PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série tome 47 (61), 1990, 5–12

ALL GENERAL SOLUTIONS OF FINITE EQUATIONS

Dragić Banković

Abstract. Recently Prešić determined in [**4**] all reproductive solutions of the finite equation, where a finite equation is an equation the solution set of which is a subset of a given finite set. In this paper we determine all general solutions of such equations. Especially we also get all reproductive solutions.

Let E be a given non-empty set and $q(x)$ be any x-equation (x is an unknown element of E and q is a given unary relation of E) supposing that $q(x)$ has at least one solution.

Definition 1. Let $f : E \to E$ be a given function. The formula $x = f(t)$ represents a general solution of x-equation $q(x)$ if and only if $(\forall t \in E)q(f(t)) \wedge (\forall x \in E)(q(x) \implies (\exists t \in E)x = f(t))$.

Definition 2. Let $q : E \to E$ be a given function. The formula $x = g(t)$ represents a reproductive solution of x-equation $q(x)$ if and only if $(\forall t \in E)q(g(t)) \wedge (\forall t \in E)(q(t) \implies x = g(t))$.

Let $B = \{b_0, b_1, \dots, b_m\}$ be a given set of $m+1$ elements and $S = \{0, 1\}$. Define the operation x^y by

$$x^y = \begin{cases} 1, & \text{if } x = y \\ 0, & \text{otherwise} \end{cases} \qquad (x, y \in B \cup S).$$

The standard Boolean operations $+$ and $\cdot$ ("or" and "and") are described by the following tables:

$+$	0	1
0	0	1
1	1	1

$\cdot$	0	1
0	0	0
1	0	1

Extend these operations to the partial operations of the set $B \cup S$ by

$$x + 0 = x, \quad 0 + x = x, \quad x \cdot 0 = 0, \quad x \cdot 1 = x, \quad 0 \cdot x = 0, \quad 1 \cdot x = x, \quad (x \in B \cup S).$$

AMS Subject Classification (1985): Primary 03 G 50

We consider the x-equation

$$s_0 x^{b_0} + s_1 x^{b_1} + \cdots + s_m x^{b_m} = 0 \tag{1}$$

where $s_i \in \{0,1\}$ are given elements and $x \in B$ is unknown.

Obviously the equation (1) is consistent if and only if

$$s_0 s_1 \cdots s_m = 0. \tag{2}$$

Definition 3. Let $(a_0, a_1, \dots, a_m) \in S^{m+1}$. Then the set $Z(a_0, \dots, a_m)$ ("the zero-set of $(a_0, \dots, a_m)$") is defined by

$$b_i \in Z(a_0, \dots, a_m) \iff a_i = 0 \qquad (i = 0, 1, \dots, m). \tag{3}$$

For instance, if $m = 3$ we have

$$Z(1,1,1,1) = \varnothing, \quad Z(1,0,1,0) = \{b_1, b_3\}, \quad Z(0,0,0,0) = \{b_0, b_1, b_2, b_3\}.$$

Let $M = \{0, 1, 2, \dots, m\}$.

Definition 4. Let $s_0 \cdots s_m = 0$. A function $A : B \to B$ of the form

$$A(x) = A_0(s_0, \dots, s_m) x^{b_0} + \cdots + A_m(s_0, \dots, s_m) x^{b_m}$$

is a repro-function if and only if each coefficient $A_k(s_0, \dots, s_m)$ is determined by some equality of the form

$$A_k(s_0, \dots, s_m) = b_k s_k^0 + \sum_{a_k \neq 0,\, a_0 \cdots a_m = 0} F_k(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m},$$

where

$$(\forall k \in M)(\forall a_0, \dots, a_m \in S)(a_k \neq 0 \wedge a_0 \cdots a_m = 0 \\ \implies F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)).$$

Definition 5. Let $s_0 \cdots s_m = 0$. A function $A : B \to B$ of the form

$$A(x) = A_0(s_0, \dots, s_m) x^{b_0} + \cdots + A_m(s_0, \dots, s_m) x^{b_m}$$

is a gener-function if and only if there is a function $\psi : M \xrightarrow{1-1} M$ such that each coefficient $A_k(s_0, \dots, s_m)$ is determined by some equality of the form

$$A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_{a_{\psi(k)} \neq 0,\, a_0 \cdots a_m = 0} F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m},$$

where

$$(\forall k \in M)(\forall a_0, \dots, a_m \in S)\big(a_{\psi(k)} \neq 0 \wedge a_0 \cdots a_m = 0 \\ \implies F_k(a_0, \dots, a_m) \in Z(a_0, \dots, a_m)\big).$$

Let (1) be denoted by $g(x) = 0$.

LEMMA 1. *Let* $A(x) = A_0(s_0, \dots, s_m)x^{b_0} + \cdots + A_m(s_0, \dots, s_m)x^{b_m}$ *assuming that* $A_0(s_0, \dots, s_m), \dots, A_m(s_0, \dots, s_m) \in B$. *Then the formula*

$$(\forall x \in B)\big(g(x) = 0 \implies (\exists t \in B) x = A(t)\big)$$

holds if and only if there is a function $\psi : M \xrightarrow{1-1} M$ *such that each coeficient* $A_k(s_0, \dots, s_m)$ *is determined by the equality*

$$A_k(s_0, \dots, s_m) = b_{\psi(k)} s_{\psi(k)}^0 + \sum_{a_{\psi(k)} \neq 0,\, a_0, \dots, a_m = 0} F_{\psi(k)}(a_0, \dots, a_m) s_0^{a_0} \cdots s_m^{a_m},$$

where

$$(\forall k \in M)(\forall a_0, \dots, a_m \in S)\big(a_{\psi(k)} \neq 0 \wedge a_0 \cdots a_m = 0 \\ \implies F_{\psi(k)}(a_0, \dots, a_m) \in B\big).$$

The proof follows from the following equivalences.

$$\begin{aligned}
&(\forall x \in B)\big(g(x) = 0 \implies (\exists t \in B) x = A(t)\big) \\
\iff &(\forall x \in B)\big(x \in Z(s_0, \dots, s_m) \implies (\exists t \in B) x = A(t)\big) \\
\iff &(\forall x \in Z(s_0, \dots, s_m))(\exists t \in B) x = A(t) \\
\iff &(\exists \bar{f} : Z(s_0, \dots, s_m) \to B)(\forall x \in Z(s_0, \dots, s_m)) x = A(\bar{f}(x))
\end{aligned}$$

(by the axiom of choice)

$$\iff (\exists \bar{f} : Z(s_0, \dots, s_m) \xrightarrow{1-1} B)(\forall x \in Z(s_0, \dots, s_m)) x = A(\bar{f}(x)).$$

(Assuming $b_p \in Z(s_0, \dots, s_m)$, $b_r \in Z(s_0, \dots, s_m)$, $b_p \neq b_r$ and $\bar{f}(b_p) = \bar{f}(b_r) = b_u$, we get from $x = A_0(f(x))^{b_0} + \cdots + A_m(f(x))^{b_m}$ the following implications:

$$x = b_p \implies b_p = A_u, \quad x = b_r \implies b_r = A_u \quad \text{i.e.} \quad b_p = b_r.$$

Thus $\bar{f}$ is $\xrightarrow{1-1}$).

$$\iff (\exists f : B \xrightarrow{1-1} B)(\forall x \in Z(s_0, \dots, s_m)) x = A(f(x))$$

(f is an extension of $\bar{f}$)

$$\begin{aligned}
\iff &(\exists f : B \xrightarrow{1-1} B)(\forall x \in B)\big(x \in Z(s_0, \dots, s_m) \implies x = A(f(x))\big) \\
\iff &(\exists f : B \xrightarrow{1-1} B)(\forall x \in B)\big(s_0 x^{b_0} + \cdots + s_m x^{b_m} \\
&\implies x = A_0(s_0, \dots, s_m)(f(x))^{b_0} + \cdots \\
&+ A_m(s_0, \dots, s_m)(f(x))^{b_m} \wedge (\forall k \in M) A_k(s_0, \dots, s_m) \in B\big)
\end{aligned}$$

$$\begin{aligned}
&\iff (\exists f : B \xrightarrow{1-1} B)(\forall k \in M)\big(s_0 b_k^{b_0} + \cdots + s_m b_k^{b_m} = 0 \\
&\qquad \implies b_k = A_0(s_0, \ldots, s_m)(f(b_k))^{b_0} + \cdots \\
&\qquad + A_m(s_0, \ldots, s_m)(f(b_k))^{b_m} \wedge A_k(s_0, \ldots, s_m) \in B\big) \\
&\iff (\exists \varphi : M \xrightarrow{1-1} M)(\forall k \in M)\big(s_k = 0 \\
&\qquad \implies b_k = A(\varphi(k))(s_0, \ldots, s_m) \wedge A_k(s_0, \ldots, s_m) \in B\big)
\end{aligned}$$

(φ is defined by $(\forall c, d \in M)(\varphi(c) = d \iff f(b_c) = b_d)$)

$$\begin{aligned}
&\iff (\exists \varphi : M \xrightarrow{1-1} M)(\forall k \in M)\Big(A_{\varphi(k)}(s_0, \ldots, s_m) \\
&\qquad = b_k s_k^0 + \sum_{a_k \neq 0\ a_0 \cdots a_m = 0} F_k(a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \\
&\qquad \wedge \forall a_0, \ldots, a_m \in S\big(a_k \neq 0 \wedge a_0 \cdots a_m = 0 \implies F_k(a_0, \ldots, a_m) \in B\big)\Big) \\
&\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M)\Big(A_k(s_0, \ldots, s_m) \\
&\qquad = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m} \\
&\qquad \wedge\ (\forall a_0, \ldots, a_m \in S)\big(C \implies F_{\psi(k)}(a_0, \ldots, a_m) \in B\big)\Big)
\end{aligned}$$

(ψ is φ^{-1}, C is the conjunction $a_{\psi(k)} \neq 0 \wedge a_0 \cdots a_m = 0$ and $\sum_C$ means the sum over all $(a_0, \ldots, a_m) \in S^{m+1}$ such that C is satisfied).

THEOREM 1. *Let* $A(x) = A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m}$ *and* $A_0(s_0, \ldots, s_m), \ldots, A_m(s_0, \ldots, s_m) \in B$. *If*

$$s_0 x^{b_0} + \cdots + s_m x^{b_m} = 0 \tag{1}$$

is a consistent equation then the formula $x = A(t)$ (t *is any element of* B) *represents a general solution of* (1) *and only if the function* A *is a gener-function.*

Proof. It we denote (1) by $g(x) = 0$ we have

$$\begin{aligned}
&(\forall x \in B) g(A(x)) = 0 \wedge (\forall x \in B)\big(g(x) = 0 \implies (\exists t \in B) x = A(t)\big) \\
&\iff (\forall x \in B) s_0\big(A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m}\big)^{b_0} + \cdots \\
&\qquad + s_m\big(A_0(s_0, \ldots, s_m)x^{b_0} + \cdots + A_m(s_0, \ldots, s_m)x^{b_m}\big)^{b_m} = 0 \\
&\qquad \wedge (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M)\Big(A_k(s_0, \ldots, s_m) \\
&\qquad = b_{\psi(k)} s_{\psi(k)}^0 + \sum_C F_{\psi(k)}(a_0, \ldots, a_m) s_0^{a_0} \cdots s_m^{a_m}
\end{aligned}$$

$$\wedge(\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\Big)$$

(by Lemma 1)

$$\iff (\forall i,k \in M) s_i A_k^{b_i}(s_0,\dots,s_m) = 0$$

$$\wedge\, (\exists \psi : M \xrightarrow{1-1} M)(\forall k \in M)\Big(A_k(s_0,\dots,s_m)$$

$$= b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m) s_0^{a_0}\cdots s_m^{a_m}$$

$$\wedge\, (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\Big)$$

(This part of the proof is based on the following general facts:

If $a_0,\dots,a_n,b$, are any elements of B then

1° $\big(a_0x^{b_0} + \cdots + a_mx^{b_m}\big)^b = a_0^b x^{b_0} + \cdots a_m x^{b_m}$ (for all $x \in B$)

2° $(\forall x \in B) a_0x^{b_0} + \cdots + a_mx^{b_m} = 0 \implies (\forall i \in M) a_i = 0$.)

$$\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Big(s_i A_k^{b_i}(s_0,\dots,s_m) = 0$$

$$\wedge\ (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m)\big)$$

$$\wedge\, A_k(s_0,\dots,s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m) s_0^{a_0}\cdots s_m^{a_m}\Big)$$

$$\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Big(s_i \Big(b_{\psi(k)} s^0_{\psi(k)}$$

$$+ \sum_C F_{\psi(k)}(a_0,\dots,a_m) s_0^{a_0}\cdots s_m^{a_m}\Big)^{b_i} = 0$$

$$\wedge\ (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)$$

$$\wedge\, A_k(s_0,\dots,s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)}(s_0,\dots,s_m) s_0^{a_0}\cdots s_m^{a_m}\Big)$$

$$\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Big(s_i \Big(\sum_C F^{b_i}_{\psi(k)}(a_0,\dots,a_m) s_0^{a_0}\cdots s_m^{a_m}\Big) = 0$$

$$\wedge\, (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)$$

$$\wedge\, A_k(s_0,\dots,s_m) = b_{\psi(k)} s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m) s_0^{a_0}\cdots s_m^{a_m}\Big)$$

(we have used the identity $s_p s_r^0 b_r^{b_p} = 0$)

$$\iff (\exists \psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Big(\sum_{(a_0,\dots,a_m)\in S^{m+1}} a_i \cdot s_0^{a_0}\cdots s_m^{a_m}$$

$$
\begin{aligned}
&\cdot \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m} = 0\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\\
&\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Bigg)
\end{aligned}
$$

(because of the identity $s_i = \sum_{(a_0,\dots,a_m)\in S^{m+1}} a_i s_0^{a_0}\cdots s_m^{a_m}$)

$$
\begin{aligned}
\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Bigg(&\sum_C a_i F^{b_i}_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m} = 0\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\\
&\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Bigg)\\
\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Bigg(&(\forall a_0,\dots,a_m \in S)\\
&\big(C \implies a_i F_{\psi(k)}(a_0,\dots,a_m) = 0\big)\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\\
&\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Bigg)\\
\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall i,k \in M)\Bigg(&(\forall a_0,\dots,a_m \in S)\big(C \implies a_i b^{b_i}_{h(k)} = 0\big)\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\\
&\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}c^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Bigg)
\end{aligned}
$$

(we have used the following facts:
(I) $(\forall k \in M)F_{\psi(k)} \in B \iff (\forall k \in M)(\exists j \in M)F_{\psi(k)} = b_j$;
(II) $(\forall k \in M)(\exists j \in M)F_{\psi(k)} = b_j \iff (\exists h : M \to M)(\forall k \in M)F_{\psi(k)} = b_{h(k)}$,
by the axiom of choice)

$$
\begin{aligned}
\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k \in M)\Bigg(&(\forall i \in M)(\forall a_0,\dots,a_m \in S)\big(C \implies a_i b^{b_i}_{h(k)} = 0\big)\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)\\
&\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Bigg)\\
\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k \in M)\Bigg(&(\forall a_0,\dots,a_m \in S)\big(C \implies a_{h(k)} = 0\big)\\
&\wedge (\forall a_0,\dots,a_m \in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m) \in B\big)
\end{aligned}
$$

$$\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Big)$$

$$\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k\in M)\Big((\forall a_0,\dots,a_m\in S)$$

$$\big(C \implies b_{h(k)} \in Z(a_0,\dots,a_m)\big)$$

$$\wedge\,(\forall a_0,\dots,a_m\in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m)\in B\big)$$

$$\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Big)$$

(Definition 3)

$$\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k\in M)\Big((\forall a_0,\dots,a_m\in S)$$

$$\big(C \implies F_k(a_0,\dots,a_m) \in Z(a_0,\dots a_m)\big)$$

$$\wedge\,(\forall a_0,\dots,a_m\in S)\big(C \implies F_{\psi(k)}(a_0,\dots,a_m)\in B\big)$$

$$\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Big)$$

$$\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k\in M)\Big((\forall a_0,\dots,a_m\in S)(\implies$$

$$F_k(a_0,\dots,a_m) \in Z(a_0,\dots,a_m) \wedge F_{\psi(k)}(a_0,\dots,a_m)\in B)$$

$$\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Big)$$

(because $(p_1 \implies p_2)\wedge(p_1 \implies p_3) \iff (p_1 \implies p_2\wedge p_3)$ is a tautology)

$$\iff (\exists\psi : M \xrightarrow{1-1} M)(\forall k\in M)\Big((\forall a_0,\dots,a_m\in S)$$

$$\big(C \implies F_k(a_0,\dots,a_m) \in Z(a_0,\dots,a_m)\big)$$

$$\wedge A_k(s_0,\dots,s_m) = b_{\psi(k)}s^0_{\psi(k)} + \sum_C F_{\psi(k)}(a_0,\dots,a_m)s_0^{a_0}\cdots s_m^{a_m}\Big)$$

$\iff A$ is a gener-function.

The following Theorem 2 can be obtained from Theorem 1 if we assume that $\psi : B \to B$ is the identical mapping.

Theorem 2. [**4**] *If*

$$s_0x^{b_0} + \cdots s_mx^{b_m} = 0 \tag{1}$$

is a consistent equation, then the formula

$$x = A(p) \qquad (p \textit{ is any element of } B\,)$$

represents a reproductive solution of the equation (1) *if and only if the function* A *is a repro-function.*

Example 1. Let $s_0x^{b_0} + s_1x^{b_1} + s_2x^{b_2} = 0$ be a consistent equation i.e. $s_0s_1s_2 = 0$. If $\psi = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ then the formula $x = A_0P^{b_0} + A_1P^{b_1} + A_2P^{b_2}$ represents a general solution of $s_0x^{b_0} + s_1x^{b_1} + s_2x^{b_2} = 0$ provided

$$\begin{aligned} A_0 &= b_2s_2^0 + F_2(0,0,1)s_0^0s_1^0s_2^1 + F_2(0,1,1)s_0^0s_1^1s_2^1 + F_2(1,0,1)s_0^1s_1^0s_2^1 \\ A_1 &= b_0s_0^0 + F_0(1,0,0)s_0^1s_1^0s_2^0 + F_0(1,0,1)s_0^1s_1^0s_2^1 + F_0(1,1,0)s_0^1s_1^1s_2^0 \\ A_2 &= b_1s_1^0 + F_1(0,1,0)s_0^0s_1^1s_2^0 + F_1(0,1,1)s_0^0s_1^1s_2^1 + F_1(1,1,0)s_0^1s_1^1s_2^0 \end{aligned}$$

i.e.

$$\begin{aligned} A_0 &= b_2s_2^0 + (b_0 \text{ or } b_1)s_0^0s_1^0s_2^1 + b_0s_0^0s_1^1s_2^1 + b_1s_0^1s_1^0s_2^1 \\ A_1 &= b_0s_0^0 + (b_1 \text{ or } b_2)s_0^1s_1^0s_2^0 + b_1s_0^1s_1^0s_2^1 + b_2s_0^1s_1^1s_2^0 \\ A_2 &= b_1s_1^0 + (b_0 \text{ or } b_2)s_0^0s_1^1s_2^0 + b_0s_0^0s_1^1s_2^1 + b_2s_0^1s_1^1s_2^1. \end{aligned}$$

Remark 1. Let $p = 2^n - 1$ (n is a natural number), $\{0,1\}^n = \{D_0, \dots, D_p\}$, $f : \{0,1\}^n \to \{0,1\}$ be a Boolean function and

$$f(x_1, \dots, x_n) = 0 \tag{4}$$

be a consistent Boolean equation, i.e. $\prod_{i=0}^{p} f(D_i) = 0$.

In accordance with Theorem 1 one can effectively find all general solutions of (4) in the form

$$X = A_0(f(D_0), \dots, f(D_p))T^{D_0} \cup \dots \cup A_p(f(D_0), \dots, f(D_p))T^{D_p},$$

where $X = (x_1, \dots, x_n)$ and $T = (t_1, \dots, t_n)$.

REFERENCES

[**1**] D. Banković, *Formulas of the general solutions of Boolean equations*, Publ. Inst. Math. Beograd **44** (**58**) (1988), 3–7.

[**2**] S. Prešić, *Une méthode de resolution des équations dont toutes les solutions appartiennent à un ensemble fini donné*, C. R. Acad. Sci. Paris Ser. **A 272** (1971), 654–657.

[**3**] S. Prešić, *Ein Satz über reproduktive Lösungen*, Publ. Inst. Math. Beograd **14** (**28**) (1973), 133–136.

[**4**] S. Prešić, *All reproductive solutions of finite equations*, Publ. Inst. Math. Beograd (in print).

[**5**] S. Rudeanu, *Boolean Functions and Equations*, North-Holland Amsterdam/London and Elsevier, New York, 1974.

Prirodno-matematički fakultet
34000 Kragujevac, pp. 60,
Yugoslavia

(Received 27 01 1989)

ERRATUM

The reference [**4**] from above should have appeared as reference [**7**] in my paper *All general reproductive solutions of Boolean equations*, Publ. Inst. Math. (Beograd) **46** (**60**) (1989), 13–19, and the reference [**7**] should have been reference [**8**].