PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série tome 54 (68), 1993, 1–12

INTENSIONAL LOGIC WITH DEEP CASES

Marica D. Prešić

Abstract. Deep cases may be treated as a type of unary operations which transform nouns (or noun phrases) to the corresponding noun case forms. As noun phrases usually occur in the form of noun cases and as intensional logic is one of the most important logical tools for the treatment of natural language, it seems reasonable to introduce deep case operations into the syntax of intensional logic.

In this paper the logic CIL (Case Intensional Logic), an extension of Montague intensional logic IL [Montague, 1970] is described. The main characteristic of CIL is that operations (2) corresponding to the deep noun cases are explicitly introduced into its syntax. This logic is very convenient for translating natural language locutions, particlularly for the languages with free word order. The role of participant is expressed in an explicit form. Thus the underlying structure, especially the structure TR (tectogrammatical representation) [Sgall, Hajičova & Penevova, 1986], is much more closer to the corresponding intensional logic formula. The idea for such an approach can be found in inflective languages in which deep cases are usually expressed by the corresponding morpholofical forms. We show that such an extension is not unnatural and that the main features of intensional logic IL are not violated. In the sequel, we develop the syntax and semantics of CIL, define generalized semantics and, for the given axiomatization, prove the generalized completeness theorem.

Introduction. Three main lines can be followed in the explanation of ordinary proper names: *denotative* [Names are mere labels serving simply to stand for a person, place or thing, but not to describe them. Thus, names have only detonation and no sense.], *descriptive* [Names have both denotation and sense, where sense is determined by some definite description or some set of definite descriptions], *predicative* [Names are a special sort of unary predicates possessed by a unique individual.]

On the line of *denotative* approach from Mill [1843] to Kripke [1979] proper names have only denotation and no sense. Thus there is a sharp distinction between names and definite description [which in praxis is sometimes not so easy to draw]. In Ziff's view [Ziff, 1960] proper names are in some sense not even part of the language. For Kripke they are 'rigid designator' having the same reference in all possible worlds [which in the real world may be fixed by means of a definite description]; the semantic account

AMS Subject Classification (1990): Primary 03 B 65

of names is supplemented by causal explanations of the pragmatics of naming, by the chain of communication which goes back to the individual designated by the name in an initial 'baptism'.

A typical classification of the roles which an individual, denoted by a proper name may play in the real or some possible world is the one made on the basis of deep cases [Fillmore, 1968 and Sgall, Hajičova & Penevova, 1986]. Thus for the individual named Peter we can talk about:

- Situations in which Peter is an actant,
- Situations in which Peter is an objective,
- Situations in which Peter is an addressee,...

And so fort, running over all the deep cases. Which and how many of these situations are taken into account for the identification of Peter, depends on the corresponding approach to proper names.

The line of *descriptive* approach has several directions. Frege [1892] regards proper names as having sense as well as reference, and equates the sense of an ordinary name with the sense of a definite description referring to the same object. And the cosignative definitive description is one that the speaker has in mind or happens to know. Russell [1905, 1956], like Frege identifies the meaning of ordinary proper names with the meaning of some relevant definite description [although his view of meaning and his explanation of definite description differs from Frege's]. Again the meaning of the name is variable between speakers. This difficulty could be avoided by identifying the meaning of the name with the set of all descriptions true of the bearer. Wittgenstein in his *Philosophische Untersuchungen* [1953] suggested that rather than a fixed meaning a name has instead some sort of unequivocal, fuzzy meaning and is loosely associated with a set of descriptions of its bearer.

Similar to classification of situations all definite descriptions related to a proper name can be classified according to the corresponding deep cases. Thus, for somebody named *Peter* the related classification would be for example:

- the man who writes poems, the man who gets up early in the morning, the man who loves travelling,...
- the man whom people love, the man whom I expect tonight,...
- the man to whom somebody gives something, brings something,...

And so forth: Definite descriptions corresponding to the same deep case thus belong to the same class.

Both classifications suggest that one specific role (function, portion) of Peter is relevant and related to each deep case and that these portions differ from each other. This is similar to the mathematical examples of a vector and its coordinates, or a natural number and its prime factors. The vector can be identified by the sequence of its coordinates and the number by the sequence of the corresponding exponets of primes.

What the previous discussion suggests is that the name could be determined by the sequence of its deep cases. This is the idea we would like to develop in this paper. Thus the deep cases are treated as operations which project the noun (or noun phrase) into the corresponding case forms. Suppose that the total number of deep cases is k and that they are ordered in some given way. The ordering, for example, in which they usually appear in the underlying structure [Seidlovà, 1983]:

Actor, Addressee, Objective, Origin, Effect, Manner,
Directional (from where), Accompaniment, Means, (1)
Directional (which way), Directional (where to), Locative, ..., $Case_k$

where with Case_k the last deep case is denoted. As symbols for deep cases we use the Roman superscripts:

$$^{\mathrm{I}},{}^{\mathrm{II}},{}^{\mathrm{III}},{}^{\mathrm{IV}},\ldots,{}^{\mathrm{K}} \tag{2}$$

corresponding to the standard sequence (1) respectively.

To conclude our discussion we just mention that in accordance with the *predicative* approach names are in fact reduced to the predicates of the second order having the property of "being satisfied for exactly one individual" [Montague, 1973]. This is a typical case of representation, so that the whole classification modulo deep cases can be reformulated in terms of this specific kind of predicates.

1. We now present the logic CIL. Throughout the paper the notation from Gallin [1975] is adopted. We start with the definition of the set T of *types* of CIL.

Types. Let $e_1, e_2, \ldots, e_k, e, t, s$ be any $k+3$ symbols, none of which is an order pair. T is the smallest set satisfying:

(i) $t \in T$, (iii) $\alpha, \beta \in T$ imply $(\alpha, \beta) \in T$,
(ii) $e, e_1, e_2, \ldots, e_k \in T$, (iv) $\alpha \in T$ implies $(s, \alpha) \in T$.

Short notation is often used: $\alpha\beta$, $s\alpha$ is written instead of (α, β), (s, α).

Primitive Symbols. Same as in Gallin [1975:11] plus the case operations (2).

Terms. To the Gallin [1975:11] definition of terms one more line is added:

$$A \in \mathrm{Tm}_e \text{ implies } A^I \in \mathrm{Tm}_{e_1}, A^{\mathrm{II}} \in \mathrm{Tm}_{e_2}, \ldots, A^K \in \mathrm{Tm}_{e_k} \tag{3}$$

where $A^I, A^{\mathrm{II}}, \ldots A^K$ are terms obtained by means of the operations (2) and they are called: *first, second,* ..., K^{th} *case of* A respectively.

Semantics. Let $D_1, D_2, \ldots D_k$ and I be non-empty sets. By a *standard frame based on these sets* the indexed family $(M_a)_{\alpha \in T}$ of sets is understood, where:

(i) $M_{e_1} = D_1, M_{e_2} = D_2, \ldots, M_{e_k} = D_k$
(ii) $M_e = D_1 \mathrm{x} D_2 \mathrm{x} \ldots \mathrm{x} D_k$
(iii) $M_t = \{0, 1\}$
(iv) $M_{\alpha\beta} = M_\beta^{M_\alpha} = \{F | F : M_\alpha \to M_\beta\}$
(v) $M_{s\alpha} = M_\alpha^I = \{F | F : I \to M_\alpha\}$

A (standard) model of CIL based on $D_1, D_2, \ldots, D_k$ and I is a system $M = (M_\alpha, m)_{\alpha \in T}$, where:

(i) $(M_\alpha)_{\alpha\in T}$ is a standard frame based on $D_1, D_2, \dots, D_k$ and I,

(ii) m (the *meaning* function) is a mapping which assigns to each constant c_α of CIL a function from I into M_α (an element of M_α^I).

The definition of *assignment over* M, the assignment $a(x/X)$ and the set $\mathrm{As}(M)$ of all assignment over M is same as in Gallin [1975:12].

The definition of *value* $V_{i,a}^M(A_\alpha)$ *in* M *of the term* A_α *with respect to the index* i *and the assignment* a is similar to the corresponding Gallin [1975:13] definition but in addition it has the part corresponding to the case operations (2) which reads [The superscript ^{M} has been suppressed]:

$$V_{i,a}(A_e^I) = \pi_1(V_{i,a}(A_e)), \dots, V_{i,a}(A_e^K) = \pi_k(V_{i,a}(A_e)) \tag{4}$$

where $\pi_1, \dots, \pi_k$ designate the projection functions.

As CIL is an extension of IL all valid formulae of IL remain valid in CIL. In addition, it is easy to prove that the following formulae of CIL are valid:

$$A_e \equiv B_e \to A^I \equiv B^I,\ \ A_e \equiv B_e \to A^{\mathrm{II}} \equiv B^{\mathrm{II}}, \dots, A_e \equiv B_e \to A^K \equiv B^K \tag{5}$$

$$A_e \equiv B_e \to [A^I \equiv B^I \wedge A^{\mathrm{II}} \equiv B^{\mathrm{II}} \wedge \dots \wedge A^K \equiv B^K] \tag{6}$$

$$[A_e^I \equiv B_e^I \wedge A_e^{\mathrm{II}} \equiv B_e^{\mathrm{II}} \wedge \dots \wedge A_e^K \equiv B_e^K] \to A \equiv B \tag{7}$$

The definitions of modally closed terms, sentencial connectives, quantifiers and modal operators are same as those given in Gallin [1975:14–17].

Because the deep case operation are incorporated into the syntax of CIL, this logic is very convenient for translating the locutions of natural infleced languages. We give several examples from Serbo-Croatian.

The most important role of deep cases is the role of the verb participants. Thus verbs are not only relations of the given length: IV unary, TV binary and so on, but they also are characterized by the deep cases which are required for the noun phrases to which the verb is applied. In accordance with this, intransitive verbs are of the grammatical category $\mathrm{IV}(I)$,

Transitive verbs like *voleti* (to love), *očekivati* (to expect), *posmatrati* (to watch) are of grammatical category TV(I,III) [for they require an actant and an objective]. In accordance with the discussion in Materna & Sgall [1983] there are at least two verbs *pisati* (to write): one is *pisati*$_1$ of length four [write something to somebody with something] and the other is *pisati*$_2$ of length 3 [to write something with something] . Their grammatical categories would be respectively: TV(I, III, II, IX), TV(I, III, IX). Generally, the grammatical category of a transitive verb is of the form: $\mathrm{TV}(R, S, \dots, T)$ where $R, S, \dots, T$ are some of the case operation symbols (2), the first one being usually actant. The corresponding type in CIL is then:

$$e_t(\dots(e_s(e_r t))\dots) \qquad \text{[$r,s,\dots,t$ are natural numbers corresponding to the deep cases $R,S,\dots,T$ respectively]}$$

Thus the type of *voleti* [to love] is $e_3(e_1 t)$ and types of *pisati*$_1$, *pisati*$_2$ are:

$$e_9(e_2(e_3(e_1 t))), e_9(e_3(e_1 t))$$

respectively. Similarly, the type of the verb *otvarati* [to open something with something] is $e_9(e_3(e_1t))$ and the type of the corresponding intransitive verb *se otvarati* [as in the sentence: The door opens] is e_1t. We note that such treatment of intransitive verbs is in the spirit of TIL, *Transparent Intensional Logic* [Tichy, 1980], whereas in Montague [1973] approach this category is defined as IV/T, T being the category of terms [noun phrases and proper names].

Applying the verb *voleti* [to love] first on the variable x_3 of type e_3 we obtain the expression $voleti(x_3)$ which is of type e_1t, thus an intransitive verb phrase. Applying it on the variable x_1 of type e_1 we obtain the formula $voleti(x_3)(x_1)$, for which we also use the usual writing: $voleti(x_1, x_3)$. This formula serves as a CIL translation of the sentence-scheme: x_1 *voli* x_3 [x_1 loves x_3] . If instead of the variables x_1, x_3 the terms x^I, y^{III} are used [where both x, y are variables of type e] the corresponding CIL formula reads: *voleti* (x^I, y^{III}) which is the translation of the sentence-scheme: x^I voli y^{III}. Similarly, the translation of the sentence:

Jovan piše Ani [John writes to Ann]

would be the following formula of CIL:

$$(\exists x)(\exists y)pisati(j^I, x^{\mathrm{III}}, a^{\mathrm{II}}, y^{\mathrm{IX}})$$

where j, a are the individual constants of CIL corresponding to the proper names *Jovan, Ana* respectively. And the translation of the sentence:

Ani Jovan piše [To Ann John writes.]

would be the formula:

$$(\exists x)(\exists y)pisati(a^{\mathrm{II}}, x^{\mathrm{III}}, j^I, y^{\mathrm{IX}})$$

in which the verb *pisati* of category TV(II, III, I, IX) appears and the original word order of the noun phrases has been preserved which was possible thanks to the deep case operations.

[It is assumed that the translation into CIL of any word from the lexicon is the same word unless otherwise stated.]

We recall that in Gallin [1975], following Montague's idea, proper names are treated as second ordered properties. Thus, the translation of the n^{th} case of Jovan, for example, would be:

$$\lambda P_n[P_n\{j^N\}] \tag{8}$$

where P_n is a variable of type $s(e_nt)$, j is non-logical constant of type e corresponding to *Jovan* and $A\{B\}$ stands for $[{}^{\vee}A](B)$ [A is of type $s(\alpha\beta)$, B is of type α]. If we apply this expression to the intransitive verb *hodati* [which is of type e_1t in which case we chose $n = 1$], we obtain the formula *hodati* (j^I) which is a translation of the sentence: *Jovan hoda* [John walks]. Similarly, starting from the formula $voleti(x_1, x_3)$ and applying λ operator on x_3 we obtain the expression: $\lambda x_3\ voleti(x_1, x_3)$ having the type e_3t.

We note: If the expression of natural language has the functional form $f(g)$ then generally, following Montague [1973], the corresponding translation into CIL is $f'(^\wedge(g'))$ where f' and g' are the translations of f, g respectively. Thus applying on the intension of $\lambda x_3 voleti(x_1, x_3)$ the translation for *Jovan* (in the objective case), we obtain: $\lambda P_3[P_3\{j^{\mathrm{III}}\}](^\wedge\lambda x_3 voleti(x_1, x_3))$, which, by λ conversion on P_3, is equivalent to the expression: $\lambda x_3[voleti(x_1, x_3)](j^{\mathrm{III}})$ and by λ conversion on x_3 is equivalent to the formula: $voleti(x_1, j^{\mathrm{III}})$. This formula is a translation of the sentence-scheme x_1 *voli Jovana* [x_1 loves John]. The translation of the verb phrase voleti Jovana [to love John] is then: $\lambda x_1 voleti(x_1, j^{\mathrm{III}})$ The translation of the determiner *svaki* [every] into CIL is the following:

$$\lambda P_m[\lambda Q_n \forall x[P_m\{x^M\} \to Q_n\{x^N\}]] \tag{9}$$

where P_m, Q_n are variables of type $s(e_m t)$, $s(e_n t)$ respectively, x is a variable of type e. It is easy to see that *every* is of the following type:

$$(s(e_m t))((s(e_n t))t)$$

The idea is that *every* is usually employed to express the fact that every x which has a property P_m has also a property Q_n. To emphasize the dependence of the determiner from two deep cases, we use two superscripts and write $svaki^{M,N}$. To translate the noun phrase *svaki čovek* [every man] into CIL we recall that this is in fact the short form of *svaki onaj koji je čovek* [every one who is a man] , so that *čovek* is in fact the predicate *je čovek* [is a man] having the type $e_1 t$. Bearing this mind the translation of the n^{th} case of the phrase *svaki čovek* would be the following: $(svaki^{I,N})'(^\wedge((čovek)'))$. Using the translation (9) for $svaki^{I,N}$, λ conversion, valid formula $^{\vee\wedge}A \equiv A$, this expression can be transformed into the following one, of type $(s(e_n t))t$:

$$\lambda Q_n \forall x\ [čovek(x^I) \to Q_n\{x^N\}]$$

If we apply the determiner *svaki čovek* (in nominative) to the verb *trčati* [run] which is of the category IV(I) we obtain the sentence: *Svaki čovek trči* [Every man runs] Translates of this into CIL is the following formula:

$$(svaki^{I,I})'(^\wedge((čovek)'))(^\wedge(trčati)')$$

which can easily be transformed into the equivalent one:

$$\forall x[čovek(x^I) \to trčati(x^I)]$$

Similarly we can translate the following noun phrase, where a is an individual constant of CIL corresponding to the proper name *Ann*:

Sve što Ana zaradi [Everything which Ann earns]

in which the verb *zaraditi* [to earn] of category TV(I,III) occurs. Starting from the sentence $zaraditi(a^I, x_3)$ we obtain predicate phrase:

$$\lambda x_3 zaraditi(a^I, x_3) \qquad [x_3 \text{ is a variable of type } e_3]$$

which is of type e_3t wherefrom applying $svaki^{\text{III, N}}$ it can easily be deduced:

$$\lambda P_3[\lambda Q_n \forall x[P_3\{x^{\text{III}}\} \to Q_n\{x^N\}]](^\wedge \lambda x_3 zaraditi(a^I, x_3))$$

which is equivalent to:

$$\lambda Q_n \forall x[(^\wedge \lambda x_3 zaraditi(a^I, x_3))\{x^{\text{III}}\} \to Q_n\{x^N\}]$$

and further equivalent to the expression of type $(s(e_nt))t$:

$$\lambda Q_n \forall x\,[zaraditi(a^I, x^{\text{III}}) \to Q_n\{x^N\}]$$

Applying this to the intension of the predicate phrase *b potroši* [*b* spends] where b is an individual constant of CIL corresponding to the proper name Bill, we obtain the expression of type t, i.e. the formula:

$$\lambda Q_3 \forall x\,[zaraditi(a^I, x^{\text{III}}) \to Q_3\{x^{\text{III}}\}](^\wedge \lambda y_3 potro\check{s}iti(b^I, y_3)$$

which after several transformations can be reduced to the equivalent formula:

$$\forall x[zaraditi(a^I, x^{\text{III}}) \to potro\check{s}iti(b^I, x^{\text{III}})]$$

And this is a CIL translation of the sentence:

Sve što Ana zaradi Bil potroši [Everything which Ann earns Bill spends]

The last example is two readings of the sentence ['de dicto' and 'de re']:

Jovan veruje da Ana voli nekog čoveka [John believes that Ann loves a man]

which would translate into the following formulae of CIL:

$$veruje(j, ^\wedge \exists x[\check{c}ovek(x^I) \wedge voleti(a^I, x^{\text{III}})])$$

$$\exists[\check{c}ovek(x^I) \wedge veruje(j^I, ^\wedge[voleti(a^I, x^{\text{III}})])]$$

We note that, on the basis of the discussion in Montague [1973] and Gallin [1975], the intensional verbs like *verovati* [to believe] are of the category TV(I,t) and the corresponding type in CIL would be the following: $(st)(e_1t)$.

2. In this part of the paper we define *generalized semantics*, give an *axiomatization* of CIL and prove the *generalized completeness theorem.*

Generalized Semantics. Let $D_1, D_2, \ldots D_k$ and I be non-empty sets. By a *frame based on these sets* the indexed family $(M_a)_{\alpha \in T}$ of sets is understood, where:

(i) $M_{e_1} = D_1, M_{e_2} = D_2, \ldots, M_{e_k} = D_k$

(ii) M_e is a non-empty subset of $D_1 \mathrm{x} D_2 \mathrm{x} \ldots \mathrm{x} D_k$

(iii) $M_t = \{0, 1\}$

(iv) $M_{\alpha\beta}$ is a non-empty subset of $M_\beta^{M_\alpha}$

(v) $M_{s\alpha}$ is a non-empty subset of M_α^I

A general model (g-model) of CIL *based on* $D_1, D_2, \ldots, D_k$ *and* I is a system $M = (M_\alpha, m)_{\alpha \in T}$, where:

(i) $(M_\alpha)_{\alpha \in T}$ is a frame based on $D_1, D_2, \ldots, D_k$ and I

(ii) m (the meaning function) is a mapping which assigns to each constant c_α a function from I into M_α

(iii) There exists a function V^M (the value function) which assigns, to each $i \in I$, $a \in \mathrm{As}(M)$, and $A_\alpha \in \mathrm{Tm}_\alpha$, a value $V^M_{i,a}(A_\alpha) \in M_\alpha$ in such a way as to satisfy the recursive conditions (1) through (7) on page 13 of Gallin [1975] as well as the condition (3) of this paper.

The definitions of *g-model, g-semantical consequence, g-valid formula, g-satisfiable set of formula* do not differ from the corresponding definitions in Gallin [1975: 17-18].

The Theory CIL. This theory is obtained from the corresponding theory of IL by adding formula (7) as a new axiom. If instead axiom AS4 of IL the Henkin type schemata AS4.1 through AS4.7 are supposed, for the logic CIL the following axioms should be added to this list:

AS4.8$_N$ $\qquad (\lambda x_\alpha A_e^N) B_\alpha \equiv [(\lambda x A) B]^N$

Also, Rule R can be replaced by the eight rules $R1$ through $R8$ of Gallin [1975: 20] to which the rules:

$R9_N$ $\qquad$ From $A_e \equiv B_e$ to infer $A^N \equiv B^N$

are added, where ^{N} can be any of the case operations (2).

All metatheorems $T1$ through $T69$ proved in Gallin [1975:21-24] remain metatheorems for CIL.

From the deduction theorem it follows that all formulae (5) are theorems of CIL. Further, by (5), $T11$ and tautology $[A \to B] \wedge [A \to C] \to [A \to B \wedge C]$ we conclude that (6) is a theorem of CIL. Combining (6) and (7) the theorem below follows immediately:

$$A_e \equiv B_e \longleftrightarrow [A^I \equiv B^I \wedge A^{\mathrm{II}} \equiv B^{\mathrm{II}} \wedge A^K \equiv B^K] \tag{10}$$

The key lemma 3.2 of Gallin [1975: 25-29] together with the given proof can be completely transferred into the theory CIL. We can now prove the following theorem:

THEOREM (Generalized Completeness Theorem for CIL). *Let* Σ *be any set of formulae. Then,* Σ *is consistent in CIL implies that* Σ *is g-satisfiable in CIL.*

Proof. Supposing Σ is consistent in CIL we prove that Σ is g-satisfiable in g-model $M = (M_\alpha, m)_{\alpha \in T}$ of CIL based on sets $D_1, D_2, \ldots, D_k$ and I, where I is denumerable and $D_1, D_2, \ldots, D_k$ and each domain M_α are at most denumerable. Let $\overline{\Sigma} = (\overline{\Sigma}_i)_{i \in \omega}$ be a sequence of sets of formulae satisfying (i) through (vi) of Lemma 3.2 of Gallin [1975:25]. The relation $\simeq$ (mod i) between terms of type α is defined by:

$$A_\alpha \simeq B_\alpha (\mathrm{mod}\ i) \ \text{ if and only if } \ [A \equiv B] \in \overline{\Sigma}_i \tag{11}$$

where $\alpha \in T$, $i \in \omega$. This relation is independent of $i \in \omega$ for variables x, y and in this case we write $x \simeq y$. Let $D_1, D_2, \ldots, D_k$ be defined as:

$$D_1 = \mathrm{Var}_{e_1}/\simeq, \ldots, D_k = \mathrm{Var}_{e_k}/\simeq \tag{12}$$

and $I = \omega$. Then I is denumerable and each $D_1, D_2, \ldots, D_k$ is at most denumerable. By recursion on $\alpha \in T$, we simultaneously define a set M_α and a mapping μ_α for Tm_α into M_α^I, satisfying the following three conditions:

$$\text{For } i, j \in I,\ x \in \mathrm{Var}_\alpha : \mu_\alpha(x)(i) = \mu_\alpha(x)(j) \tag{13}$$

$$\text{For every } X \in M_\alpha \text{ there exists } x \in \mathrm{Var}_\alpha \text{ such that } X = \mu_\alpha(x)(i) \tag{14}$$

$$\text{For } i \in I, A, B \in \mathrm{Tm}_\alpha : \mu_\alpha(A)(i) = \mu_\alpha(B)(i) \text{ if and only if } A \simeq B \pmod i \tag{15}$$

The conditions (13) through (15) are the same as the Gallin [1975] conditions (1) through (3) on page 31.

$\underline{\alpha = e_n}(n = 1, 2, \ldots, k)$: Let $M_n = D_n = \mathrm{Var}_{e_n}/\simeq$ and define μ_{e_n} as follows:

$$\mu_{e_n}(A_{e_n})(i) = x/\simeq \text{ if and only if } A \simeq x \pmod i$$

where x is a variable of type e_n. By theorem $T29$ such a variable exists and the definition does not depend on the choice of x. In the case A is variable x: $\mu_{e_n}(x)(i) = x/\simeq$, from which it follows that conditions (13) and (14) hold. The verification of (15) follows immediately.

$\underline{\alpha = e}$: Let $A \in \mathrm{Tm}_e$. Then $A^I \in \mathrm{Tm}_{e_1}$, $A^{\mathrm{II}} \in \mathrm{Tm}_{e_2}, \ldots, A^K \in \mathrm{Tm}_{e_k}$. Suppose that $M_{e_1}, M_{e_2}, \ldots, M_{e_k}$ and $\mu_{e_1}, \mu_{e_2}, \ldots, \mu_{e_k}$ have already been defined so that the conditions (13) through (15) hold for $e_1, e_2, \ldots, e_k$. Firstly, we define the mapping μ_e from Tm_e into $(M_{e_1}\mathrm{x}M_{e_2}\mathrm{x}\ldots\mathrm{x}M_{e_k})^I$ as follows:

$$\mu_e(A)(i) = (\mu_{e_1}(x^1)(i), \ldots, \mu_{e_k}(x^k)(i)) \text{ iff } A^I \simeq x^1 (\mathrm{mod}\, i), \ldots, A^K \simeq x^K (\mathrm{mod}\, i) \tag{16}$$

where $x^1 \in \mathrm{Var}_{e_1}, \ldots, x^k \in \mathrm{Var}_{e_k}$. Such variables exist by $T29$ and the definition does not depend on the choice of sequence $(x^1, \ldots, x^k)$. We verify the conditions (13) through (15). To prove (13), suppose A is a variable $x \in \mathrm{Var}_e$. Then for $i \in I$ there exist variables $y^1, \ldots, y^k, y^1 \in \mathrm{Var}_{e_1}, \ldots, y^k \in \mathrm{Var}_{e_k}$, such that:

$$x^I \simeq y^1 (\mathrm{mod}\, i), \ldots, x^K \simeq y^k (\mathrm{mod}\, i)$$

i.e. such that $x^I \equiv y^1, \ldots, x^K \equiv y^k$ belong to $\overline{\Sigma}_i$. As these formulae are modally closed, by $T39$ it follows that $\Box[x^I \equiv y^1]$, $\Box[x^K \equiv y^k]$ belong to $\overline{\Sigma}_i$, from which it follows that $x^I \equiv y^1, \ldots, x^K \equiv y^k$ belong to $\overline{\Sigma}_j$ for every $j \in I$. This yields:

$$x^I \simeq y^1 (\mathrm{mod}\ j), \ldots, x^K \simeq y^k (\mathrm{mod}\, j), \text{ for every } j \in I \tag{17}$$

By applying definition (16) to the variable x by (17) we conclude:

$$\mu_e(x)(j) = (\mu_{e_1}(y^1)(j), \ldots, \mu_{e_k}(y^k)(j)), \text{ for every } j \in I \tag{18}$$

As condition (13) holds for the variables $y^1, \ldots, y^k$, we deduce from (18) $\mu_e(x)(i) = \mu_e(x)(j)$, for every $i, j \in I$, which proves (13).

To prove (15), let $A, B \in \mathrm{Tm}_e$, $i \in I$. Then:

$$
\begin{aligned}
A \simeq B \pmod{i} \ &\text{implies}\ A \equiv B \in \overline{\Sigma}_i \\
&\text{implies}\ \overline{\Sigma}_i \vdash A \equiv B \\
&\text{implies}\ \overline{\Sigma}_i \vdash A^I \equiv B^I \wedge \ldots \wedge A^K \equiv B^K \\
&\qquad (\text{By }(5)) \\
&\text{implies}\ \overline{\Sigma}_i \vdash A^I \equiv B^I, \ldots, \overline{\Sigma}_i \vdash A^K \equiv B^K \\
&\text{implies}\ A^I \simeq B^I \pmod{i}, \ldots, A^K \simeq B^K \pmod{i} \\
&\text{implies}\ [A^I \simeq x^1 \pmod{i} \text{ iff } B^I \simeq x^1 \pmod{i}], \\
&\qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\qquad [A^K \simeq x^k \pmod{i} \text{ iff } B^K \simeq x^k \pmod{i}] \\
&\qquad (\text{For any variables } x^1 \in \mathrm{Var}_{e_1}, \ldots, x^k \in \mathrm{Var}_{e_k}) \\
&\text{implies}\ [[A^I \simeq x^1 \pmod{i}, \ldots, A^K \simeq x^k \pmod{i}] \\
&\qquad \text{iff } [B^I \simeq x^1 \pmod{i}, \ldots, B^K \simeq x^k \pmod{i}]] \\
&\text{implies}\ [\mu_e(A)(i) = (\mu_{e_1}(x^1)(i), \ldots, \mu_{e_k}(x^k)(i)) \\
&\qquad \text{iff } \mu_e(B)(i) = (\mu_{e_1}(x^1)(i), \ldots, \mu_{e_k}(x^k)(i))] \\
&\text{implies}\ \mu_e(A)(i) = \mu_e(B)(i)
\end{aligned}
$$

We have thus proved:

$$A \simeq B \pmod{i} \ \text{implies}\ \mu_e(A)(i) = \mu_e(B)(i) \tag{19}$$

To complete the verification of (15) it suffices to prove:

$$\mu_e(A)(i) = \mu_e(B)(i) \ \text{implies}\ A \simeq B \pmod{i} \tag{20}$$

A proof is the following:

$$
\begin{aligned}
\mu_e(A)(i) = \mu_e(B)(i) \ &\text{implies}\ [\mu_e(A)(i) = (\mu_{e_1}(x^1)(i), \ldots, \mu_{e_k}(x^k)(i)), \\
&\qquad \mu_e(B)(i) = (\mu_{e_1}(x^1)(i), \ldots, \mu_{e_k}(x^k)(i))] \\
&\qquad (\text{For some } x^1 \in \mathrm{Var}_{e_1}, \ldots, x^k \in \mathrm{Var}_{e_k}) \\
&\text{implies}\ [A^I \simeq x^1 \pmod{i}, \ldots, A^K \simeq x^k \pmod{i}, \\
&\qquad B^I \simeq x^1 \pmod{i}, \ldots, B^K \simeq x^k \pmod{i}] \\
&\text{implies}\ A^I \simeq B^I \pmod{i}, \ldots, A^K \simeq B^K \pmod{i} \\
&\text{implies}\ A^I \equiv B^I \in \overline{\Sigma}_i, \ldots, A^K \equiv B^K \in \overline{\Sigma}_i \\
&\text{implies}\ \overline{\Sigma}_i \vdash A^I \equiv B^I, \ldots, \overline{\Sigma}_i \vdash A^K \equiv B^K \\
&\text{implies}\ \overline{\Sigma}_i \vdash A^I \equiv B^I \wedge \ldots \wedge A^K \equiv B^K \\
&\text{implies}\ \overline{\Sigma}_i \vdash A \equiv B \\
&\qquad (\text{By the axiom (7) and T19}) \\
&\text{implies}\ A \simeq B \pmod{i}
\end{aligned}
$$

which completes the proof of (20) and thus the proof of (15).

We can now set

$$M_e = \{\mu_e(x)(i) | x \in \mathrm{Var}_e\} \subseteq M_{e_1}\mathrm{x}\ldots\mathrm{x}M_{e_k} \tag{21}$$

which by condition (13) is independent of $i \in I$, and the condition (14) will be satisfied.

The definitions of all other sets M and the corresponding mappings μ_α are the same as in Gallin [1975:331-33], as well as the definition of $V_{i,a}^M$. To prove that V^M is a value function in M, it suffices to prove that condition (4) holds.

Let $A \in \mathrm{Tm}_e$, $a \in \mathrm{As}(M)$ and let $x^0, \ldots, x^{m-1}$ be all distinct free variables of A, having the types $\alpha_0, \ldots, \alpha_{m-1}$. We choose a representing sequence $y^0, \ldots, y^{m-1}$ for A and a, i.e. a sequence satisfying the conditions:

$$\mu_\alpha(y^n)(i) = a(x^n) \;\; (n = 0, \ldots, m-1; \;\; \text{independent of } \; i \in I) \tag{22}$$

$$y^n \;\text{ is free for }\; x^n \;\text{ in }\; A \;\; (n = 0, \ldots, m-1) \tag{23}$$

By definition of $V_{i,a}$ we have:

$$V_{i,a}(A) = \mu(\overline{A})(i) = (\mu_{e_1}(z^1)(i), \ldots, \mu_{e_k}(z^k)(i)) \tag{24}$$

where $\overline{A}$ is $A(y^0, \ldots, y^{m-1})$ and $z^1 \in \mathrm{Var}_{e_1}, \ldots, z^k \in \mathrm{Var}_{e_k}$ are variables satisfying the conditions:

$$A^I \simeq z^1 (\operatorname{mod} i), \ldots, A^K \simeq z^k (\operatorname{mod} i) \tag{25}$$

It is obvious that $\overline{A}^I = \overline{A^I}, \ldots, \overline{A}^K = \overline{A^K}$. Thus the conditions (25) become:

$$\overline{A^I} \simeq z^1 (\operatorname{mod} i), \ldots, \overline{A^K} \simeq z^k (\operatorname{mod} i)$$

from which we obtain:

$$\mu_{e_1}\left[\overline{A^I}\right](i) = \mu_{e_1}(z^1)(i), \ldots, \mu_{e_k}\left[\overline{A^K}\right](i) = \mu_{e_K}(z^k)(i)$$

In other words:

$$V_{i,a}(A^I) = \mu_{e_1}(z^1)(i), \ldots, V_{i,a}(A^K) = \mu_{e_k}(z^k)(i)$$

which yields:

$$(V_{i,a}(A^I), \ldots, V_{i,a}(A^K)) = (\mu_{e_1}(z^1)(i), \ldots \mu_{e_k}(z^k)(i)) \tag{26}$$

Using (24) and (26) we conclude that condition (4) is satisfied. This completes the proof that M is a g-model of CIL. If $a \in \mathrm{As}(M)$ is defined by:

$$a(x_\alpha) = \mu_\alpha(x)(i) \;\; (\text{independent of } \; i)$$

then for any formula A of CIL we have:

$$M, i, a \;\text{ sat }\; A \;\text{ iff }\; A \in \overline{\Sigma}_i$$

Since $\Sigma \subseteq \overline{\Sigma}_o$, it follows that:

$$M, 0, a \text{ sat } \Sigma$$

which completes the proof of the Theorem.

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Matematički fakultet,
Univerzitet u Beogradu,
Studentski trg 16,
11000 Beograd,
Jugoslavija

(Received 05 04 1993)