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TATJANA OSTROGORSKI

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Tatjana Ostrogorski was born in Belgrade, in a family of distinguished intellectuals. Both of her parents were historians, university professors and members of the Serbian Academy of Sciences and Arts. Her mother Fanula Papazoglu came from a Greek family from Bitolj, in Macedonia. Her father Georgije Ostrogorski, a famous byzantologist and author of the book *History of the Byzantine State*, was born in St. Petersburg. He was one of the most prominent Russian intellectuals who found a home in Serbia after the Russian Revolution. Tatjana was married to the logician Kosta Došen, also a member of the Mathematical Institute. Their daughter Ana is now 14 years old. Tatjana is also survived by a brother and a sister.

Tatjana went to primary and secondary school of a classical orientation (with ancient Greek and Latin and not much mathematics). From her father, Tatjana learned Russian as her first language. Her mother and her grandmother taught her modern Greek. She was also fluent in French and English, and of course Serbian. In 1973, she graduated from the University of Belgrade at the Department

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of Mathematics. She obtained her Master's degree in 1978 with a thesis entitled *Saturation Problems in the Theory of Approximation of Real Functions*. Tatjana spent the academic year 1980/81 in Moscow at Moscow State University, where she worked under the supervision of Sergei Borisovich Stechkin. In 1987, she obtained her doctoral degree, with the thesis *L^p Inequalities with Weights on Cones in R^n and H^p Spaces on Half-Planes in R^n*. For both theses her advisor was Slobodan Aljančić.

After graduating in 1973, Tatjana took up a position in Belgrade at the Mathematical Institute, where she remained until the beginning of her long illness in 2003.

At the beginning of her career she also had teaching assistantships in differential geometry at the Department of Mathematics and in analysis at the Faculty of Mechanical Engineering of the University of Belgrade. In the academic year 1988/89, she taught a graduate course on Lie groups at the Faculty of Mathematics in Belgrade. From 1994 until 1998 Tatjana was a visiting professor at the University of Montpellier III, in France.

In 1982, Tatjana was appointed as Secretary (in effect, Managing Editor) of *Publications de l'Institut Mathématique*. This appointment was based on exceptional qualities she had for this position: a patient and hard worker, with an excellent mathematical and general education, reliable, and an extremely decent and kind person. She and Dragan Blagojević, who was appointed at the same time as Technical Editor of the journal, made a very good team for many years. Due to their efforts, the level and the appearance of the journal improved significantly.

Tatjana was the Editor of the special issue of Publications de l'Institut Mathématique, published in 2002, celebrating the centenary of Jovan Karamata's birth. She was also a member of the Editorial Board for the publication of the works of Karamata, planned for 2002 (but still unpublished).

Tatjana's research was in mathematical analysis. Her first papers dealt with Fourier analysis and approximation theory, while later ones are in regular variation, harmonic analysis and Lie groups. The results from her Master's thesis were published in [1], where she investigated saturation of convolution operators (singular integrals) with kernels of the form $k_r(x) = rk(rx)$, $r \in R$, $k \in L(R)$, and she proved local and global saturation theorems for the spaces of temperate distributions (which are larger than L^p spaces).

In her doctoral thesis, Tatjana investigated Hardy's inequality for positive functions on R:

$$\int_0^{+\infty} \left(x^{-1} \int_0^x f(t)\,dt \right)^p x^a dx \leqslant C \int_0^{+\infty} f^p(x) x^a dx$$

($a \leqslant p-1$, C- a certain const.), which says that the operator $Hf(x) = x^{-1}\int_0^x f(t)dt$ is continuous in the space $L^p(0,\infty)$, $1 \leqslant p < \infty$, with the weight x^a. Her aim there was not only to find an n-dimensional analogue of Hardy's inequality, but more generally, to determine a class of operators as wide as possible, and weights such that an inequality, analogous to Hardy's, holds. She considered integral operators defined by

$$Kf(x) = \int_V k(x,y) f(y)\,dy, \quad x \in R^n$$

where $k : V \to R^+$ is the kernel of the operator and f is a positive function on a self-adjoint cone V in R^n. Her central result is that if K is a homogeneous operator of order b, and for some c

$$\int_V k^r(x_0, y) s^{-rc+r-1}(y)\,dy < \infty$$

holds, then

$$\|Kf\|_{q,c-b-1} \leqslant C\|f\|_{p,c},$$

where the norm is $\|f\|_{p,\mu} = (\int_V |f(x)|^p s^{\mu p-1}(x)dx)^{1/p}$, and $1 \leqslant p \leqslant q \leqslant \infty$, $r^{-1} = p^{-1} + q^{-1}$, C- a certain const. This was applied to several n-dimensional generalizations of classical operators such as those of Hardy, Laplace, Riemann–Liouville, Hilbert and Stieltjes. The obtained inequalities, expressing the continuity of the related operators in some L^p spaces with weights, reduce to the usual inequalities when $n = 1$. These results were published in [7] and [8]. For weighted H^p spaces, which are n-dimensional generalizations of Hardy spaces on half-planes, she proved a theorem of the Paley–Wiener type in [9].

In the late 1970s, Slobodan Aljančić, Dušan Adamović and Dragoljub Arandjelović organized a special course on regular variation at the Department of Mathematics in Belgrade. Since then, Tatjana's research was often connected with that topic. Her first papers in which regular variation appeared were [2] and [3], where she investigated the integrability of the sum g of the sine (and also of the cosine) series, depending on its coefficients. In the case of quasimonotone coefficients, she presented in [2] sufficient conditions on the upper and lower indices of an O-regularly varying function K such that $K(1/x)g^p(x) \in L(0,\pi)$ if and only if $\sum n^{p-2}K(n)a_n^p < \infty$, where $1 < p < \infty$. An analogous result was obtained for the cosine series too. In the case of positive coefficients, a similar theorem (with $p = 1$) was obtained in [3].

Multidimensional generalizations of regularly varying functions occupy the central part of Tatjana's investigation. The first results in this direction are Theorems of the Abelian and Tauberian type for different integral transforms of multivariate regularly varying functions in the sense of Yakymiv[1], published in [4], [5] and [6]. Bajšanski and Karamata[2] defined regularly varying functions on an arbitrary topological group. In [11], Tatjana studied a special case when this topological group is the group of automorphisms of a homogeneous cone V in R^n. She also considered there regular variation with respect to the additive group of the vector

[1] A. L. Yakymiv, *Multidimensional Tauberian theorems and their applications to the Bellman-Harris branching processes*, Mat. USSR Sb. 43 (1982), 413–425. See also Chapter 1.1 of Yakymiv's book *Probabilistic applications of Tauberian theorems*, Modern Probability and Statistics, VSP, Utrecht, 2005.

[2] B. Bajšanski and J. Karamata, *Regularly varying functions and the principle of equicontinuity*, Publ. Ramanujan Inst. 1 (1968–69), 235–246.

space R^n and established the relationship between the two classes by an exponential mapping of the Lie group of the cone. The uniform convergence theorem and the representation theorem were proved in this setup. In [12] and [13] regularly varying functions defined by Bajšanski and Karamata were studied on the light cone and on the cone R^n_+. This definition of regular variation on R^n_+ is different than that of Yakymiv. In [18], she explained the relationship between the Banach–Steinhaus theorem and the uniform convergence theorem for regularly varying functions, gave a simple proof of the uniform convergence theorem for continuous functions and demonstrated that this proof can be carried on to general topological groups and to Banach spaces.

In [10], Tatjana presented some results concerning Vinberg's[3] theory of homogeneous cones and homogeneous integrals, which she used in [14] and [15] to prove n-dimensional generalizations of the Abelian type theorems for the Laplace transform and, amongst others, for the Riemann–Liouville operator and for the Stieltjes transform. In [16], she proved an n-dimensional generalization of Hardy's inequality for homogeneous cones.

In [17] and [21] regularly bounded functions were introduced. Various properties, analogous to the properties of classical regularly varying functions and O-regularly varying functions, are proved for them: a uniform boundedness theorem, a representation theorem, Abelian-type theorems for integral operators, a Hardy-type inequality with regularly bounded weights.

The article [19] grew out of a problem posed by E. Omey, which is related to the fact that the class of slowly varying functions is not closed under subtraction. The question was: "If the truncated variance of a random variable is slowly varying, are its components in the positive and in the negative half line slowly varying too; or, in other words, given a nondecreasing slowly varying function L, which is the sum of two nondecreasing functions $L = F + G$, does it follow that the components F and G are necessarily slowly varying too?" In [19] one finds a necessary and sufficient condition for the components to be slowly varying. The class of functions satisfying this condition turned out to be rather small. In order to enlarge this class, analogous problems, but with various stronger monotonicity conditions, were treated in [22], [24] and [25]. The case where the functions above are nonincreasing was presented in [20].

With Tatjana's untimely death, the Mathematical Institute in Belgrade has lost a precious member, an excellent mathematician and the dearest of colleagues.

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