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SOME SPECIAL SUBSPACES OF A FINSLER SPACE

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Abstract. In the present paper are studied such subspaces of a Finsler space for which he absolute differential of the tangent or normal vectors have special positions.

1. Introduction. The equation of a subspace F_m of a Finsler space F_n, the definitions of the tangent vectors B^i_α, the normal vectors $\underset{\mu}{N}^i$, and the induced and intrinsic connection coefficients and curvature tensors are the same as in [**6**], [**2**] and [**3**]; so they are omitted. The induced connection coefficients and curvature tensors shall be denoted as usual by —.

Let us denote by $T_H(P)$ the subspace of the tangent space of F_n at $P(x, \dot{x}) = (X^i(u^\alpha),^i_\alpha \dot{u}^\alpha)$ spanned by B^i_α and by $T_V(P)$ the subspace spanned by $\underset{\mu}{N}^i$.

The object of the present paper is to study special subspaces which satisfy some of the following conditions at a fixed P for every displacement $(du^\alpha, d\dot{u}^\alpha)$ on the subspace F_m:

$$\text{1)}\quad DB^i_\alpha \in T_H \Leftrightarrow D\underset{\mu}{N}^i \in T_V \Leftrightarrow (DB^i_\alpha \in T_H) \wedge (D\underset{\mu}{N}^i \in T_V)$$

$$\text{1a)}\quad D\underset{\mu}{N}^i = 0 \Rightarrow DB^i_\alpha \in T_H$$

$$\text{1b)}\quad DB^i_\alpha \Rightarrow D\underset{\mu}{N}^i \in T_V$$

$$\text{2)}\quad DB^i_\alpha \in T_V$$

$$\text{3)}\quad D\underset{\mu}{N}^i \in T_H$$

$$\text{2a)} = \text{3a)}\quad (DB^i_\alpha \in TV) \wedge (D\underset{\mu}{N}^i \in T_H)$$

for every $\alpha = 1, 2, \dots, m$, $\mu = m + 1, \dots, n$.

Cases 1a) and 1b) are special cases of 1); 2a) = 3a) is a special case of 2) or 3).

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For the case 1) the induced and intrinsic connection coefficients are the same and the normal curvature $\overset{\nu}{N}(u,\dot{u}) = 0$ for every curve $u^\alpha = u^\alpha(s)$ trough P. Theorem 1.1 gives equivalent conditions for F_m to satisfy the conditions of case 1) for a fixed u and every $\dot{u}$.

For case 2) the subspace F_m is Riemannian with

$$^0\overline{R}_{\alpha\ \beta\gamma}^{\ \delta} = 0, \quad ^0\overline{P}_{\alpha\ \beta\gamma}^{\ \delta} = 0, \quad ^0\overline{S}_{\alpha\ \beta\gamma}^{\ \delta} = 0.$$

For case 3) we have

$$^1\overline{R}_{\mu\ \beta\gamma}^{\ \nu} = 0, \quad ^1\overline{P}_{\mu\ \beta\gamma}^{\ \nu} = 0, \quad ^1\overline{S}_{\mu\ \beta\gamma}^{\ \nu} = 0.$$

2. Case 1). $DB^i_\alpha \in T_H$. For any subspace F_m of F_n we have

$$DB^i_\alpha = (\overline{\Gamma}^{*\delta}_{\alpha\ \beta} du^\beta + \overline{A}_{\alpha\ \beta}^{\ \delta} \overline{D} l^\beta) B^i_\delta + (\overline{\theta}^{*\mu}_{\alpha\ \beta} du^\beta + \overline{A}_{\alpha\ \beta}^{\ \mu}) \underset{\mu}{N}^i,$$

$$D\underset{\mu}{N}^i = (-\overline{\theta}^{*\delta}_{\ \ \mu\beta} du^\beta - \overline{A}^{\ \delta}_{\ \ \mu\beta} \overline{D} l^\beta) B^i_\delta + (\overline{\lambda}^{*\nu}_{\mu\ \beta} du^\beta + \overline{A}_{\mu\ \beta}^{\ \nu} \overline{D} l^\beta) \underset{\nu}{N}^i.$$

In the case 1) these formulae become

$$DB^i_\alpha = (\overline{\Gamma}^{*\delta}_{\alpha\ \beta} du^\beta + \overline{A}_{\alpha\ \beta}^{\ \delta} \overline{D} l^\beta) B^i_\delta, \tag{2.1}$$

$$D\underset{\mu}{N}^i = (\overline{\lambda}^{*\nu}_{\mu\ \beta} du^\beta + \overline{A}_{\mu\ \beta}^{\ \nu} \overline{D} l^\beta) \underset{\nu}{N}^i. \tag{2.2}$$

In this case $\overline{\theta}^{*\mu}_{\alpha\ \beta} du^\beta + \overline{A}_{\alpha\ \beta}^{\ \mu} \overline{D} l^\beta = 0$, for every du^β and $\overline{D} l^\beta$, so

$$\overline{\theta}^{*\mu}_{\alpha\ \beta} du^\beta = 0, \quad \overline{A}_{\alpha\ \beta}^{\ \mu} \overline{D} l^\beta = 0 \tag{2.3}$$

for all

$$\alpha, \beta = 1, 2, \dots, m \ \ \mu = m+1, \dots, n.$$

From (2.1), (2.3) and

$$\overline{\theta}^*_{\mu\alpha\beta} = -\overline{\theta}^*_{\mu\alpha\beta}, \quad \overline{A}_{\mu\alpha\beta} = -\overline{A}_{\mu\alpha\beta}$$

we obtain

$$\overline{\theta}^*_{\mu\alpha\beta} = 0 \quad \overline{A}_{\mu\alpha\beta} = 0 \tag{2.4}$$

for all

$$\alpha, \beta = 1, 2, \dots, m \ \ \mu = m+1, \dots, n.$$

As for any subspace F_m we have

$$Dl^k = B^k_\alpha D l^\alpha + \overline{H}^k_\beta du^\beta$$

and for case 1)

$$Dl^k = D(B^k_\alpha l^\alpha) = B^k_\alpha \overline{D} l^\alpha$$

we conclude that in this case

$$\overline{H}^k_\beta du^\beta = 0.$$

The above equation is true for any du^β so that in case 1)

$$\overline{H}^k_\beta = 0, \quad k = 1, \dots, n \;\; \beta = 1, \dots, m. \tag{2.5}$$

From (2.5) it follows that the corresponding equations for $\overline{\Gamma}^{*\delta}_{\alpha\ \beta}$ and $\overline{\lambda}^{*\nu}_{\mu\ \beta}$ reduce to

$$\overline{\Gamma}^*_{\alpha\gamma\beta} = g_{ir} B^r_\gamma (B^{\ i}_{\alpha\ \beta} + \Gamma^{*i}_{j\ k} B^{j\ k}_{\alpha\ \beta}) \tag{2.6}$$

$$\overline{\lambda}^{*\nu}_{\mu\ \beta} = g_{ir} \underset{\nu}{N}^r (\partial_\beta \underset{\nu}{N}^i - \delta 0 t \partial_\delta \underset{\nu}{N}^i \overline{\Gamma}^{*\delta}_\alpha + \overline{\Gamma}^{*i}_{j\ k} \underset{\nu}{N}^j B^k_\beta). \tag{2.7}$$

Tensors $\overline{A}_{\alpha\beta\gamma}$ and $\overline{A}_{\mu\nu\gamma}$ are determined by

$$\overline{A}_{\alpha\beta\gamma} = A_{ijk} B^{i\ j\ k}_{\alpha\ \beta\ \gamma} = L(u, \dot{u}) 2^{-1} \dot{\partial}_\gamma g_{\alpha\beta}(u, \dot{u}) \tag{2.8}$$

$$\overline{A}_{\mu\nu\gamma} = g_{ij} \underset{\nu}{N}^j L \dot{\partial}_\gamma \underset{\mu}{N}^i + A_{ijk} \underset{\mu}{N}^i \underset{\nu}{N}^j B^k_\gamma. \tag{2.9}$$

The normal curvature $\overset{\nu}{N}$ of a curve $u^\alpha = u^\alpha(s)$ of the subspace F_m in the direction of $\overset{\nu}{N}_i$ is given by

$$\overset{\nu}{N}(u, \dot{u}) = L^{-2}(u, \dot{u}) \overline{\theta}^{*\nu}_{\alpha\ \beta} \dot{u}^\alpha \dot{u}^\beta \qquad (\dot{u}^\alpha = du^\alpha / du^s)$$

From (2.3) if follows that

$$\overset{\nu}{N}(u, \dot{u}) = 0 \tag{2.10}$$

for every curve $u^\alpha = u^\alpha(s)$ through the point (u).

From (2.1) and (2.2) we obtain

$$\begin{aligned}[\Delta D] B^i_\alpha = &\overline{\Omega}^\delta_\alpha (d, \delta) B^i_\delta = \{2^{-1} {}^0\overline{R}^{\ \delta}_{\alpha\beta\gamma} [du^\beta \delta u^\gamma] + \\ &{}^0\overline{P}_\alpha{}^\delta{}_{\beta\gamma} [du^\beta \overline{\Delta} l^\gamma] + 2^{-1} {}^0\overline{S}_\alpha{}^\delta{}_{\beta\gamma} [\overline{D} l^\beta \overline{\Delta} l^\gamma]\} B^i_\delta\end{aligned} \tag{2.11}$$

$$\begin{aligned}[\Delta D] \underset{\mu}{N}^i = &\overline{\Omega}^\nu_\mu (d, \delta) \underset{\nu}{N}^i = \{2^{-1} {}^1\overline{R}_\mu{}^\nu{}_{\beta\gamma} [du^\beta \delta u^\gamma] + \\ &{}^1\overline{P}_\mu{}^\nu{}_{\beta\gamma} [du^\beta \overline{\Delta} l^\gamma] + 2^{-1} {}^1\overline{S}_\mu{}^\nu{}_{\beta\gamma} [\overline{D} l^\beta \overline{\Delta} l^\gamma]\} \underset{\nu}{N}^i\end{aligned} \tag{2.12}$$

It may be seen that in case 1)

$${}^0\overline{R}_\alpha{}^\mu{}_{\beta\gamma}, \quad {}^0\overline{P}_\alpha{}^\mu{}_{\beta\gamma}, \quad {}^0\overline{S}_\alpha{}^\mu{}_{\beta\gamma}$$

The definitions of curvature tensors given above and in the sequel are given in [**6**].

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Some vector field $\xi^i(x(u) B_\alpha \dot{u}^\alpha)$ defined on the subspace F_m, may be decomposed in the following way

$$\xi^i = B^i_\alpha \xi^\alpha + \underset{\nu}{N}^i \xi^\mu$$

Using the known formulae

$$
\begin{aligned}
[\Delta D]\xi^i &= \{2^{-1}R_j{}^i{}_{hk}[dx^h\delta x^k] + P_j{}^i{}_{hk}[dx^h\Delta l^k] + 2^{-1}S_j{}^i{}_{hk}[Dl^h\Delta l^k]\}\xi^j\\
dx^h &= B^h_\alpha du^\alpha
\end{aligned}
$$

and for case 1)

$$Dl^h = B^h_\alpha \overline{D} l^\alpha$$

we get

$$R_j{}^i{}_{hk}\xi^j B^{h\,k}_{\beta\,\gamma} = {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\xi^\alpha B^i_\varepsilon + {}^1\overline{R}_\mu{}^\nu{}_{\beta\gamma}\xi^\mu \underset{\nu}{N}{}^i \tag{2.14}$$

The above formula is true for tensors P and S. Comparing the coefficients of ξ^α and ξ^μ we obtain

$$
\begin{aligned}
&\text{a)} && R_j{}^i{}_{hk}B^{j\,h\,k}_{\alpha\,\beta\,\gamma} = {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon\\
&\text{b)} && R_j{}^i{}_{hk}\underset{\mu}{N}{}^j B^{;h\,k}_{\beta\,\gamma} = {}^1\overline{R}_\mu{}^\nu{}_{\beta\gamma}\underset{\nu}{N}{}^i\\
&\text{c)} && P_j{}^i{}_{hk}B^{j\,h\,k}_{\alpha\,\beta\,\gamma} = {}^0\overline{P}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon\\
&\text{d)} && P_j{}^i{}_{hk}\underset{\mu}{N}{}^j B^{;h\,k}_{\beta\,\gamma} = {}^1\overline{P}_\mu{}^\nu{}_{\beta\gamma}\underset{\nu}{N}{}^i\\
&\text{e)} && S_j{}^i{}_{hk}B^{j\,h\,k}_{\alpha\,\beta\,\gamma} = {}^0\overline{S}_\alpha{}^\varepsilon{}_{\beta\gamma}B^i_\varepsilon\\
&\text{f)} && S_j{}^i{}_{hk}\underset{\mu}{N}{}^j B^{;h\,k}_{\beta\,\gamma} = {}^1\overline{S}_\mu{}^\nu{}_{\beta\gamma}\underset{\nu}{N}{}^i
\end{aligned}
\tag{2.15}
$$

If we define the induced covariant differentiations $\overset{1}{\scriptstyle\top}$ and $\overset{1}{\top}$ for some mixed tensor $T^{\beta\,\mu}_{\alpha\,\nu}$ in the form

$$
\begin{aligned}
T^{\beta\,\mu}_{\alpha\,\nu}\overset{1}{\scriptstyle\top}_\gamma = {}&\partial_\gamma T^{\beta\,\mu}_{\alpha\,\nu} - \dot\partial_\varkappa T^{\beta\,\mu}_{\alpha\,\nu}\overline{\Gamma}^{*\varkappa}{}_\gamma - T^{\beta\,\mu}_{\varkappa\,\nu}\overline{\Gamma}^{*\;\varkappa}_{\alpha\,\gamma} +\\
&T^{\beta\,\mu}_{\varkappa\,\nu}\overline{\Gamma}^{*\;\beta}_{\varkappa\,\gamma} - T^{\beta\,\mu}_{\alpha\,\zeta}\overline{\lambda}^{*\;\zeta}_{\nu\,\gamma} + T^{\beta\,\mu}_{\alpha\,\nu}\overline{\lambda}^{*\;\mu}_{\xi\,\gamma}\\
T^{\beta\,\mu}_{\alpha\,\nu}\overset{1}{\top}_\gamma = {}&L\dot\partial_\gamma T^{\beta\,\mu}_{\alpha\,\nu} - T^{\beta\,\mu}_{\varkappa\,\nu}\overline{A}^{\;\varkappa}_{\alpha\,\gamma} + T^{\beta\,\mu}_{\varkappa\,\nu}\overline{A}^{\;\beta}_{\varkappa\,\gamma} -\\
&T^{\beta\,\mu}_{\alpha\,\xi}\overline{A}^{\;\zeta}_{\nu\,\gamma} + T^{\beta\,\zeta}_{\alpha\,\nu}\overline{A}^{\;\mu}_{\zeta\,\gamma}.
\end{aligned}
$$

then the Bianchi identities ([**6**], (3.1)—(3.3)) for the case 1) reduce to

$$
\begin{aligned}
&\text{a)} && {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\overset{1}{\top}_\delta + {}^0\overline{P}_\alpha{}^\varepsilon{}_{[\gamma|\delta|}\overset{1}{\scriptstyle\top}_{\beta]} + {}^0\overline{\mathrm{R}}_\alpha{}^\varepsilon{}_{\varkappa[\gamma}\mathrm{A}^{\varkappa}_{\beta]}{}_\delta +\\
&&& {}^0\overline{S}_\alpha{}^\varepsilon{}_{\delta\varkappa}{}^0\overline{K}_o{}^\varkappa{}_{\beta\gamma} - {}^0\overline{P}_\alpha{}^\varepsilon{}_{[\gamma|\varkappa}\dot\partial_\delta\overline{\Gamma}_\iota{}^{*\varkappa}{}_{|\beta]}\dot u^\iota = -l_\delta\overline{A}^\varepsilon_{\alpha\varkappa}{}^0\overline{K}_o{}^\varkappa{}_{\beta\gamma},\\
&\text{b)} && {}^1\overline{R}_\mu{}^\nu{}_{\beta\gamma}\overset{1}{\top}_\delta + {}^1\overline{P}_\mu{}^\nu{}_{[\gamma|\delta|}\overset{1}{\scriptstyle\top}_{\beta]} + {}^1\overline{\mathrm{R}}_\mu{}^\nu{}_{\varkappa[\gamma}\mathrm{A}^{\varkappa}_{\beta]}{}_\beta +\\
&&& {}^1\overline{S}_\mu{}^\nu{}_{\delta\varkappa}{}^0\overline{K}_o{}^\varkappa{}_{\beta\gamma} - {}^1\overline{P}_\mu{}^\nu{}_{[\gamma|\varkappa}\dot\partial_\delta\overline{\Gamma}_\iota{}^{*\varkappa}{}_{|\beta]}\dot u^\iota = -l_\delta\overline{A}^\nu_{\mu\varkappa}{}^0\overline{K}_o{}^\varkappa{}_{\beta\gamma},\\
&\text{c)} && ({}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\overset{1}{\scriptstyle\top}_\delta + {}^0\overline{\mathrm{P}}_\alpha{}^\varepsilon{}_{\beta\varkappa}{}^0\overline{\mathrm{K}}_{\mathrm{o}}{}^\varkappa{}_{\gamma\delta}) + \mathrm{cycl}(\beta\gamma\delta) = 0,\\
&\text{d)} && ({}^1\overline{R}_\mu{}^\nu{}_{\beta\gamma}\overset{1}{\scriptstyle\top}_\delta + {}^1\overline{\mathrm{P}}_\mu{}^\nu{}_{\beta\varkappa}{}^0\overline{\mathrm{K}}_{\mathrm{o}}{}^\varkappa{}_{\gamma\delta}) + \mathrm{cycl}(\beta\gamma\delta) = 0,
\end{aligned}
\tag{2.16}
$$

e) $\quad {}^0\overline{P}_{\alpha}{}^{\varepsilon}{}_{\beta[\gamma|\delta|}\overset{1}{\top}_{\beta]} + A^{\varkappa}_{\alpha]}{}_{[\delta}{}^0\overline{P}_{|\alpha}{}^{\varepsilon}{}_{\varkappa|\gamma]} + {}^0\overline{S}_{\alpha}{}^{\varepsilon}{}_{\gamma\beta}\overset{1}{\top}_{\beta} + {}^0\overline{\mathrm{S}}_{|\alpha}{}^{\varepsilon}{}_{\iota[\gamma]}\dot{\partial}_{\delta]}\overline{\Gamma}^{*\iota}_{\varkappa\beta}\dot{\mathrm{u}}^{\varkappa} = Ll_{[\delta}\dot{\partial}_{\gamma]}\overline{\Gamma}^{*\varepsilon}_{\alpha\beta},$

f) $\quad {}^1\overline{P}_{\mu}{}^{\nu}{}_{\beta[\gamma|\delta|}\overset{1}{\top}_{\beta]} + A^{\varkappa}_{\beta]}{}_{[\delta}{}^1\overline{P}_{|\mu}{}^{\nu}{}_{\varkappa|\gamma]} + {}^1\overline{S}_{\mu}{}^{\nu}{}_{\gamma\beta}\overset{1}{\top}_{\beta} + {}^1\overline{\mathrm{S}}_{|\mu}{}^{\nu}{}_{\iota[\gamma]}\dot{\partial}_{\delta}\overline{\Gamma}^{*\iota}_{\varkappa\beta}\dot{\mathrm{u}}^{\varkappa} = Ll_{[\delta}\dot{\partial}_{\gamma]}\overline{\lambda}^{*\nu}_{\mu\beta}.$

If we denote by D_i the absolute differential in F_n which corresponds to the displacement $(d_i u^{\alpha}, d_i\dot{u}^{\alpha})$ $(i = 1, 2)$ in F_m then from (2.11), (2.12), (2.15a), (2.15b) we have

$$\begin{aligned}([D_2D_1]R_j{}^i{}_{hk})B^{j\,h\,k}_{\alpha\,\beta\,\gamma} = {}&{}^0\overline{R}_{\alpha}{}^{\varepsilon}{}_{\beta\gamma}\overline{\Omega}^{\delta}_{\varepsilon}(d_1,d_2)B^i_{\delta} - {}^0\overline{R}_{\varepsilon}{}^{\delta}{}_{\beta\gamma}\overline{\Omega}^{\varepsilon}_{\alpha}(d_1,d_2)B^i_{\delta} - {}^0\overline{R}_{\alpha}{}^{\delta}{}_{\varepsilon\gamma}\overline{\Omega}^{\varepsilon}_{\beta}(d_1,d_2)B^i_{\delta} -\\ &{}^0\overline{R}_{\alpha}{}^{\delta}{}_{\beta\varepsilon}\overline{\Omega}^{\varepsilon}_{\gamma}(d_1,d_2)B^i_{\delta},\end{aligned}\tag{2.17}$$

$$\begin{aligned}([D_2D_1]R_j{}^i{}_{hk})\underset{\mu}{N}^j B^{h\,k}_{\beta\,\gamma} = {}&{}^1\overline{R}_{\mu}{}^{\nu}{}_{\beta\gamma}\overline{\Omega}^{\psi}_{\nu}(d_1,d_2)\underset{\psi}{N}^i - {}^1\overline{R}_{\psi}{}^{\nu}{}_{\beta\gamma}\overline{\Omega}^{\psi}_{\mu}(d_1,d_2)\underset{\nu}{N}^j - {}^1\overline{R}_{\mu}{}^{\nu}{}_{\varepsilon\gamma}\overline{\Omega}^{\varepsilon}_{\beta}(d_1,d_2)\underset{\nu}{N}^j -\\ &{}^1\overline{R}_{\mu}{}^{\nu}{}_{\beta\varepsilon}\overline{\Omega}^{\varepsilon}_{\gamma}(d_1,d_2)\underset{\varepsilon}{N}^j.\end{aligned}\tag{2.18}$$

Formulae of type (2.17), (2.18) are satisfied for tensors P and S and we may get them substituting the letter R with P and S.

If the space F_n satisfies the relation

$$[D_2D_1]R_j{}^i{}_{hk} = 0\tag{2.19}$$

then from (2.17) and (2.18) we have

$$\begin{aligned}&{}^0\overline{R}_{\alpha}{}^{\varepsilon}{}_{\beta\gamma}\overline{\Omega}^{\delta}_{\varepsilon}(d_1,d_2) - {}^0\overline{R}_{\varepsilon}{}^{\delta}{}_{\beta\gamma}\overline{\Omega}^{\varepsilon}_{\alpha}(d_1,d_2) -\\ &{}^0\overline{R}_{\alpha}{}^{\delta}{}_{\varepsilon\gamma}\Omega^{\varepsilon}_{\beta}(d_1,d_2) - {}^0\overline{R}_{\alpha}{}^{\delta}{}_{\beta\varepsilon}\overline{\Omega}^{\varepsilon}_{\gamma}(d_1,d_2) = 0,\end{aligned}\tag{2.20}$$

$$\begin{aligned}&{}^1\overline{R}_{\mu}{}^{\psi}{}_{\beta\gamma}\overline{\Omega}^{\nu}_{\psi}(d_1,d_2) - {}^1\overline{R}_{\psi}{}^{\nu}{}_{\beta\gamma}\overline{\Omega}^{\psi}_{\mu}(d_1,d_2) -\\ &{}^1\overline{R}_{\mu}{}^{\nu}{}_{\varepsilon\gamma}\overline{\Omega}^{\varepsilon}_{\beta}(d_1,d_2) - {}^1\overline{R}_{\mu}{}^{\nu}{}_{\beta\varepsilon}\overline{\Omega}^{\varepsilon}_{\gamma}(d_1,d_2) = 0.\end{aligned}\tag{2.21}$$

If the space F_n satisfies

$$\text{a)}\quad [D_2D_1]P_j{}^i{}_{hk} = 0 \qquad \text{or} \qquad \text{b)}\quad [D_2D_1]S_j{}^i{}_{hk} = 0\tag{2.19}$$

then the induced curvature tensors of the subspace ${}^0\overline{P}_{\alpha}{}^{\delta}{}_{\beta\gamma}$, ${}^1\overline{P}_{\mu}{}^{\nu}{}_{\beta\gamma}$, ${}^0\overline{S}_{\alpha}{}^{\delta}{}_{\beta\gamma}$, ${}^1\overline{S}_{\mu}{}^{\nu}{}_{\beta\gamma}$ satisfy the equations of type (2.20) and (2.21) and we get these equations when the letter R is substituted by P or S.

If (2.19) is true for every D_1, D_2, i.e., the tensor R is parallel on the subspace F_m, then from (2.20) and (2.21) we obtain

$$(2.23)\quad \begin{aligned}
&\text{a)}\quad {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\,{}^0\overline{R}_\varepsilon{}^\delta{}_{\iota\varkappa} - {}^0\overline{R}_\varepsilon{}^\delta{}_{\beta\gamma}\,{}^0\overline{R}_\alpha{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\varepsilon\gamma}\,{}^0\overline{R}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\beta\varepsilon}\,{}^0\overline{R}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0\\
&\text{b)}\quad {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\,{}^0\overline{P}_\varepsilon{}^\delta{}_{\iota\varkappa} - {}^0\overline{R}_\varepsilon{}^\delta{}_{\beta\gamma}\,{}^0\overline{P}_\alpha{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\varepsilon\gamma}\,{}^0\overline{P}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\beta\varepsilon}\,{}^0\overline{P}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0\\
&\text{c)}\quad {}^0\overline{R}_\alpha{}^\varepsilon{}_{\beta\gamma}\,{}^0\overline{S}_\varepsilon{}^\delta{}_{\iota\varkappa} - {}^0\overline{R}_\varepsilon{}^\delta{}_{\beta\gamma}\,{}^0\overline{S}_\alpha{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\varepsilon\gamma}\,{}^0\overline{S}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^0\overline{R}_\alpha{}^\delta{}_{\beta\varepsilon}\,{}^0\overline{S}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0\\
&\text{d)}\quad {}^1\overline{R}_\mu{}^\psi{}_{\beta\gamma}\,{}^1\overline{R}_\psi{}^\nu{}_{\iota\varkappa} - {}^1\overline{R}_\psi{}^\nu{}_{\beta\gamma}\,{}^1\overline{R}_\mu{}^\psi{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\varepsilon\gamma}\,{}^1\overline{R}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\beta\varepsilon}\,{}^0\overline{R}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0\\
&\text{e)}\quad {}^1\overline{R}_\mu{}^\psi{}_{\beta\gamma}\,{}^1\overline{P}_\psi{}^\nu{}_{\iota\varkappa} - {}^1\overline{R}_\psi{}^\nu{}_{\beta\gamma}\,{}^1\overline{P}_\mu{}^\psi{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\varepsilon\gamma}\,{}^1\overline{P}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\beta\varepsilon}\,{}^0\overline{P}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0\\
&\text{f)}\quad {}^1\overline{R}_\mu{}^\psi{}_{\beta\gamma}\,{}^1\overline{S}_\psi{}^\nu{}_{\iota\varkappa} - {}^1\overline{R}_\psi{}^\nu{}_{\beta\gamma}\,{}^1\overline{S}_\mu{}^\psi{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\varepsilon\gamma}\,{}^1\overline{S}_\beta{}^\varepsilon{}_{\iota\varkappa} - {}^1\overline{R}_\mu{}^\nu{}_{\beta\varepsilon}\,{}^0\overline{S}_\gamma{}^\varepsilon{}_{\iota\varkappa} = 0
\end{aligned}$$

If (2.23) is true for every D_1, D_2, then we easily obtain equations similar to (2.22) for the tensors P and S.

We shall examine what form the intrinsic connection coefficients take for case 1. In the subspace F_m with respect to the intrinsic connection coefficients DB^i_α and $D\underset{\mu}{N}^i$ take the form

$$\begin{aligned}
DB^i_\alpha &= [(\Gamma^{*}{}_{\alpha}{}^{\delta}{}_{\beta} + \Lambda_\alpha{}^\delta{}_\beta)du^\beta + A_\alpha{}^\delta{}_\beta)du^\beta Dl^\beta]B^i_\delta + (\theta^{*}{}_\alpha{}^\mu{}_\beta du^\beta + A_\alpha{}^\mu{}_\beta Dl^\beta)\underset{\mu}{N}^i\\
D\underset{\mu}{N}^i &= -(\theta^{*\delta}{}_{\mu\beta}du^\beta + A^\delta{}_{\alpha\beta}Dl^\beta)D^i_\delta + (\lambda^{*}{}_\mu{}^\nu{}_\beta du^\beta + A_\mu{}^\nu{}_\beta Dl^\beta)\underset{\nu}{N}^i.
\end{aligned}$$

As

$$\begin{aligned}
\theta_\alpha{}^\mu{}_\beta &= \overline{\theta}^{*}{}_\alpha{}^\mu{}_\beta - A_\alpha{}^\mu{}_\varkappa A^\varkappa{}_{\nu\beta}\overset{\nu}{N},\\
A_{\alpha\beta\gamma} &= \overline{A}_{\alpha\beta\gamma}, \quad A_{\alpha\mu\beta} = \overline{A}_{\alpha\mu\beta},\\
\overline{D}l^\beta &= Dl^\beta = -A^\beta_{\nu\gamma}\overset{\nu}{N}du^\gamma
\end{aligned}$$

we have in case 1)

$$\theta^{*}{}_\alpha{}^\mu{}_\beta = \overline{\theta}^{*}{}_\alpha{}^\mu{}_\beta = 0 \tag{2.24}$$

$$\begin{aligned}
A_{\alpha\mu\beta} &= \overline{A}_{\alpha\mu\beta} = 0\\
\overline{D}l^\beta &= Dl^\beta
\end{aligned} \tag{2.25}$$

From the last equation and

$$Dl^k = B^k_\alpha Dl^\alpha = H^k_\beta du^\beta$$

it follows that

$$H^k_\beta = 0$$

From

$$\Lambda_\alpha{}^\delta{}_\beta = -A_{hkj}B^j_\beta g^{\rho\delta}(H^h_\alpha B^k_\rho - B^h_\alpha H^k_\rho)$$

and $H^k_\beta = 0$ we get immediately

$$\Lambda_{\alpha\ \beta}^{\ \delta} = 0 \tag{2.26}$$

As $\overline{\Gamma}^*_{\alpha\rho\beta}$ and $\Gamma^*_{\alpha\rho\beta}$ are connected by

$$\Gamma^*_{\alpha\rho\beta} = \overline{\Gamma}^*_{\alpha\rho\beta} + A_{ikj}B^j_\beta(H^i_\alpha B^k_\rho - B^i_\alpha H^k_\rho) - A_{\alpha\rho\delta}A^\delta_{\ \nu\beta}\overset{\nu}{N}$$

using $H^k_\beta = 0$, $\overset{\nu}{N} = 0$ we have

$$\Gamma_{\alpha\rho\beta} = \overline{\Gamma}^*_{\alpha\rho\beta}. \tag{2.27}$$

As

$$A_{\mu\ \beta}^{\ \nu} = \overline{A}_{\mu\ \beta}^{\ \nu} \tag{2.28}$$

for any subspace from (2.24) — (2.28) we have:

THEOREM 2.1. *If the subspace F_m of the Finsler space F_n has the property $DB^i_\alpha \in T_H$ for the mixed lineelement $P(u, \dot{u})$ and every $(du^\alpha, d\dot{u}^\alpha)$, then the induced and intrinsic conneetion coefficients are the same, from which it follows that the induced and intrinsic curvature tensors are the same, and satisfy the same equations at P.*

In all previous equations every quantity and tensor was considered at the fixed lineelement $P(u, \dot{u})$. Let us denote by HF_m the subspace of case 1) for all lineelements $(u, \dot{u})$ where u is a fixed point and $\dot{u}$ is any direction in the subspace. Then we have the following:

THEOREM 2.2. *The subspace F_m of the Finsler space F_n is HF_m iff one of the following equivalent equations* (2.1) (2.5) *or* (2.10) *is satisfied for all directions $\dot{u}$ at fixed point $\dot{u}$.*

Proof. From the definition it is obvious that the subspace F_m is HF_m iff (2.1) for fixed u and $\dot{u}$. Furthermore

$$(2.1) \Rightarrow (2.4) \Rightarrow (2.5)$$

To prove (2.5) $\Rightarrow$ (2.1) from $l^k = B^k_\alpha l^\alpha$, $g_{ij}(x, \dot{x})\underset{\mu}{N}^i l^j = 0$ we have

$$g_{ij}D\underset{\mu}{N}^i l^j + g_{ij}\underset{\mu}{N}^i Dl^j = 0$$

From (2.5) and the equation above we obtain $g_{ij}DN^iB^j_\alpha l^\alpha = 0$ for all l^α; so

$$D\underset{\mu}{N}^i Dl^j = (\lambda^{*\ \nu}_{\mu\ \beta} du^\beta + \overline{A}_{\mu\ \beta}^{\ \nu}\overline{D}l^\beta)\underset{\nu}{N}^i$$

from which (2.1) follows.

To prove (2.5) $\Leftrightarrow$ (2.10) i. e., $\overline{H}^k_\alpha = 0 \Leftrightarrow \overset{\mu}{N} = 0$ for all $\dot{u}$ and $\mu = m+1, \dots, n$ we have the relation

$$\overline{H}^i_\beta l^\beta = \overline{\theta}^{*\ \mu}_{\alpha\ \beta} l^\alpha l^\beta \underset{\mu}{N} = \overset{\mu}{N}(u, \dot{u})\underset{\mu}{N}^i.$$

3. Case 2). $DB^i_\alpha \in T_V$.

In this case the absolute differentials of tangent and normal vectorc take the form:

$$DB^i_\alpha = (\overline{\theta}^{*\,\mu}_{\alpha\,\beta} du^\beta + \overline{A}_{\alpha\,\beta}^{\ \mu} \overline{D} l^\beta) \underset{\mu}{N}^i \tag{3.1}$$

$$D\underset{\mu}{N}^i = (\overline{\theta}^{*\,\delta}_{\mu\,\beta} du^\beta + A_{\alpha\,\beta}^{\ \mu} \overline{D} l^\beta) B^i_\delta + (\overline{\lambda}^{*\,\nu}_{\mu\,\beta} du^\beta + \overline{A}_{\mu\,\beta}^{\ \nu} \overline{D} l^\beta) \underset{\nu}{N}^i. \tag{3.2}$$

As in this case $A_{\alpha\,\beta}^{\ \mu} = 0$, we have:

$$\overline{A}_{\alpha\beta\gamma} = 2^{-1} L(u,\dot{u}) \dot{\partial}_\gamma g_{\alpha\beta}(u,\dot{u}) = 0, \tag{3.3}$$

from which we conclude that the metric tensor of the subspace is not a function of the direction $\dot{u}$, i. e.,

$$g_{\alpha\beta} = g_{\alpha\beta}(u)$$

and the subspace F_m of the Finsler space F_n is Riemannian. From the equations

$$\begin{gathered} \Gamma_{\alpha\beta\gamma} + \Lambda_{\alpha\beta\gamma} = \overline{\Gamma}^*_{\alpha\beta\gamma} - \overline{A}_{\alpha\beta\delta} \overline{A}^{\delta}_{\ \nu\gamma} \overset{\nu}{N} \\ \overline{A}_{\alpha\beta\gamma} = 0, \quad \overline{\Gamma}^*_{\alpha\beta\gamma} = 0, \end{gathered} \tag{3.4}$$

we obtain that in case 2) the intrinsic connection coefficient is the tensor $-\Lambda_{\alpha\beta\gamma}$, i.e. $\Gamma_{\alpha\beta\gamma} = -\Lambda_{\alpha\beta\gamma}$.

The other connection coefficients are obtained from the same formulae as in any other subspace.

Using the equations $\overline{A}_{\alpha\,\beta}^{\ \delta} = 0$, $\overline{\Gamma}^{*\,\delta}_{\alpha\,\beta} = 0$ for case 2) we get

$$\begin{gathered}
[\Delta D] B^i_\alpha = \{2^{-1} \overline{\theta}^{*\,\delta}_{\alpha\,[\beta} \overline{\theta}^{*\,\delta}_{|\mu|\,\gamma]} [du^\beta \delta u^\gamma] + (\overline{\theta}^{*\,\mu}_{\alpha\,\beta} \overline{A}_{\mu\,\gamma}^{\ \delta} - \overline{\theta}^{*\,\delta}_{\mu\,\beta} \overline{A}_{\alpha\,\gamma}^{\ \mu}) [du^\beta \overline{\Delta} l^\gamma] + \\
2^{-1} \overline{A}_{\alpha\,[\beta}^{\ \mu} \overline{A}_{|\mu|\,\gamma]}^{\ \delta} [\overline{D} l^\beta \overline{\Delta} l^\gamma]\} B^i_\delta + \{2^{-1} (\partial_{[\gamma} \overline{\theta}^{*\,\mu}_{|\alpha|\,\beta]} + \overline{\theta}^{*\,\nu}_{\alpha\,[\beta} \overline{\lambda}^{*\,\mu}_{|\nu|\,\gamma]}) [du^\beta \delta u^\gamma] + \\
(L \dot{\partial}_\gamma \overline{\theta}^{*\,\mu}_{\alpha\,\beta} - \partial_\beta A_{\alpha\,\gamma}^{\ \mu} - \overline{A}_{\alpha\,\beta}^{\ \nu} \overline{\lambda}^{*\,\mu}_{\nu\,\gamma} + \overline{\theta}^{*\,\mu}_{\alpha\,\beta} \overline{A}_{\mu\,\gamma}^{\ \nu}) [du^\beta \overline{\Delta} l^\gamma] + \\
2^{-1} (L \dot{\partial}_{[\gamma} \overline{A}_{|\alpha|\,\beta]}^{\ \mu} + \overline{A}_{\alpha\,[\beta}^{\ \nu} \overline{A}_{|\nu|\,\gamma]}^{\ \mu}) [\overline{D} l^\beta \overline{\Delta} l^\gamma]\} \underset{\mu}{N}^i. \\
[\Delta D] \underset{\mu}{N}^i = \{2^{-1} (\partial_{[\gamma} \overline{\theta}^{*\,\delta}_{|\mu|\,\beta]} + \overline{\theta}^{*\,\delta}_{|\nu|\,[\gamma]} \overline{\lambda}^{*\,\nu}_{|\mu|\,\beta]}) [du^\beta \delta u^\gamma] + \\
2^{-1} (L \dot{\partial}_\gamma \overline{\theta}^{*\,\delta}_{\mu\,\beta} - \partial_\beta \overline{A}_{\mu\,\gamma}^{\ \delta} + \overline{A}_{\nu\,\gamma}^{\ \delta} \overline{\lambda}^{*\,\nu}_{\mu\,\beta} - \overline{A}_{\mu\,\gamma}^{\ \nu} \overline{\theta}^{*\,\delta}_{\nu\,\beta}) [du^\beta \overline{\Delta} l^\gamma] + \\
2^{-1} (L \dot{\partial}_{[\gamma} \overline{A}_{|\mu|\,\beta]}^{\ \delta} + \overline{A}_{\mu\,[\beta}^{\ \nu} \overline{A}_{|\nu|\,\gamma]}^{\ \delta}) [\overline{D} l^\beta \overline{\Delta} l^\gamma]\} B^i_\beta \\
\{2^{-1} (\partial_{[\gamma} \overline{\lambda}^{*\,\nu}_{|\mu|\,\beta]} + \overline{\theta}^{*\,\delta}_{\mu\,[\beta} \overline{\theta}^{*\,\nu}_{|\delta|\,\gamma]} + \overline{\lambda}^{*\,\psi}_{\mu\,[\beta} \overline{\lambda}^{*\,\nu}_{|\psi|\,\gamma]}) [du^\beta \delta u^\gamma] + \\
(L \dot{\partial}_\gamma \overline{\lambda}^{*\,\nu}_{\mu\,\beta} - \partial_\beta A_{\mu\,\gamma}^{\ \nu} + \overline{\lambda}^{*\,\psi}_{\mu\,\beta} A_{\psi\,\gamma}^{\ \nu} - \overline{A}_{\mu\,\gamma}^{\ \psi} \overline{\lambda}^{*\,\nu}_{\psi\,\beta} - \overline{A}_{\mu\,\gamma}^{\ \delta} \overline{\theta}^{*\,\nu}_{\delta\,\beta} + \overline{\theta}^{*\,\delta}_{\mu\,\beta} \overline{A}_{\delta\,\gamma}^{\ \nu}) [du^\beta \overline{\Delta} l^\gamma] + \\
2^{-1} (L \dot{\partial}_{[\gamma} \overline{A}^{*\,\nu}_{|\mu|\,\beta]} - A_{\mu\,[\beta}^{\ \psi} \overline{A}_{|\psi|\,\gamma]}^{\ \nu} + \overline{A}_{\mu\,[\beta}^{\ \delta} \overline{A}_{|\delta|\,\gamma}^{\ \nu}) [du^\beta \overline{\Delta} l^\gamma].
\end{gathered} \tag{3.5}$$

Comparing the above formulae with those in [**6**] we obtain that in case 2) the curvature tensors

$${}^0\overline{R}_{\alpha\ \beta\gamma}^{\ \delta}, \quad {}^0\overline{P}_{\alpha\ \beta\gamma}^{\ \delta}, \quad {}^0\overline{S}_{\alpha\ \beta\gamma}^{\ \delta}.$$

and some others are reduced, because of $\overline{\Gamma}^{*\ \delta}_{\ \alpha\beta}=0$, $\overline{A}_{\alpha\beta}^{\ \ \delta}=0$.

4. Case 3). $D\underset{\mu}{N}^i \in T_H$.

In this case the absolute differentials of tangent and normal vectors take the form:

$$DB^i_\alpha = (\overline{\Gamma}^{*\ \delta}_{\alpha\beta}du^\beta + \overline{A}_{\alpha\beta}^{\ \ \delta}Dl^\beta)B^i_\delta + (\overline{\theta}^{*\ \mu}_{\alpha\beta}du^\beta + \overline{A}_{\alpha\beta}^{\ \ \mu}Dl^\beta)\underset{\mu}{N}^i, \tag{4.1}$$

$$DN^i = (\overline{\theta}^{*\ \delta}_{\mu\beta}du^\beta + \overline{A}_{\mu\beta}^{\ \ \delta}Dl^\beta)B^i_\delta \tag{4.2}$$

Also

$$\overline{\lambda}^{*\ \nu}_{\mu\gamma}=0, \quad \overline{A}_{\mu\gamma}^{\ \ \nu}=0,$$

hence

$$\overline{\lambda}^{*\ \nu}_{\mu\gamma} = \overset{\nu}{N}_i(\partial_\gamma \underset{\mu}{N}^i - \partial_\delta \underset{\mu}{N}^i \overline{\Gamma}^{*\delta}_{\ \ \gamma} + \overline{\Gamma}^{*i}_{\ jk}\underset{\mu}{N}^j B^k_\gamma + A^i_{jk}\underset{\mu}{N}^j \overline{H}^k_\gamma) = 0,$$

$$\overline{A}_{\mu\gamma}^{\ \ \nu} = \overset{\nu}{N}_i(L\dot{\partial}_\gamma \underset{\mu}{N}^i + A^i_{jk}\underset{\mu}{N}^j B^k_\gamma) = 0.$$

The other connection coefficients we get from the same formulae as in any other subspace.

We also have that the absolute differentials of tangent and normal vectors take the form:

$$\begin{gathered}
[\Delta D]B^i_\alpha = \{2^{-1}({}^0\overline{R}_{\alpha\ \beta\gamma}^{\ \varepsilon} + \overline{\theta}^{*\ \mu}_{\alpha[\beta}\overline{\theta}^{*\ \varepsilon}_{|\mu|\gamma]})[du^\beta\delta u^\gamma] + \\
({}^0\overline{P}_{\alpha\ \beta\gamma}^{\ \varepsilon}\overline{\theta}^{*\ \mu}_{\alpha\beta}\overline{A}_{\mu\gamma}^{\ \ \varepsilon} - \overline{A}_{\alpha\gamma}^{\ \ \mu}\overline{\theta}^{*\ \varepsilon}_{\mu\beta})[du^\beta\overline{\Delta}l^\gamma] + 2^{-1}{}^0\overline{S}_{\alpha\ \beta\gamma}^{\ \varepsilon}\overline{A}_{\alpha[\beta}^{\ \ \nu}\overline{A}_{|\nu|\gamma]}^{\ \ \varepsilon})[\overline{D}l^\beta\overline{\Delta}l^\gamma]\}B^i_\varepsilon + \\
2^{-1}{}^0\overline{R}_{\alpha\ \beta\gamma}^{\ \mu}[du^\beta\delta u^\gamma] + {}^0\overline{P}_{\alpha\ \beta\gamma}^{\ \mu}[du^\beta\Delta u^\gamma] + 2^{-1}{}^0\overline{S}_{\alpha\ \beta\gamma}^{\ \mu}[\overline{D}l^\beta\overline{\Delta}l^\gamma]\underset{\mu}{N}^i, \\
[\Delta D]\underset{\mu}{N}^i = \{2^{-1}({}^0\overline{R}_{\mu\ \beta\gamma}^{\ \varepsilon}[du^\beta\delta u^\gamma] + {}^0\overline{P}_{\mu\ \beta\gamma}^{\ \varepsilon}[du^\beta\Delta u^\gamma] + {}^0\overline{S}_{\mu\ \beta\gamma}^{\ \varepsilon}[du^\beta\Delta u^\gamma]\}B^i_\varepsilon + \\
2^{-1}(\overline{\theta}^{*\ \varkappa}_{\mu[\beta}(\overline{\theta}^{*\ \nu}_{|\varkappa|\gamma]}[du^\beta\Delta u^\gamma] + (\overline{A}_{\varkappa\gamma}^{\ \ \nu}\overline{\theta}^{*\ \varkappa}_{\mu\beta} - \overline{A}_{\mu\gamma}^{\ \ \varkappa}\overline{\theta}^{*\ \nu}_{\varkappa\beta})[du^\beta\overline{\Delta}l^\gamma] + \\
2^{-1}\overline{A}_{\mu[\beta}^{\ \ \varkappa}\overline{A}_{|\varkappa|\gamma]}^{\ \ \nu} + [\overline{D}l^\beta\overline{\Delta}l^\gamma]\}\underset{\nu}{N}^i.
\end{gathered}$$

Finally we have

$${}^1\overline{R}_{\mu\ \beta\gamma}^{\ \nu}=0, \quad {}^1\overline{P}_{\mu\ \beta\gamma}^{\ \nu}=0, \quad {}^1\overline{S}_{\mu\ \beta\gamma}^{\ \nu}=0.$$

5. Case 2a) or 3a) $(DB^i_\alpha \in T_V)\wedge(DN^i \in T_H)$.

In this case we have

$$\overline{\Gamma}^{*\ \beta}_{\alpha\gamma}=0, \quad \overline{A}_{\alpha\gamma}^{\ \ \beta}=0, \quad \overline{A}_{\mu\beta}^{\ \ \nu}=0, \quad \overline{\lambda}^{*\ \nu}_{\mu\beta}=0 \tag{5.1}$$

and

$$DB^i_\alpha = (\overline{\theta}^{*\ \mu}_{\alpha\beta}du^\beta + \overline{A}_{\alpha\beta}^{\ \ \mu}\overline{D}l^\beta)\underset{\mu}{N}^i \tag{5.2}$$

$$D\underset{\mu}{N}_i = (\overline{\theta}^{*\ \alpha}_{\mu\beta}du^\beta + \overline{A}_{\alpha\beta}^{\ \ \mu}\overline{D}l^\beta)B^i_\alpha \tag{5.2}$$

For the absolute differential of tangent and normal vectors we obtain:

$$
\begin{aligned}
(5.4)\qquad [\Delta D]B^i_\alpha = \{&2^{-1}\overline{\theta}^{*\ \delta}_{\alpha\ [\beta}\overline{\theta}^{*\ \delta}_{|\mu|\ \gamma]}[du^\beta\delta u^\gamma] + (\overline{\theta}^{*\ \mu}_{\alpha\ \beta}\overline{A}_{\mu\ \gamma}^{\ \delta} - \overline{\theta}^{*\ \delta}_{\mu\ \beta}\overline{A}_{\alpha\ \gamma}^{\ \mu})[du^\beta\overline{\Delta}l^\gamma]+\\
&2^{-1}\overline{A}_{\alpha\ [\beta}^{\ \mu}\overline{A}_{|\mu|\ \gamma]}^{\ \delta}[\overline{D}l^\beta\overline{\Delta}l^\gamma]\}B^i_\delta+\\
\{&2^{-1}(\partial_{[\gamma}\overline{\theta}^{*\ \mu}_{|\alpha|\ \beta]}[du^\beta\overline{\Delta}l^\gamma] + (L\dot{\partial}_\gamma\overline{\theta}^{*\ \mu}_{\alpha\ \beta} - \partial_\beta A_{\alpha\ \beta}^{\ \mu})[du^\beta\overline{\Delta}l^\gamma]+\\
&2^{-1}(L\dot{\partial}_{[\gamma}\overline{A}_{|\alpha|\ \beta]}^{\ \mu})[\overline{D}l^\beta\overline{\Delta}l^\gamma]\}\underset{\mu}{N}^i,
\end{aligned}
$$

$$
\begin{aligned}
(5.5)\qquad [\Delta D]\underset{\mu}{N}^i = \{&2^{-1}(\partial_{[\gamma}\overline{\theta}^{*\ \alpha}_{|\mu|\ \beta]}[du^\beta\delta u^\gamma] + (L\dot{\partial}_\gamma\overline{\theta}^{*\ \alpha}_{\mu\ \beta} - \partial_\beta\overline{A}_{\mu\ \delta}^{\ \alpha})[du^\beta\overline{\Delta}l^\gamma]+\\
&2^{-1}L\dot{\partial}_{[\gamma}\overline{A}_{|\mu|\ \beta]}^{\ \alpha}[\overline{D}l^\beta\overline{\Delta}l^\gamma]\}B^i_\alpha+\\
\{&2^{-1}\overline{\theta}^{*\ \delta}_{\mu\ [\beta}\overline{\theta}^{*\ \nu}_{|\delta|\ \gamma]}[du^\beta\delta u^\gamma] + (\overline{\theta}^{*\ \delta}_{\mu\ \beta}\overline{A}_{\delta\ \gamma}^{\ \nu} - \overline{\theta}^{*\ \nu}_{\delta\ \beta}\overline{A}_{\mu\ \gamma}^{\ \delta})[du^\beta\overline{\Delta}l^\gamma]+\\
&2^{-1}A_{\mu\ [\beta}^{\ \delta}\overline{A}_{|\delta|\ \gamma]}^{\ \nu}[\overline{D}l^\beta\overline{\Delta}l^\gamma]\}\underset{\nu}{N}^i.
\end{aligned}
$$

We also have:

$$
\begin{aligned}
{}^0\overline{R}_{\mu\ \beta\gamma}^{\ \delta} = 0,\quad {}^0\overline{P}_{\mu\ \beta\gamma}^{\ \delta} = 0,\quad {}^0\overline{S}_{\mu\ \beta\gamma}^{\ \delta} = 0\\
{}^1\overline{R}_{\mu\ \beta\gamma}^{\ \nu} = 0,\quad {}^1\overline{P}_{\mu\ \beta\gamma}^{\ \nu} = 0,\quad {}^1\overline{S}_{\mu\ \beta\gamma}^{\ \nu} = 0.
\end{aligned}
$$

The intrinsic connection coefficients are:

$$\Gamma^*_{\alpha\beta\gamma} = -\Lambda_{\alpha\beta\gamma},\quad \lambda_{\mu\nu\beta} = 0,\quad A_{\mu\nu\beta} = 0,\quad A_{\alpha\beta\gamma} = 0$$

and the corresponding equations for the intrinsic curvature tensors are the same as (4.6), (4.7) except for

$${}^0\overline{R}_{\mu\ \beta\gamma}^{\ \delta} = -\partial_{[\gamma}\Lambda_{|\alpha|\ \beta]}^{\ \delta} + \Lambda_{\alpha\ [\beta}^{\ \varkappa} + \Lambda_{|\varkappa|\ \gamma]}^{\ \delta}.$$

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