, and Guardasoni give a numerical study of energetic BEM-FEM applied to wave propagation in 2D multidomains. L^p-convergence of Bernstein–Durrmeyer operators with respect to an arbitrary measure and the convergence in capacity of rational approximants of meromorphic functions are considered by Elena Berdysheva and Bing-Zheng Li and by Hans-Peter Blatt, respectively. Len Bos, Stefano De Marchi, and Norm Levenberg study Fekete points for ridge function interpolation and hyperbolic potential theory. New integral representations in the linear theory of viscoelastic materials with voids are given by Cialdea, Dolce, Leonessa and Malaspina. An algebraic exposition of umbral calculus with some applications in interpolation is presented by Francesco Aldo Costabile and Elisabetta Longo. Victor Didenko and Bernd Silbermann deal with the Coburn–Simonenko theorem for some classes of Wiener–Hopf plus Hankel operators. Silvia Falletta and Giovanni Monegato investigate the exact nonreflecting boundary conditions for exterior wave equation problems, and Fermo, Van der Mee, and Seatzu consider emerging problems in approximation theory for the numerical solution of the nonlinear Schrödinger equation. A fast algorithm for the numerical solution of an integral equation with logarithmic kernel is developed by Flemming and Junghanns. Gonska, Raşa, and Elena-Dorina Stanila deal with Lagrange-type operators, and Kopotun, Leviatan, and Igor Shevchuk introduce new moduli of smoothness. Doron Lubinsky studies weighted Markov–Bernstein inequalities for entire functions of exponential type. Generalized coherent pairs on the unit circle and Sobolev orthogonal polynomials are considered by Francisco Marcellán and Natalia Pinzón-Cortés. Milovanović, Stanić, and Tomović consider trigonometric multiple orthogonal polynomials of semi-integer degree and the corresponding quadrature formulas. Finally, Incoronata Notarangelo investigates a conjecture of Paul Nevai, and Szili and Vértesi deal with a convergent process of S. Bernstein.

It is a privilege and an honor to have edited these papers dedicated to an eminent mathematician and friend – Professor Giuseppe Mastroianni.

Finally, we wish to express our deepest appreciation to all the mathematicians from the international mathematical community, who contributed their papers for publication in this issue dedicated to Giuseppe Mastroianni, as well as to the referees for their careful reading of the manuscripts.

Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia
gvm@mi.sanu.ac.rs

Department of Mathematics, Computer Science and Economics
University of Basilicata, Potenza, Italy
donatella.occorsio@unibas.it
mariagrazia.russo@unibas.it