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LINE GRAPHS WITH EXACTLY TWO POSITIVE EIGENVALUES

Bojana Borovićanin

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Abstract. All connected line graphs whose third largest eigenvalue does not exceed 0 are characterized. Besides, all minimal line graphs with third largest eigenvalue greater than 0 are determined. Finally, all connected line graphs with exactly two positive eigenvalues are characterized.

1. Introduction

Let G be a simple graph with n vertices and $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n$ be the eigenvalues of its 0-1 adjacency matrix. We also use the notation $\lambda_i = \lambda_i(G)$ $(i = 1, 2, \ldots n)$.

There are several results concerning graphs with small number of positive eigenvalues (e.g., [3], [4], [5]). Smith [5] determined all simple graphs with exactly one positive eigenvalue. All simple graphs with at most two nonnegative eigenvalues and all minimal simple graphs with exactly three nonnegative eigenvalues are determined in [4]. Characterization of graphs with exactly two positive eigenvalues is still unsolved problem in the general case. In [4] this problem was solved in the class of connected bipartite graphs. We give the solution in the class of connected line graphs.

We explicitly characterize all connected line graphs with property $\lambda_3(G) \leqslant 0$ and prove that a connected line graph G has this property if and only if G is an induced subgraph of some of the 4 graphs displayed in Fig. 2. Two of them represent in fact classes of graphs.

We also determine all minimal line graphs with the property $\lambda_3(G) > 0$. There are exactly 13 such graphs and they are presented in Fig. 3. Finally, we characterize connected line graphs having exactly two positive eigenvalues.

A graph property P is called hereditary if the following implication holds for any graph G: if G has property P, then the same applies for any induced subgraph

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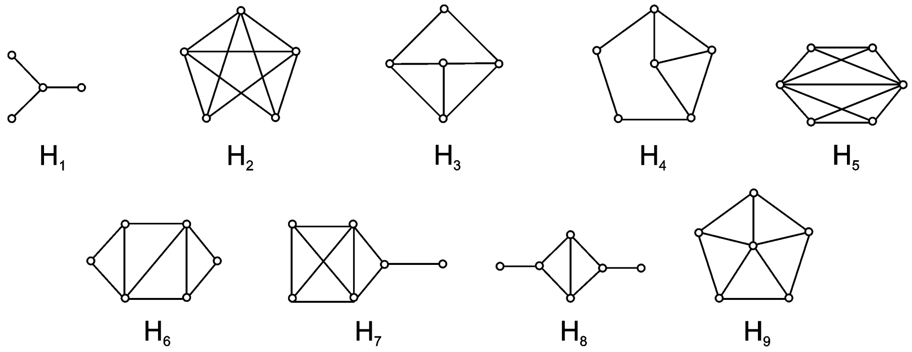

FIGURE 1

of G. A graph H is forbidden for a property P if it does not have property P. If a graph G contains the forbidden subgraph H (for a property P) as an induced subgraph, then G does not have property P. A forbidden subgraph H is called minimal if all vertex deleted subgraphs $H-i$ have the property P.

Throughout the paper $H \subset G$ will denote that H is an induced subgraph of a graph G.

Beineke characterized line graphs by showing that there are exactly nine minimal nonline graphs.

LEMMA 1. [1] *A graph G is a line graph if and only if does not contain any of the* 9 *graphs in Fig.* 1 *as an induced subgraph.*

2. Main results

Denote by F_1, F_2, $F_3 = F_3(n)$ $(n \geqslant 3)$ and $F_4 = F_4(p,m,n)$ $(2p+m+n \geqslant 2)$ the line graphs displayed in Fig. 2.

THEOREM 1. *The graphs F_1–F_4 from Fig.* 2 *have the property $\lambda_3(F_i) \leqslant 0$ $(i = 1,2,3,4)$.*

PROOF. By direct calculation we verify that $\lambda_3(F_i) \leqslant 0$ $(i = 1,2)$.

It is not difficult to see that all eigenvalues of the graph $F_3 = F_3(n)$ $(n \geqslant 3)$, different from -2, -1 and 0, are determined by the equation

$$\lambda^3 - (n+3)\lambda^2 + (n-4)\lambda + 6n = 0$$

that has exactly two roots greater than 0.

In order to verify this we show that for all values of $n \geqslant 3$, in the sequence $(1, -(n+3), n-4, 6n)$ there are exactly two sign changes. This has to be done separately for $n = 3$ and $n \geqslant 4$.

Further, all eigenvalues of the graph $F_4 = F_4(p,m,n)$ $(p \geqslant 1,\ m \geqslant n \geqslant 1)$, different from -2, -1 and 0, are determined by the equation

$$c_0\lambda^5 + c_1\lambda^4 + c_2\lambda^3 + c_3\lambda^2 + c_4\lambda + c_5 = 0$$

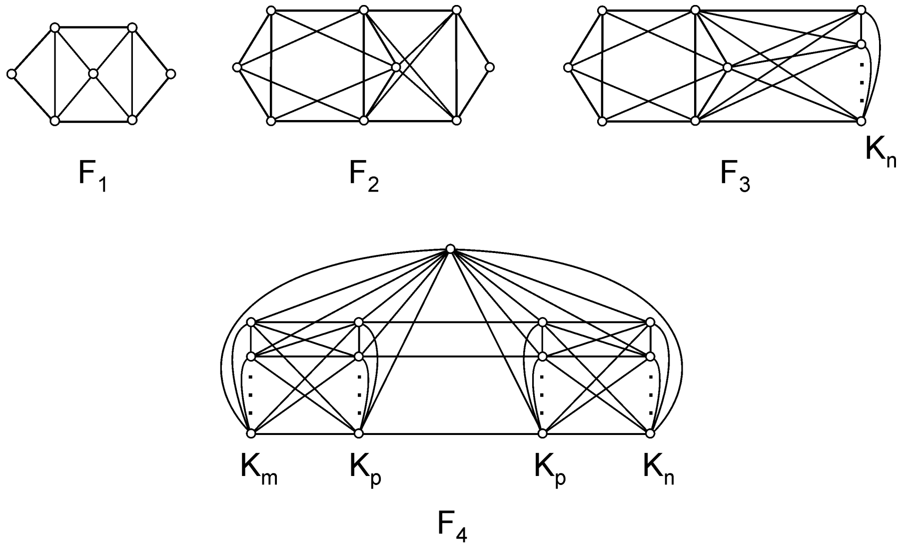

FIGURE 2

where $c_0 = 1$, $c_1 = -(m + n + 2p - 4)$, $c_2 = (p - 4)(m + n) + p^2 - 8p + 5 + mn$, $c_3 = (4p-5)(m+n)+4p^2-14p+2+4mn$, $c_4 = (5p-2)(m+n)+5p^2-12p+4mn$, $c_5 = 2p(m + n + p - 2)$. This equation has exactly two roots greater than 0. In order to verify this we show that for all values of $p, m, n \geqslant 1$, in the sequence $(c_0, c_1, c_2, c_3, c_4, c_5)$ there are exactly two sign changes. If $n \leqslant m$, then this has to be done separately for $p = m = n = 1$, $p = n = 1$ & $m = 2$, $p = n = 1$ & $m \geqslant 3$, $p = 1$ & $n \geqslant 2$ and $p \geqslant 2$. The details of these elementary considerations are not reproduced here.

From the fact that F_4 has exactly two positive eigenvalues it follows that $\lambda_3(F_4(p,m,n)) \leqslant 0$ $(p \geqslant 1,\ m \geqslant n \geqslant 1)$. By the Interlacing theorem we then have $\lambda_3(F_4(p,0,0)) \leqslant 0$ for $p \geqslant 1$, $\lambda_3(F_4(p,m,0)) \leqslant 0$ for $p, m \geqslant 1$, $\lambda_3(F_4(0,m,n)) \leqslant 0$ for $m, n \geqslant 1$, and $\lambda_3(F_4(0,m,0)) \leqslant 0$ for $m \geqslant 2$. □

In the sequel we shall determine all connected line graphs G with the property

$$\lambda_3(G) \leqslant 0 \tag{1}$$

The hereditary property (1) implies that there are minimal line graphs that do not satisfy (1), i.e., minimal forbidden subgraphs. In the set of all line graphs with at most 6 vertices, there are exactly 13 forbidden subgraphs (10 connected and 3 disconnected) (Fig. 3).

Now, let Λ denote the set of all connected line graphs G such that G does not contain as an induced subgraph any of the graphs G_1–G_{13} in Fig. 3.

LEMMA 2. *If $G \in \Lambda$ contains no triangle, then G is an induced subgraph of the graph F_1 displayed in Fig. 2.*

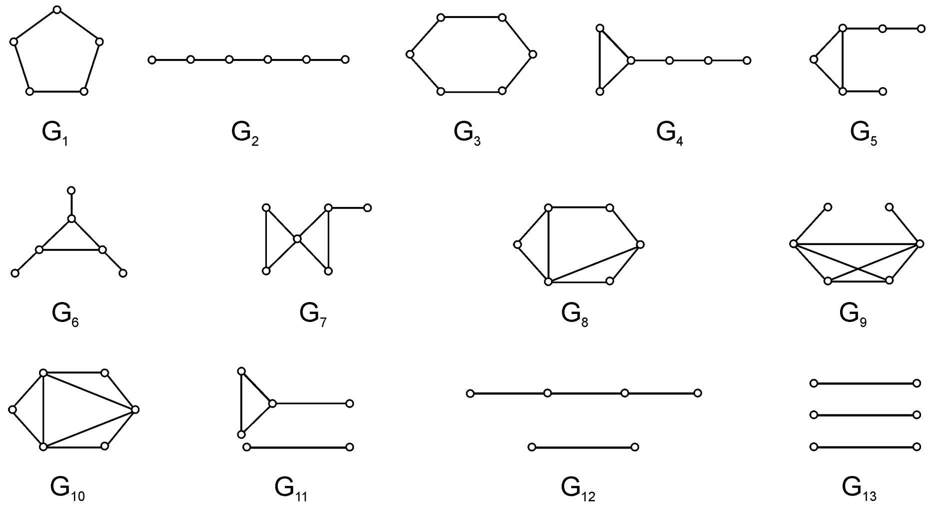

FIGURE 3

PROOF. If $G \in \Lambda$ contains no triangle, then the degree of each vertex of G is less than or equal 2 (in the opposite case we would have $H_1 \subset G$ or $K_3 \subset G$, which is a contradiction [1]). It follows that G is a circuit C_n $(n > 3)$ or a path P_m $(m \geqslant 2)$. Since $n \leqslant 4$ $(G_1 \subset G \vee G_2 \subset G \vee G_3 \subset G)$ and $m \leqslant 5$ $(G_2 \subset G)$, we conclude that G is an induced subgraph of the graph F_1. This completes the proof. $\square$

Next, let K_n be a maximal clique which $G \in \Lambda$ contains and let $V(K_n) = \{x_1, x_2, \ldots, x_n\}$. Denote by T the set $V(G) \smallsetminus V(K_n)$. The vertices from T can be adjacent to at most two vertices from the graph K_n $(H_2 \subset G \vee K_{n+1} \subset G)$. Hence, we have

$$T = T_0 \cup T_1 \cup T_2$$

where T_i $(i = 0, 1, 2)$ is the set of vertices which are adjacent to exactly i vertices from K_n. We also have

$$T_1 = T_{x_1} \cup T_{x_2} \cup \cdots \cup T_{x_n}$$

and

$$T_2 = T_{x_1x_2} \cup T_{x_1x_3} \cup \cdots \cup T_{x_{n-1}x_n}$$

where T_{x_i} is the set of vertices from T_1 that are adjacent to a vertex x_i, and $T_{x_ix_j}$ is the set of vertices from T_2 that are adjacent to vertices x_i and x_j of the graph K_n.

LEMMA 3. *If $G \in \Lambda$ has maximal clique size 3, then G is an induced subgraph of some of the graphs F_1, F_2 and F_4 displayed in Fig. 2.*

PROOF. In the proof we distinguish the following three cases:

(A) $T_1 \neq \emptyset \wedge T_2 = \emptyset$; (B) $T_1 = \emptyset \wedge T_2 \neq \emptyset$; (C) $T_1 \neq \emptyset \wedge T_2 \neq \emptyset$.

[1] We shall often denote this sentence by $H_1 \subset G \vee K_3 \subset G$, for short

Case A. The vertices of the set T have the following properties:

(1) The set T_{x_i} does not contain nonadjacent vertices ($H_1 \subset G$) and $|T_{x_i}| \leqslant 2$ ($K_4 \subset G$);

(2) $|T_{x_i}| = 2 \Rightarrow T_{x_j} = \emptyset$ $(j \neq i)$ $(H_3 \subset G \vee G_7 \subset G \vee G_8 \subset G)$;

(3) If $|T_{x_1}| = |T_{x_2}| = |T_{x_3}| = 1$, i.e., $|T_1| = 3$, then the graph which is induced by the set of vertices T_1 is the complete graph or it has exactly one pair of adjacent vertices $(H_1 \subset G \vee G_6 \subset G)$;

(4) If the graph induced by the set of vertices T_1 is complete, then $|T_1 \cup T_0| \leqslant 3$ and the graph induced by the set of vertices $T_1 \cup T_0$ is a complete graph $(H_1 \subset G \vee G_4 \subset G \vee G_5 \subset G \vee G_7 \subset G \vee G_{11} \subset G \vee K_4 \subset G)$. In the opposite case we have $T_0 = \emptyset$ $(G_1 \subset G \vee G_5 \subset G)$.

By properties (1)–(4) we conclude that the graph G is an induced subgraph of some of the graphs F_1, F_2 and F_4 displayed in Fig. 2.

Case B. The vertices of the set T have the following properties:

(1) $|T_{x_i x_j}| \leqslant 1$ $(H_1 \subset G \vee K_4 \subset G)$;

(2) If $|T_2| = 3$, then the graph induced by the set of vertices T_2 is the complete graph or it has exactly one pair of adjacent vertices $(H_3 \subset G \vee G_{10} \subset G)$;

(3) Let $|T_2| = 1$. If $T_0 \neq \emptyset$, then at least one vertex from the set T_0 is adjacent to a vertex from the set T_2 (in the opposite case the graph G is not connected). The vertices from the set T_0 which are adjacent to a vertex from the set T_2 are adjacent $(H_1 \subset G)$ and the number of these vertices is less than or equal two $(K_4 \subset G)$. Let $x \in T_0$ be a vertex of the set T_0 which is adjacent to a vertex from the set T_2, and $y \in T_0$ a vertex from the set T_0 which is not adjacent to a vertex from the set T_2. Then the vertices x and y are adjacent (in the opposite case, since y is not an isolated vertex, there exists a vertex $z \in T_0$ which is adjacent to the vertex y, and that is impossible because $G_4 \subset G$ or $G_5 \subset G$ or $G_{11} \subset G$). There exists at most one vertex of the set T_0 which is not adjacent to a vertex from the set T_2 $(H_1 \subset G \vee G_4 \subset G)$.

We conclude that $|T_0| \leqslant 3$ and the graph that is induced by the set of vertices T_0 is the complete graph. In the case $|T_0| = 3$ exactly two of these three vertices are adjacent to the vertex from the set T_2;

(4) Let $|T_2| = 2$ and let the vertices of the set T_2 be nonadjacent. If $T_0 \neq \emptyset$, then a vertex from the set T_0 is adjacent to exactly one vertex from the set T_2 $(G_1 \subset G \vee G_5 \subset G \vee G_{11} \subset G)$. In this case we have $|T_0| \leqslant 1$ $(H_1 \subset G \vee G_1 \subset G \vee G_4 \subset G \vee G_5 \subset G)$;

(5) Let $|T_2| = 2$ and let vertices of the set T_2 be adjacent. If $T_0 \neq \emptyset$, then a vertex from the set T_0 is either adjacent to both vertices from the set T_2 or nonadjacent to both vertices of the set T_2 $(H_1 \subset G)$. Since the graph G is connected, if $T_0 \neq \emptyset$, then there exists exactly one vertex of the set T_0 which is adjacent to both vertices from the set T_2 $(H_1 \subset G \vee K_4 \subset G)$. Let $x \in T_0$ be the vertex which is adjacent to both vertices of the set T_2 and $y \in T_0$ a vertex which is nonadjacent to both vertices of the set T_2. Then the vertices x and y are adjacent (in the opposite case, since y is not an isolated vertex, there exists a vertex $z \in T_0$ which is adjacent to the vertex y, but this is impossible because then we have $G_4 \subset G$ or $G_{11} \subset G$).

The set T_0 contains at most one vertex which is nonadjacent to both vertices of the set T_2 ($H_1 \subset G \vee G_7 \subset G$).

We conclude that $|T_0| \leqslant 2$ and the graph that is induced by the set of vertices T_0 is the complete graph. If $|T_0| = 2$, then one of these two vertices is adjacent to both vertices from the set T_2 and the other is nonadjacent to both vertices of the set T_2;

(6) If $|T_2| = 3$ and the graph induced by the set of vertices T_2 is complete, then $T_0 = \emptyset$ ($K_4 \subset G$).

By properties (1)–(6) we conclude that the graph G is an induced subgraph of some of the graphs F_1 and F_2 displayed in Fig. 2.

Case C. Beside the properties from cases A and B, the vertices of the set T have the following properties:

(1) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_i} \neq \emptyset$, then a vertex from the set T_{x_i} is adjacent to a vertex from the set $T_{x_ix_j}$ ($H_1 \subset G$), and $|T_{x_i}| \leqslant 1$ ($K_4 \subset G$);

(2) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_k} \neq \emptyset$, then a vertex from the set T_{x_k} is nonadjacent to a vertex from the set $T_{x_ix_j}$ ($H_3 \subset G$);

(3) The graph induced by the set of vertices T_2 is complete ($H_6 \subset G \vee H_9 \subset G$) and $|T_2| \leqslant 2$ ($K_4 \subset G$);

(4) If $T_{x_i} \neq \emptyset$, $T_{x_j} \neq \emptyset$ and $T_{x_ix_j} \neq \emptyset$, then a vertex from the set T_{x_i} is adjacent to a vertex from the set T_{x_j} ($G_{10} \subset G$);

(5) If $T_{x_i} \neq \emptyset$, $T_{x_k} \neq \emptyset$ and $T_{x_ix_j} \neq \emptyset$, then a vertex from the set T_{x_i} is nonadjacent to a vertex from the set T_{x_k} ($G_1 \subset G$);

(6) If $T_0 \neq \emptyset$, then a vertex from the set T_0 is nonadjacent to a vertex from the set T_2 ($H_1 \subset G \vee H_6 \subset G \vee H_8 \subset G \vee G_1 \subset G$);

(7) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_jx_k} \neq \emptyset$, then $T_{x_j} = \emptyset$ ($K_4 \subset G$);

(8) By the property (4) from the case (A) we conclude that if the graph that is induced by the set of vertices T_1 is the complete graph, then $|T_0| \leqslant 1$ ($G_4 \subset G \vee G_7 \subset G$) and in the opposite case we have $T_0 = \emptyset$.

By the properties (1)–(8) we conclude that the graph G is an induced subgraph of some of the graphs F_1 and F_2 displayed in Fig. 2. □

LEMMA 4. *If a graph $G \in \Lambda$ has maximal clique size n ($n \geqslant 4$), then G is an induced subgraph of some of the graphs F_1, F_2, F_3 and F_4 from Fig.* 2.

PROOF. In the proof we distinguish the following three cases:

$$(A)\quad T_1 \neq \emptyset \wedge T_2 = \emptyset; \quad (B)\quad T_1 = \emptyset \wedge T_2 \neq \emptyset; \quad (C)\quad T_1 \neq \emptyset \wedge T_2 \neq \emptyset.$$

Case A. The vertices of the set T have the following properties:

(1) The graph which is induced by the set of vertices $T_1 \cup T_0$ is the complete graph ($H_1 \subset G \vee G_4 \subset G \vee G_7 \subset G \vee G_9 \subset G \vee G_{11} \subset G$) and $|T_1 \cup T_0| \leqslant n$ ($K_{n+1} \subset G$);

(2) $|T_{x_i}| \geqslant 2 \Rightarrow T_{x_j} = \emptyset$ $(i \neq j)$ ($H_3 \subset G$);

(3) $|T_{x_i}| = 2 \Rightarrow |T_0| \leqslant 1$ ($H_7 \subset G$);

(4) $|T_{x_i}| \geqslant 3 \Rightarrow T_0 = \emptyset$ ($H_2 \subset G$);

(5) $|T_{x_i}| \leqslant n-1$ ($K_{n+1} \subset G$).

By the properties (1)–(5) we conclude that the graph G is an induced subgraph of some of the graphs F_3 and F_4 displayed in Fig. 2.

Case B. The vertices of the set T have the following properties:

(1) $|T_{x_ix_j}| \leqslant 1$ $(H_1 \subset G \vee H_5 \subset G)$;

(2) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_jx_k} \neq \emptyset$, then a vertex from $T_{x_ix_j}$ is adjacent to a vertex from the set $T_{x_jx_k}$ $(H_1 \subset G)$;

(3) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_kx_l} \neq \emptyset$, then a vertex from the set $T_{x_ix_j}$ is nonadjacent to a vertex from the set $T_{x_kx_l}$ $(H_3 \subset G)$;

(4) $T_0 = \emptyset$ $(H_7 \subset G)$;

(5) If $T_{x_ix_j} \neq \emptyset$ and $T_{x_jx_k} \neq \emptyset$, then $T_{x_ix_l} = T_{x_kx_l} = \emptyset$ $(l \neq i, j, k)$ $(G_1 \subset G)$. Hence, for $n = 4$ the graph which is induced by the set of vertices T_2 is the complete graph K_s $(1 \leqslant s \leqslant 3)$ or the graph $K_1 \cup K_1$. For $n = 5$ the graph induced by the set of vertices T_2 is the complete graph K_s $(1 \leqslant s \leqslant 4)$ or the graph $K_1 \cup K_p$ $(1 \leqslant p \leqslant 3)$;

(6) If $n \geqslant 6$ and $T_{x_ix_j} \neq \emptyset$, then $T_{x_kx_l} = \emptyset$ $(G_9 \subset G)$. Hence, the graph which is induced by the set of vertices T_2 is the complete graph K_s $(1 \leqslant s \leqslant n-1)$.

By the properties (1)–(6) we conclude that the graph G is an induced subgraph of some of the graphs F_2, F_3 and F_4 from Fig. 2.

Case C. Beside the properties from cases A and B, the vertices of the set T have the following properties:

(1) If $T_{x_i} \neq \emptyset$ and $T_{x_ix_j} \neq \emptyset$, then a vertex from the set T_{x_i} is adjacent to a vertex from the set $T_{x_ix_j}$ $(H_1 \subset G)$ and $|T_{x_i}| \leqslant n-2$ $(K_{n+1} \subset G)$;

(2) If $T_{x_i} \neq \emptyset$ and $T_{x_jx_k} \neq \emptyset$, then a vertex from the set T_{x_i} is nonadjacent to a vertex from the set $T_{x_jx_k}$ $(H_3 \subset G)$ and $|T_{x_i}| \leqslant 1$ $(G_7 \subset G)$;

(3) $T_0 = \emptyset$ $(H_7 \subset G \vee G_5 \subset G \vee G_7 \subset G)$;

(4) If $T_{x_ix_j} \neq \emptyset$, then the sets T_{x_i} and T_{x_k} are not coexisting $(G_1 \subset G)$;

(5) $T_{x_ix_j} \neq \emptyset \wedge T_{x_jx_k} \neq \emptyset \Rightarrow T_{x_i} = T_{x_k} = \emptyset$ $(G_{10} \subset G)$;

(6) If $n \geqslant 5$ and $T_{x_ix_j} \neq \emptyset$, then $T_{x_k} = \emptyset$ $(G_9 \subset G)$;

(7) If $n \geqslant 5$, $T_{x_i} \neq \emptyset$ and $T_{x_ix_j} \neq \emptyset$, then $T_{x_kx_l} = \emptyset$ $(G_5 \subset G)$. Hence, the graph that is induced by the set of vertices $T_1 \cup T_2$ is the complete graph K_s $(2 \leqslant s \leqslant n-1)$.

By the properties (1)–(7) we conclude that the graph G is an induced subgraph of some of the graphs F_2, F_3 and F_4 from Fig. 2. □

Thus, collecting the former conclusions from Lemmas 2–4, we arrive to the following theorem.

THEOREM 2. *If a connected line graph does not contain as an induced subgraph any of the graphs G_1–G_{13} in Fig. 3, then G is an induced subgraph of some of the graphs F_1–F_4 in Fig. 2.*

THEOREM 3. *A connected line graph G has the property $\lambda_3(G) \leqslant 0$ if and only if G is an induced subgraph of some of the graphs F_1–F_4 in Fig. 2.*

PROOF. Assume that G is a connected line graph with the property $\lambda_3(G) \leqslant 0$. Then by the known Interlacing theorem we conclude that G does not contain any of graphs G_1–G_{13} in Fig. 3 as an induced subgraph. In view of Theorem 2, G must be an induced subgraph of some of the graphs F_1–F_4 in Fig. 2.

Conversely, if a connected line graph G is an induced subgraph of some of the graphs F_1–F_4 in Fig. 2, then by Theorem 1 and by the fact that the mentioned property is hereditary, we have $\lambda_3(G) \leqslant 0$. □

In the sequel we shall determine all minimal line graphs with the property $\lambda_3(G) > 0$. For this purpose the following lemma will be useful.

LEMMA 5. [3] *A connected graph has two positive eigenvalues if and only if it contains the path P_4 or the graph $(K_1 \cup K_2)\nabla K_1$ as an induced subgraph.*

THEOREM 4. *There are exactly* 13 *minimal line graphs with the property* $\lambda_3(G) > 0$. *These are the graphs G_1–G_{13} in Fig.* 3.

PROOF. By direct calculation one can easily prove that the graphs G_1–G_{13} in Fig. 3 are minimal with respect to this property.

Let G be an arbitrary connected line graph that is minimal with respect to the property $\lambda_3(G) > 0$ and that is distinct from the graphs G_1–G_{10}. Then G does not contain any of the graphs G_1–G_{10} as an induced subgraph. By Theorem 2 we get that G is an induced subgraph of some of the graphs F_1–F_4 in Fig. 2. But Theorem 1 and the Interlacing theorem also give $\lambda_3(G) \leqslant 0$, which is a contradiction. Thus, G_1–G_{10} are the only minimal connected line graphs with the property $\lambda_3(G) > 0$.

Now assume that G is an arbitrary disconnected line graph that is minimal with respect to the property $\lambda_3(G) > 0$. Then G has no isolated vertices and exactly one of the statements holds:

(1) The graph G has exactly two connected components E_1 and E_2, where $\lambda_1(E_1) > 0$, $\lambda_1(E_2) > 0$ and $\lambda_2(E_2) > 0$;

(2) The graph G has exactly three connected components E_1, E_2 and E_3, where $\lambda_1(E_1) > 0$, $\lambda_1(E_2) > 0$ and $\lambda_1(E_3) > 0$.

Hence, if the statement (1) holds we have that graph E_1 contains the graph P_2 as an induced subgraph and the graph E_2 contains the graph P_4 or the graph $(K_1 \cup K_2)\nabla K_1$ as an induced subgraph, so we get $P_2 \cup P_4 \subset G$ or $P_2 \cup (K_1 \cup K_2)\nabla K_1 \subset G$. If the statement (2) holds we have that graphs E_1, E_2 and E_3 contain the graph P_2 as an induced subgraph and $P_2 \cup P_2 \cup P_2 \subset G$. So we get that the graphs G_{11}–G_{13} are the only minimal disconnected line graphs with the property $\lambda_3(G) > 0$. □

Finally, we will characterize connected line graphs with exactly two positive eigenvalues.

Now, combining Lemma 5 and Theorem 3, we arrive at the following result.

THEOREM 5. *A connected line graph G has exactly two positive eigenvalues if and only if G is an induced subgraph of some of the graphs F_1–F_4 from Fig.* 2, *and it contains the path P_4 or the graph $(K_1 \cup K_2)\nabla K_1$ as an induced subgraph.*

References

[1] L. Beineke, *Characterization of derived graphs*, J. Comb. Theory **9** (1970), 129–135.

[2] D. Cvetković, M. Doobs and H. Sachs, *Spectra of Graphs, Theory and Application*, Academic press, New York, 1980; III edition, Barth, Heidelberg, 1995.

[3] M. Petrović, *Some classes of graphs with two, three or four positive eigenvalues*, Collection Sci. Papers Fac. **5** (1984), 31–39.

[4] M. Petrović, *Contribution to the spectral theory of graphs*, (in Serbian) Ph. D. thesis, Univ. Beograd, 1984.

[5] J. H. Smith, *Some properties of the spectrum of a graph* In: *Combinatorial Structures and Their Application* (eds. R. Guy, H. Hanani, N. Sauer, J. Schonheim), Gordon and Breach, New York, Paris, 1970, 403–406.

Prirodno-matematički fakultet
Radoja Domanovića 12
34000 Kragujevac
Serbia and Montenegro

(Received 16 01 2002)
(Revised 16 10 2002)

bojanab@knez.uis.kg.ac.yu