HYPQ

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This is a MATHEMATICA package for handling basic hypergeometric series. It provides quite a few tools for

(A) manipulating q-factorial expressions
(B) transforming q-binomial sums into basic hypergeometric notation
(C) summing basic hypergeometric series
(D) transforming basic hypergeometric series
(E) applying contiguous relations
(F) doing formal limits of basic hypergeometric expressions
(G) transforming basic hypergeometric MATHEMATICA expressions into TEX-code.

The tools for items (A), (B), (F), (G) are contained in the file `hyp.q`, the basic package. This file must be loaded at the very beginning of your MATHEMATICA session. (Ignore error messages occuring when loading `hyp.q`.) The file `hyp.q` defines the basic objects, the rules and functions for items (A), (B), (F), and (G), and predefines all the remaining ones. The tools for (C) are the contents of the file `summatio.q`, those for (D) are the contents of the files `transfor.q` and `transfor.qli`, and those for (E) are the contents of the file `contig.q`. You also have access to summation and transformation formulas in form of equations. This is the contents of the files `summatio.qgl` and `transfor.qgl`, respectively. The file `output` defines some nice MATHEMATICA output features for `Sum`, `Product`, `Integrate`, `Abs`, `Floor`, `Ceiling`, `Pi`, and `Infinity`.

However, the philosophy of this package is:

Do it by yourself!

The idea is that you should be able to control each step in a series of manipulations by yourself. So, for instance, this package does not make any attempt to sum or transform a series automatically. So, it is you who has to tell the package which command, summation, or transformation has to be applied next. Therefore a basic knowledge of basic hypergeometric series is required (cf. [1, pp. 1–6]). This handbook provides you with a list of the rules, functions, summations, transformations that are available. The main source for identities that are included in this package has been the book [1], which contains a fairly comprehensive collection of known summation and transformation formulas for basic hypergeometric series. In particular, the (almost) complete Appendix of [1] is included in this package.

Finally you should be warned that there is no guarantee that a formula that has been obtained using this package is actually valid. Many formulas or operations are only valid under certain restrictions for the parameters. This package only helps you to do calculations fast. It is up to you to check that the manipulations you are doing are actually being allowed.

For a brief summary of the main features of this package the user is referred to [2] which is the contents of the $\mathcal{AMS}$-TEX file `hyp_hypq.tex`.

Basic hypergeometric notation

All the notation and terminology is adopted from [1, pp. 1–6]. Given a (fixed) complex number q with $|q|<1$, the *basic hypergeometric series* is defined by

$$ {}_r\phi_s\left[\begin{matrix}a_1,\dots,a_r\\ b_1,\dots,b_s\end{matrix};q,z\right]=\sum_{n=0}^{\infty}\frac{(a_1;q)_n\cdots(a_r;q)_n}{(q;q)_n(b_1;q)_n\cdots(b_s;q)_n}\left((-1)^n q^{\binom{n}{2}}\right)^{s-r+1}z^n\ ,$$

where the rising q-factorial $(a;q)_n$ is given by $(a;q)_n := (1-a)(1-aq)\cdots(1-aq^{n-1})$, $n \geq 1$, $(a)_0 := 1$. Also the infinite q-factorial $(a;q)_\infty := \prod_{i=0}^{\infty}(1-aq^i)$ is used. A basic hypergeometric series ${}_{r+1}\phi_r$ is called *very well-poised* if $a_i b_i = qa_0$ for $i = 1,2,\ldots,r$, and among the parameters a_i occur both $q\sqrt{a_0}$ and $-q\sqrt{a_0}$. We use the standard abbreviation for very well-poised basic hypergeometric series,

$$ {}_{r+1}W_r(a_0;a_3,a_4,\ldots,a_r;z) := {}_{r+1}\phi_r\left[\begin{matrix} a_0, q\sqrt{a_0}, -q\sqrt{a_0}, a_3, a_4, \ldots, a_r \\ \sqrt{a_0}, -\sqrt{a_0}, qa_0/a_2, qa_0/a_3, \ldots, qa_0/a_r \end{matrix}; z\right]. $$

We also use the compact Gasper-Rahman notation

$$ (a_1,a_2,\ldots,a_r;q)_n := (a_1;q)_n\,(a_2;q)_n\,\cdots\,(a_r;q)_n. $$

The file `hyp.q`

The objects which are defined in the file hyp.q are

`AbsGreater`, `AbsSmaller`, `AbsUndetermined`, `Add`, `AmSLaTeX`, `AmSTeX`, `baszerl1`, `baszerl2`, `baszus1`, `baszus2`, `Binomialpq`, `Binomialq`, `Div`, `Drucke`, `Ers`, `erw1`, `erw2`, `Expandq`, `Factorialpq`, `Factorialq`, `Gleichung`, `GlTausche`, `Hoch`, `hypqAttributes`, `inv1`, `inv2`, `LaTeX`, `Limes`, `lina1`, `lina2`, `linz`, `LS`, `Mal`, `ManipulationsListe`, `MinusOne`, `neg1`, `neg2`, `ph`, `phCancel`, `phEinf`, `phFormat`, `phinv`, `phOrdne`, `phPerm`, `phSum`, `phTausche`, `PosListe`, `pq`, `PQ`, `pqaufl`, `pqinf`, `pqinfzerl`, `pqinfzus`, `PQSort`, `pqzerl`, `pqzus`, `RS`, `SchreibeZahl`, `SimplifyPQ`, `Sub`, `Subst`, `SumErw1`, `SumErw2`, `SumExpand`, `SumInfinity`, `Sumph`, `SumRegeln`, `SumSammle`, `SumShift`, `SumTausche`, `SumUmkehr`, `SumZerl`, `TeX`, `TeXMat`, `TeXphW`, `trans`, `W`, `zerl`, `zus1`, `zus2`, `zus3`.

These objects can be divided into 9 groups: There are the basic objects,

`Binomialpq`, `Binomialq`, `Factorialpq`, `Factorialq`, `ph`, `pq`, `pqinf`, `W`,

the rules for manipulating q-factorial expressions

`baszerl1`, `baszerl2`, `baszus1`, `baszus2`, `erw1`, `erw2`, `Expandq`, `inv1`, `inv2`, `lina1`, `lina2`, `linz`, `MinusOne`, `neg1`, `neg2`, `pqaufl`, `pqinfzerl`, `pqinfzus`, `pqzerl`, `pqzus`, `trans`, `zerl`, `zus1`, `zus2`, `zus3`,

the rules for manipulating sums and basic hypergeometric series,

`phCancel`, `phEinf`, `phFormat`, `phinv`, `phOrdne`, `phPerm`, `phSum`, `phTausche`, `SumErw1`, `SumErw2`, `SumExpand`, `SumInfinity`, `Sumph`, `SumRegeln`, `SumSammle`, `SumShift`, `SumTausche`, `SumUmkehr`, `SumZerl`,

two functions for controlled use of rules,

`Ers`, `Posliste`,

one function for substitution of an expression instead of another expression,

`Subst`,

some objects for doing limits of basic hypergeometric expressions,

`AbsGreater`, `AbsSmaller`, `AbsUndetermined`, `Limes`,

one object for simplifying arguments in basic hypergeometric expressions,

`SimplifyPQ`,

some objects for converting expressions into TEX-code,

`AmSLaTeX`, `AmSTeX`, `LaTeX`, `TeX`, `TeXMat`, `TeXphW`,

two objects for on-line help,

`hypqAttributes`, `ManipulationsListe`,

and the function

`Drucke`

which enables you to directly send an expression to the printer. Finally there are

`Add`, `Div`, `Gleichung`, `GlTausche`, `Hoch`, `LS`, `Mal`, `PQSort`, `RS`, `Sub`

for manipulating equations and writing expressions in a "normalized" form (`PQSort`) in order to be able to quickly check if two expressions agree. These objects are particularly important when using objects from `summatio.qgl` and `transfor.qgl`.

Most of the tools for manipulating expressions that are provided by this package are rules. This has the advantage that very often you do not have to specify to which part of an expression you want to apply a rule, since there is just one subexpression to which the rule applies. However, if there are more subexpressions to which a rule applies, you will sometimes want to apply the rule only to some of the subexpressions. To handle this conveniently, there are the functions `Ers` and `PosListe`.

The file `summatio.q`

This file contains most of the summation formulas of the book [1], including the (almost) complete Appendix II of [1], in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `summatio.q` are

`S1001`, `S1101`, `S2101`, `S2102`, `S2103`, `S2104`, `S2105`, `S2106`, `S2107`, `S2161`, `S2201`, `S2202`, `S3201`, `S3202`, `S3203`, `S3204`, `S3261`, `S4301`, `S4302`, `S4303`, `S4304`, `S4305`, `S4306`, `S4307`, `S4308`, `S4361`, `S5401`, `S5402`, `S5501`, `S6501`, `S6502`, `S8701`, `S8702`, `S8703`, `S8704`, `S8761`, `S10901`, `SListe`, `SumListe`.

The numbering of each rule is `S`⟨`d1`⟩⟨`d2`⟩⟨`n1`⟩ following the following system: The number ⟨`d1`⟩ is the number of the upper parameters, the number ⟨`d2`⟩ is the number of the lower parameters of the basic hypergeometric series to which the rule applies. The number ⟨n1⟩ allows to distinguish the rules applying to basic hypergeometric series with equal numbers of upper and lower parameters. ⟨n1⟩ is within the range 01–30 if the summation is a one-term summation, it is within the range 61–90 if the summation is a two- or more-term summation.

For terminating series there is a check if one of the parameters is of the form q^{-n} where n is a nonnegative integer. Depending on your input you might be asked if some expression is a nonnegative integer (see the examples for `S3201`). Be sure to give an affirmative answer only for *one* of several expressions, otherwise the package will try to find the minimum of all of these, which might cause problems. This remark also applies to other rules which put this question, e.g. `SumUmkehr`, `phSum`, or in case that automatic evaluating is active (cf. `PQ`). The rule `SListe` enables you to quickly check if one of the summation rules `S1001`–`S8761` can be directly applied.

The file `transfor.q`

This file contains most of the transformation formulas of the book [1], including the (almost) complete Appendix III of [1], in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `transfor.q` are

`T2101`, `T2102`, `T2103`, `T2104`, `T2105`, `T2106`, `T2107`, `T2108`, `T2109`, `T2110`, `T2111`, `T2112`, `T2161`, `T2162`, `T2163`, `T2201`, `T2202`, `T3101`, `T3201`, `T3202`, `T3203`, `T3204`, `T3205`, `T3206`, `T3207`, `T3208`, `T3209`, `T3210`, `T3211`, `T3212`, `T3213`, `T3214`, `T3215`, `T3216`, `T3217`, `T3261`, `T3262`, `T3263`, `T3264`, `T3265`, `T3266`, `T3267`, `T3268`, `T3269`, `T4301`, `T4302`, `T4303`, `T4304`, `T4305`, `T4306`, `T4307`, `T4308`, `T4309`, `T4310`, `T4311`, `T4312`, `T4361`, `T4362`, `T5401`, `T5402`, `T5403`, `T5404`, `T5405`, `T5461`, `T5462`, `T5463`, `T5464`, `T5465`, `T5466`, `T5467`, `T5468`, `T5469`, `T6501`, `T7601`, `T7701`, `T8701`, `T8702`, `T8703`, `T8704`, `T8705`, `T8706`, `T8707`, `T8708`, `T8709`, `T8710`, `T8711`, `T8761`, `T8762`, `T8763`, `T8764`, `T10901`, `T10902`, `T10903`, `T10904`, `T10905`, `T10906`, `T10907`, `T10961`, `T10962`, `T10963`, `T121101`, `T121102`, `T121103`, `T121104`, `T121105`, `T121106`, `T121161`, `TListe`, `TransListe`.

The comments for the file `summatio.q` regarding the numbering of the rules and optional questions for input also apply here. The rule `TListe` enables you to quickly check if one of the transformation rules `T2101`–`T10961` can be directly applied.

The file `transfor.qli`

Each of the objects of this file corresponds to a transformation rule of the file `transfor.q`. Each object gives a list of all the outcomes under application of a particular transformation after before having permuted the upper and lower parameters of the involved basic hypergeometric series. All the objects in this file are rules. These rules help to prove conjectured transformation formulas quickly. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `transfor.qli` are

`Tli2101`, `Tli2102`, `Tli2103`, `Tli2104`, `Tli2105`, `Tli2106`, `Tli2107`, `Tli2108`, `Tli2109`, `Tli2110`, `Tli2111`, `Tli2112`, `Tli2161`, `Tli2162`, `Tli2163`, `Tli2201`, `Tli2202`, `Tli3101`, `Tli3201`, `Tli3202`, `Tli3203`, `Tli3204`, `Tli3205`, `Tli3206`, `Tli3207`, `Tli3208`, `Tli3209`, `Tli3210`, `Tli3211`, `Tli3212`, `Tli3213`, `Tli3214`, `Tli3215`, `Tli3216`, `Tli3261`, `Tli3262`, `Tli3263`, `Tli3264`, `Tli3265`, `Tli3266`, `Tli3267`, `Tli3268`, `Tli3269`, `Tli4301`, `Tli4302`, `Tli4303`, `Tli4304`, `Tli4305`, `Tli4306`, `Tli4307`, `Tli4308`, `Tli4309`, `Tli4310`, `Tli4311`, `Tli4312`, `Tli4361`, `Tli4362`, `Tli5401`, `Tli5402`, `Tli5403`, `Tli5404`, `Tli5405`, `Tli5461`, `Tli5462`, `Tli5463`, `Tli5464`, `Tli5465`, `Tli5466`, `Tli5467`, `Tli5468`, `Tli5469`, `Tli6501`, `Tli7601`, `Tli8701`, `Tli8702`, `Tli8703`, `Tli8704`, `Tli8705`, `Tli8706`, `Tli8707`, `Tli8708`, `Tli8709`, `Tli8710`, `Tli8711`, `Tli8761`, `Tli8762`, `Tli8763`, `Tli8764`, `Tli10901`, `Tli10902`, `Tli10903`, `Tli10904`, `Tli10905`, `Tli10906`, `Tli10907`, `Tli10961`, `Tli10962`, `Tli10963`, `Tli121101`, `Tli121102`, `Tli121103`, `Tli121104`, `Tli121105`, `Tli121106`, `Tli121161`.

The files `summatio.qgl` and `transfor.qgl`

These files contain the same summations, respectively transformations, as `summatio.q`, respectively `transfor.q`, but in form of equations. You do not have to load these files by hand since they are loaded automatically once an object of this file is called. The respective objects are

`Sgl1001`, `Sgl1101`, `Sgl2101`, `Sgl2102`, `Sgl2103`, `Sgl2104`, `Sgl2105`, `Sgl2106`, `Sgl2107`, `Sgl2161`, `Sgl2201`, `Sgl2202`, `Sgl3201`, `Sgl3202`, `Sgl3203`, `Sgl3204`, `Sgl3261`, `Sgl4301`, `Sgl4302`, `Sgl4303`, `Sgl4304`, `Sgl4305`, `Sgl4306`, `Sgl4307`, `Sgl4308`, `Sgl4361`, `Sgl5401`, `Sgl5402`, `Sgl5501`, `Sgl6501`, `Sgl6502`, `Sgl8701`, `Sgl8702`, `Sgl8703`, `Sgl8704`, `Sgl8761`, `Sgl10901`, `SumListe$gl`,

and

`Tgl2101`, `Tgl2102`, `Tgl2103`, `Tgl2104`, `Tgl2105`, `Tgl2106`, `Tgl2107`, `Tgl2108`, `Tgl2109`, `Tgl2110`, `Tgl2111`, `Tgl2112`, `Tgl2161`, `Tgl2162`, `Tgl2163`, `Tgl2201`, `Tgl2202`, `Tgl3101`, `Tgl3201`, `Tgl3202`, `Tgl3203`, `Tgl3204`, `Tgl3205`, `Tgl3206`, `Tgl3207`, `Tgl3208`, `Tgl3209`, `Tgl3210`, `Tgl3211`, `Tgl3212`, `Tgl3213`, `Tgl3214`, `Tgl3215`, `Tgl3216`, `Tgl3261`, `Tgl3262`, `Tgl3263`, `Tgl3264`, `Tgl3265`, `Tgl3266`, `Tgl3267`, `Tgl3268`, `Tgl3269`, `Tgl4301`, `Tgl4302`, `Tgl4303`, `Tgl4304`, `Tgl4305`, `Tgl4306`, `Tgl4307`, `Tgl4308`, `Tgl4309`, `Tgl4310`, `Tgl4311`, `Tgl4312`, `Tgl4361`, `Tgl4362`, `Tgl5401`, `Tgl5402`, `Tgl5403`, `Tgl5404`, `Tgl5405`, `Tgl5461`, `Tgl5462`, `Tgl5463`, `Tgl5464`, `Tgl5465`, `Tgl5466`, `Tgl5467`, `Tgl5468`, `Tgl5469`, `Tgl6501`, `Tgl7601`, `Tgl8701`, `Tgl8702`, `Tgl8703`, `Tgl8704`, `Tgl8705`, `Tgl8706`, `Tgl8707`, `Tgl8708`, `Tgl8709`, `Tgl8710`, `Tgl8711`, `Tgl8761`, `Tgl8762`, `Tgl8763`, `Tgl8764`, `Tgl10901`, `Tgl10902`, `Tgl10903`, `Tgl10904`, `Tgl10905`, `Tgl10906`, `Tgl10907`, `Tgl10961`, `Tgl10962`, `Tgl10963`, `Tgl121101`, `Tgl121102`, `Tgl121103`, `Tgl121104`, `Tgl121105`, `Tgl121106`, `Tgl121161`, `TransListe$gl`.

When calling one of these objects you will be put two questions. If the variables of the called summation or transformation are undefined, the first question is

```
Do you want to set values for the equation?  [y|n]:
```

Enter `y` if you want to set values, even only for some of them, if you do not need to set values enter `n`. If some of the variables of the called summation or transformation are already defined, you will be asked as first question

```
Some variables have a value.  Should the variables
{[V,a,r,i,a,b,l,e,s]} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:
```

Now you have four options depending on if you want to set values or not and if you want to clear the already defined variables or not. For example, if you want to set values but do not want to clear the defined variables, enter **nv**.

The second question concerns the base `q`. Either it is

```
Do you want to set a value for q in the equation?  [y|n]:
```

or

```
q has a value.  Should q be cleared?
Do you want to set a value for q in the equation (v)?  [y|n|yv|nv]:
```

The explanations for the first question also apply here. (Cf. the examples in `Sgl2101`).

In addition there are the functions and variables

`Add, Div, Gleichung, GlTausche, Hoch, LS, Mal, RS, Sub,`

for manipulating equations. In fact, once you have called one of the objects `Sgl*` or `Tgl*`, the right-hand side of the displayed equation will have been assigned to `RS`, the left-hand side to `LS`, and thus the equation itself to `Gleichung` (cf. the example in `Gleichung`). The functions `Add, Div, GlTausche, Hoch, Mal, Sub`, and also `Ers`, allow you to manipulate the equation.

The file `contig.q`

This file contains a vast number of contiguous relations in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `contig.q` are

`C01`, `C02`, `C11`, `C12`, `C13`, `C14`, `C15`, `C16`, `C17`, `C18`, `C19`, `C20`, `C21`, `C22`, `C23`, `C24`, `C25`, `C26`, `C27`, `C28`, `C29`, `C30`, `C31`, `C32`, `C33`, `C34`, `C35`, `C36`, `C37`, `C38`, `C39`, `C40`, `C41`, `C42`, `C43`, `C44`, `C45`, `C46`, `C47`, `C48`, `C49`, `C50`, `C51`, `C52`, `C53`, `C54`, `C55`, `C56`, `C57`, `C58`, `C59`, `C60`, `C61`, `C62`, `C63`, `C64`, `C65`, `C66`, `C67`, `C68`, `C69`, `C70`, `C71`, `C72`, `C73`, `C74`, `C75`, `C76`, `C77`, `C78`, `C79`, `C80`, `C81`, `C82`, `C83`, `C84`, `C85`, `C86`, `C87`, `C88`, `C89`, `C90`, `C91`, `C92`, `C93`, `C94`, `C95`, `C96`, `C97`, `C98`, `C99`, `C100`, `C101`, `C102`, `C103`, `C104`, `C105`, `C106`, `C107`, `C108`, `C109`, `C110`, `C111`, `C112`, `C113`, `C114`, `C115`, `C116`, `C117`, `C118`, `C119`, `C120`, `C121`, `ContigListe`.

Simultaneous use of HYP and HYPQ

It is possible to load both packages, HYP and HYPQ. In this case, the objects of the package that is loaded last will override the respective objects of the other package which have identical names. However, you can use the overrided objects by calling them by their *full* names. To determine the full name of an object the following rule applies:

If the object `Object` is defined in the file `File.ext`, then the full name of `Object` is ``File`ext`Object``.

For instance, if you load `hyp.q` first and then `hyp.m` and want to use `Limes` with a basic hypergeometric expression, then you have to type ``Hyp`q`Limes`` instead of `Limes`. (Calling `Limes` would invoke the *ordinary* hypergeometric `Limes`.) For information on contexts in MATHEMATICA confer [4].

On-line help

For each object of this package on-line help is supported in the usual way. For instance, quick information about

`Limes` (not having the manual at hand) is available in the following way.

```
In[1]:= ?Limes
Description:  Function for doing formal limits of basic hypergeometric
  expressions.  If required for taking the limit, you will be asked whether
  or not the absolute value of some variable or expression is smaller than
  1.  Your decision is stored for the rest of your MATHEMATICA session.  If
  you want to change your decision later, use "AbsGreater", "AbsSmaller",
  or "AbsUndetermined", respectively.
  By default the absolute value of q is defined to be smaller than 1.  Also
  this can be changed by "AbsGreater", "AbsSmaller", or "AbsUndetermined",
  respectively.
Warning:  This function uses primitive algebraic techniques to do the
  limit.  There is no check if taking the limit is actually allowed.  So it
  is left to you to check the validity of a result of "Limes".
Usage:  Limes[Expr, x->x0].
See also:  AbsGreater, AbsSmaller, AbsUndetermined, MinusOne.
```

The screen output

The screen output of the examples in this manual imitates the output under usage of code tables 437, 860, 863, or 865 (cf. the `read.me`). The output under other code tables is a little bit less attractive, but similar. For instance, the examples for `ph` then would read as follows.

```
In[1]:= ph[{a,b},{c},q,z]

            [            ]
            | a, b       |
Out[1]=  ph |      ; q, z |
        2  1|  c         |
            [            ]

In[2]:= ph[{a,b,c},{d,e,0},q,z]

            [               ]
            | a, b, c       |
Out[2]=  ph |         ; q, z |
        3  3| d, e, 0       |
            [               ]
```

A brief dictionary

Most of the names of the objects are obviously German influenced. To help those who are not so familiar with German, brief German–English and English–German vocabularies are provided.

A German–English vocabulary

German	English	*Mathematica* objects in HYPQ containing the word
abspalten	split	`lina1, lina2`
auflösen	dissolve	`pqaufl`
drucken	print	`Drucke`
einfügen	insert	`phEinf`
ersetzen	replace	`Ers`
erweitern	extend	`erw1, erw2, SumErw1, SumErw2`
Gleichung	equation	`Gleichung`
"2 hoch 3"	"2 to the 3"	`Hoch`
"2 mal 3"	"2 times 3"	`Mal`
ordnen	order	`phOrdne`
Regel	rule	`SumRegeln`
sammeln	collect	`SumSammle`
schreiben	write	`SchreibeZahl`
tauschen	exchange, interchange	`SumTausche, GlTausche`
umkehren	reverse	`SumUmkehr`
Zahl	number	`SchreibeZahl`
zerlegen	split	`zerl1, pqzerl, pqinfzerl, SumZerl`
zusammenfassen	put together	`zus1, pqzus, pqinfzus`

An English–German vocabulary

English	German	*Mathematica* objects in HYPQ containing the word
collect	sammeln	`SumSammle`
dissolve	auflösen	`pqaufl`
equation	Gleichung	`Gleichung`
exchange	tauschen	`SumTausche, GlTausche`
extend	erweitern	`erw1, erw2, SumErw1, SumErw2`
insert	einfügen	`phEinf`
interchange	tauschen	`SumTausche, GlTausche`
number	Zahl	`SchreibeZahl`
order	ordnen	`phOrdne`
print	drucken	`Drucke`
replace	ersetzen	`Ers`
reverse	umkehren	`SumUmkehr`
rule	Regel	`SumRegeln`
split	abspalten	`lina1, lina2`
split	zerlegen	`zerl1, pqzerl, pqinfzerl, SumZerl`
"2 times 3"	"2 mal 3"	`Mal`
"2 to the 3"	"2 hoch 3"	`Hoch`
put together	zusammenfassen	`zus1, pqzus, pqinfzus`
write	schreiben	`SchreibeZahl`

Alphabetic List of the objects with desriptions

AbsGreater

Description: Function for declaring the absolute value of a variable or expression to be greater than 1. This declaration is used by `Limes`. By default the absolute value of `q` is defined to be smaller than 1.

Usage: `AbsGreater[Expr].`

Example(s):

```
In[1]:= Limes[pq[a,n,p],n->Infinity]
Is |p| smaller than 1?
[y|n|u]:  y

Out[1]= (a;p)
             ∞

In[2]:= Limes[pq[a*p^-n,n,p],n->Infinity]
Is n even, odd, or neither of both?
[e|o|n]:  n
Is |a| smaller than 1?
[y|n|u]:  u
Is |a| smaller than 1?
[y|n|u]:  u

The expression

                  2
    n  n  -n/2 - n /2  p
(-1)  a  p            (-;p)
                       a  ∞

was obtained.

Therefore the limit n -> ∞ could not be determined.
Here is your expression:

         a
Out[2]= (--; p)
          n    n
         p

In[3]:= AbsGreater[p]

In[4]:= Limes[pq[a*p^-n,n,p],n->Infinity]
Is 2 n even, odd, or neither of both?
[e|o|n]:  e
```

```
         a 1
Out[4]= (-;-)
         p p∞
```

See also: AbsSmaller, AbsUndetermined, Limes.

AbsSmaller

Description: Function for declaring the absolute value of a variable or expression to be smaller than 1. This declaration is used by `Limes`. By default the absolute value of `q` is defined to be smaller than 1.

Usage: `AbsSmaller[Expr]`.

Example(s):

```
In[1]:= Limes[pq[a,n,1/p],n->Infinity]
Is |p| smaller than 1?
[y|n|u]:  n

           1
Out[1]= (a;-)
           p∞

In[2]:= Limes[pq[a*p^n,n,1/p],n->Infinity]
Is n even, odd, or neither of both?
[e|o|n]:  n
Is |a| smaller than 1?
[y|n|u]:  u
Is |a| smaller than 1?
[y|n|u]:  u

The expression

                   2
    n  n  n/2 + n /2   1  1
(-1)  a  p           (---;-)
                      a p p∞

was obtained.

Therefore the limit n -> ∞ could not be determined.
Here is your expression:

            n  1
Out[2]= (a p ; -)
               p n

In[3]:= AbsSmaller[p]
```

```
In[4]:= Limes[pq[a*p^n,n,1/p],n->Infinity]
Is 2 n even, odd, or neither of both?
[e|o|n]:  e

Out[4]= (a p;p)
              ∞
```

See also: `AbsGreater, AbsUndetermined, Limes.`

AbsUndetermined

Description: Function for declaring the absolute value of a variable or expression to be neither smaller nor greater than 1. This declaration is used by `Limes`. By default the absolute value of `q` is defined to be smaller than 1.

Usage: `AbsUndetermined[Expr].`

Example(s):

```
In[1]:= AbsSmaller[p]

In[2]:= Limes[pq[a,n,p],n->Infinity]

Out[2]= (a;p)
            ∞

In[3]:= AbsGreater[p]

In[4]:= Limes[pq[a*p^-n,n,p],n->Infinity]
Is 2 n even, odd, or neither of both?
[e|o|n]:  e

         a 1
Out[4]= (-;-)
         p p ∞

In[5]:= AbsUndetermined[p]

In[6]:= Limes[pq[a,n,p],n->Infinity]
Is |p| smaller than 1?
[y|n|u]:  u

The expression

Indeterminate

was obtained.

Therefore the limit n -> ∞ could not be determined.
Here is your expression:
```

```
Out[6]= (a; p)
             n

In[7]:= Limes[pq[a*p^-n,n,p],n->Infinity]
Is |p| smaller than 1?
[y|n|u]:  u

The expression

Indeterminate

was obtained.

Therefore the limit n -> ∞ could not be determined.
Here is your expression:

         a
Out[7]= (--; p)
          n    n
         p
```

See also: AbsGreater, AbsSmaller, Limes.

Add

Description: Function that adds Expr to Gleichung.

Usage: Add[Expr].

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                      n   c
                                     a   (-; q)
                -n                        a    n
Out[1]=  phi  [ a, q   ; q, q ] == ----------
        2 1                          (c; q)
             c                              n

In[2]:= Add[pq[c/a,n]]

                                                            n   c
                                                           a   (-; q)
                -n                c          c                  a    n
Out[2]=  phi  [ a, q   ; q, q ] + (-; q)  == (-; q)   + ----------
        2 1                       a    n     a    n      (c; q)
             c                                                 n
```

```
In[3]:= Gleichung

                                                      n  c
                                                     a  (-; q)
                -n              c          c             a    n
Out[3]=  phi  [ a, q   ; q, q ] + (-; q)  == (-; q)  + ----------
            2 1                     a    n     a    n    (c; q)
              c                                                n
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Div, Sub, Hoch, GlTausche, Ers.`

AmSLaTeX

Description: Switch that changes the output of TeXForm to be usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX. By default the output of TeXForm is usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX.

Usage: `AmSLaTeX.`

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= TeXForm[ph[{a,b},{c},q,z]]

Out[2]//TeXForm=
  {} _{2} \phi _{1} \!  \left [ \matrix \let \over / a, b\\ \let \over /
   c\endmatrix ;q, {\displaystyle z} \right ]

In[3]:= AmSLaTeX

In[4]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-LaTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[5]:= TeXForm[ph[{a,b},{c},q,z]]

Out[5]//TeXForm=
  {} _{2} \phi _{1} \!  \left [ \begin{matrix} \let \over / a, b\\ \let \over /
   c\end{matrix} ;q, {\displaystyle z} \right ]
```

See also: `AmSTeX, LaTeX, TeX, TeXMat, TeXphW.`

AmSTeX

Description: Switch that changes the output of TeXForm to be usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX. By default the output of TeXForm is usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX.

Usage: `AmSTeX.`

Example(s):

```
In[1]:= TeX

In[2]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[3]:= TeXForm[ph[{a,b},{c},q,z]]

Out[3]//TeXForm=
  {} _{2} \phi _{1} \!  \left [ \matrix { \let \over / a, b\cr \let \over / c}
   ;q, {\displaystyle z} \right ]

In[4]:= AmSTeX

In[5]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[6]:= TeXForm[ph[{a,b},{c},q,z]]

Out[6]//TeXForm=
  {} _{2} \phi _{1} \!  \left [ \matrix \let \over / a, b\\ \let \over /
   c\endmatrix ;q, {\displaystyle z} \right ]
```

See also: `AmSLaTeX, LaTeX, TeX, TeXMat, TeXphW.`

---

baszerl1

Description: $(a;q)_n \to \prod_{k=0}^{m-1}(aq^k;q^m)_{n/m}$,
$(a;q)_\infty \to \prod_{k=0}^{m-1}(aq^k;q^m)_\infty$.

The parameter `m` has to be entered on request.

Usage: `Expr/.baszerl1.`

Example(s):

```
In[1]:= pq[a,n]
```

```
Out[1]= (a; q)
              n

In[2]:= %/.baszerl1
split into ?  terms:  2

             2            2
Out[2]= (a; q )    (a q; q )
               n/2          n/2

In[3]:= pq[b,4*m]

Out[3]= (b; q)
              4 m

In[4]:= %/.baszerl1
split into ?  terms:  4

             4          4       2   4       3   4
Out[4]= (b; q )  (b q; q )  (b q ; q )  (b q ; q )
               m          m           m           m

In[5]:= pqinf[c,q^(1/2)]

Out[5]= (c;Sqrt[q])
                   ∞

In[6]:= %/.baszerl1
split into ?  terms:  4

            2                2         2       3/2  2
Out[6]= (c;q )  (c Sqrt[q];q )  (c q;q )  (c q   ;q )
              ∞                ∞         ∞            ∞
```

See also: `baszerl2, baszus1, baszus2, Ers, PosListe, ManipulationsListe.`

---

baszerl2

Description: $(a;q)_n \to \prod_{k=0}^{m-1}(e^{2\pi ik/m}a^{1/m};q^{1/m})_n$,
$(a;q)_\infty \to \prod_{k=0}^{m-1}(e^{2\pi ik/m}a^{1/m};q^{1/m})_\infty$.

The parameter `m` has to be entered on request.

Usage: `Expr/.baszerl2.`

Example(s):

```
In[1]:= pq[a,n]

Out[1]= (a; q)
              n
```

```
In[2]:= %/.baszerl2
split into ?  terms:  2

Out[2]= (-Sqrt[a]; Sqrt[q])  (Sqrt[a]; Sqrt[q])
                           n                   n

In[3]:= pq[a^2,n,q^4]

          2   4
Out[3]= (a ; q )
                n

In[4]:= %/.baszerl2
split into ?  terms:  4

Out[4]= (-Sqrt[a]; q)  (-I Sqrt[a]; q)  (I Sqrt[a]; q)  (Sqrt[a]; q)
                     n                n               n             n

In[5]:= pqinf[c,q^3]

            3
Out[5]= (c;q )
              ∞

In[6]:= %/.baszerl2
split into ?  terms:  3

          1/3       (2 I)/3 π  1/3       (4 I)/3 π  1/3
Out[6]= (c   ;q)  (E           c   ;q)  (E           c   ;q)
                ∞                    ∞                    ∞
```

See also: `Ers, PosListe, ManipulationsListe.`

---

baszus1

Description: $(a;q)_n \to (a;q^{1/m})_{mn} / \prod_{k=1}^{m-1}(aq^{k/m};q)_n$,
$(a;q)_\infty \to (a;q^{1/m})_\infty / \prod_{k=1}^{m-1}(aq^{k/m};q)_\infty$.

The parameter `m` has to be entered on request. This operation is basically the inverse of `baszerl1`.

Usage: `Expr/.baszus1.`

Example(s):

```
In[1]:= pq[a,n,q^2]*pq[a*q,n,q^2]

             2          2
Out[1]= (a; q )  (a q; q )
               n          n
```

```
In[2]:= Ers[%,baszus1,{1}]
put together ?  terms:  2
put together ?  terms:  2

Out[2]= (a; q)
              2 n

In[3]:= pqinf[a/q,q^2]*pqinf[a,q^2]*pq[b,m]*pq[b*q^(1/3),m]*pq[b*q^(2/3),m]

            2    a  2               1/3         2/3
Out[3]= (a;q )  (-;q )  (b; q)  (b q   ; q)  (b q   ; q)
             ∞  q    ∞       m             m             m

In[4]:= Ers[%,baszus1,{3}]
put together ?  terms:  3
put together ?  terms:  3

            2    a  2        1/3
Out[4]= (a;q )  (-;q )  (b; q    )
             ∞  q    ∞          3 m

In[5]:= Ers[%,baszus1,{2}]
put together ?  terms:  2
put together ?  terms:  2

         a          1/3
Out[5]= (-;q)  (b; q    )
         q   ∞         3 m
```

See also: `Ers, PosListe, ManipulationsListe.`

---

baszus2

Description: $(a;q)_n \to (a^m;q^m)_n / \prod_{k=1}^{m-1}(e^{2\pi ik/m}a;q)_n$,
$(a;q)_\infty \to (a^m;q^m)_\infty / \prod_{k=1}^{m-1}(e^{2\pi ik/m}a;q)_\infty$.

The parameter `m` has to be entered on request. This operation is basically the inverse of `baszerl2`.

Usage: `Expr/.baszus2.`

Example(s):

```
In[1]:= pq[a,m,q^2]*pq[-a,m,q^2]

              2       2
Out[1]= (-a; q )  (a; q )
               m        m

In[2]:= Ers[%,baszus2,{1}]
put together ?  terms:  2
put together ?  terms:  2
```

```
           2   4
Out[2]= (a  ; q )
                m

In[3]:= pqinf[-A,q^(1/2)]*pqinf[A,q^(1/2)]*pq[-a,n]*pq[-I*a,n]*pq[I*a,n]*
        pq[a,n]

Out[3]= (-A;Sqrt[q])  (A;Sqrt[q])  (-a; q)  (-I a; q)  (I a; q)  (a; q)
                    ∞            ∞        n          n         n       n

In[4]:= Ers[%,baszus2,{3}]
put together ?  terms:  4
put together ?  terms:  4

                                     4   4
Out[4]= (-A;Sqrt[q])  (A;Sqrt[q])  (a  ; q )
                    ∞            ∞         n

In[5]:= Ers[%,baszus2,{1}]
put together ?  terms:  2
put together ?  terms:  2

           2       4   4
Out[5]= (A  ;q)  (a  ; q )
               ∞          n
```

Whenever you use a rule, that asks you for an input, within **Ers**, you will get the reqest for input twice. The value which is entered last is used.

See also: `Ers, PosListe, ManipulationsListe.`

Binomialpq

Description: `Binomialpq[n,k,q]` is the q-binomial coefficient, written in terms of q-factorial symbols `pq`. The parameter `q` is optional. It will be set equal `q` if it is omitted.

Usage: `Binomialpq[n,k,q]`
or: `Binomialpq[n,k].`

Example(s):

```
In[1]:= Binomialpq[n,k]

          1 - k + n
        (q          ; q)
                        k
Out[1]= ----------------
            (q; q)
                  k

In[2]:= Binomialpq[6,3]
```

```
           4
        (q  ; q)
               3
Out[2]= --------
        (q; q)
              3
```

See also: `Binomialq, Factorialq, Factorialpq.`

Binomialq

Description: `Binomialq[n,k,q]` is the q-binomial coefficient, expanded into a q-series, if possible. The parameter `q` is optional. It will be set equal `q` if it is omitted.

Usage: `Binomialq[n,k,q]`
or: `Binomialq[n,k].`

Example(s):

```
In[1]:= Binomialq[n+k,n-k]

        [ k + n  ]
Out[1]= [        ]
        [ -k + n ]

                q

In[2]:= Binomialq[6,3]

                    2     3     4     5     6     7    8    9
Out[2]= 1 + q + 2 q  + 3 q  + 3 q  + 3 q  + 3 q  + 2 q  + q  + q
```

See also: `Binomialpq, Factorialq, Factorialpq.`

C01

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,z\right] \longrightarrow 1+\frac{(-1)^{1-r+s}z}{(1-q)}\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}\,{}_{r+1}\phi_{s+1}\left[\begin{matrix}q,(qA)\\q^2,(qB)\end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C01.`

See also: `C64, ContigListe, Ers, PosListe.`

C02

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix}(A),q\\(B)\end{matrix};q,z\right] \longrightarrow -\frac{(-1)^{1-r+s}q^{1-r+s}}{z}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})} + \frac{(-1)^{1-r+s}q^{1-r+s}}{z}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q),q\\(B/q)\end{matrix};q,q^{-1+r-s}z\right]$$

Usage: `Expr/.C02`.

See also: `C64, ContigListe, Ers, PosListe`.

C11

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,qz\right] + (-1)^{1-r+s}z\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}{}_r\phi_s\left[\begin{matrix}(qA)\\(qB)\end{matrix};q,q^{1-r+s}z\right]$$

Usage: `Expr/.C11`.

See also: `C64, ContigListe, Ers, PosListe`.

C12

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,\frac{z}{q}\right] - \frac{(-1)^{1-r+s}z}{q}\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}{}_r\phi_s\left[\begin{matrix}(qA)\\(qB)\end{matrix};q,q^{-r+s}z\right]$$

Usage: `Expr/.C12`.

See also: `C64, ContigListe, Ers, PosListe`.

C13

Description: Contiguous relation in form of a rule.

$$\begin{aligned}{}_r\phi_s\left[\begin{matrix}(A)\\(B)\end{matrix};q,z\right] \longrightarrow & \frac{(-1)^{1-r+s}q^{1-r+s}}{z}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\(B/q)\end{matrix};q,q^{-1+r-s}z\right]\\ & - \frac{(-1)^{1-r+s}q^{1-r+s}}{z}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\(B/q)\end{matrix};q,q^{r-s}z\right]\end{aligned}$$

Usage: `Expr/.C13`.

See also: `C64, ContigListe, Ers, PosListe`.

C14

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}az}{q}\frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}{}_r\phi_s\left[\begin{matrix} a,(qA) \\ (qB) \end{matrix};q,q^{1-r+s}z\right] + {}_r\phi_s\left[\begin{matrix} a/q,(A) \\ (B) \end{matrix};q,z\right] $$

Usage: `Expr/.C14[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe`.

C15

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq,(A) \\ (B) \end{matrix};q,z\right] - (-1)^{1-r+s}az\frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}{}_r\phi_s\left[\begin{matrix} aq,(qA) \\ (qB) \end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C15[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe`.

C16

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}q^{2-p+q}}{az}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-1}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix} a,(A/q) \\ (B/q) \end{matrix};q,q^{-1+r-s}z\right] $$
$$ - \frac{(-1)^{1-r+s}q^{2-p+q}}{az}\frac{\prod_{i=1}^{s}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-1}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix} a/q,(A/q) \\ (B/q) \end{matrix};q,q^{-1+r-s}z\right] $$

Usage: `Expr/.C16[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe`.

C17

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow (-1)^{1-r+s}z\frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}{}_r\phi_s\left[\begin{matrix} a,(qA) \\ (qB) \end{matrix};q,q^{1-r+s}z\right] + {}_r\phi_s\left[\begin{matrix} a/q,(A) \\ (B) \end{matrix};q,qz\right] $$

Usage: `Expr/.C17[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe`.

C18

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq,(A) \\ (B) \end{matrix};q,\frac{z}{q}\right] - \frac{(-1)^{1-r+s}\,z}{q}\,\frac{\prod\limits_{i=1}^{r-1}(1-A_i)}{\prod\limits_{i=1}^{s}(1-B_i)}\,{}_r\phi_s\left[\begin{matrix} aq,(qA) \\ (qB) \end{matrix};q,q^{-r+s}z\right] $$

Usage: `Expr/.C18[m1].`
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C19

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}\,q^{1-r+s}}{z}\,\frac{\prod\limits_{i=1}^{s}(1-\frac{B_i}{q})}{\prod\limits_{i=1}^{r-1}(1-\frac{A_i}{q})}\,{}_r\phi_s\left[\begin{matrix} a,(A/q) \\ (B/q) \end{matrix};q,q^{-1+r-s}z\right] $$
$$ - \frac{(-1)^{1-r+s}\,q^{1-r+s}}{z}\,\frac{\prod\limits_{i=1}^{s}(1-\frac{B_i}{q})}{\prod\limits_{i=1}^{r-1}(1-\frac{A_i}{q})}\,{}_r\phi_s\left[\begin{matrix} a/q,(A/q) \\ (B/q) \end{matrix};q,q^{r-s}z\right] $$

Usage: `Expr/.C19[m1].`
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C20

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} (A) \\ bq,(B) \end{matrix};q,z\right] + \frac{(-1)^{1-r+s}\,bz}{(1-b)\,(1-bq)}\,\frac{\prod\limits_{i=1}^{r}(1-A_i)}{\prod\limits_{i=1}^{s-1}(1-B_i)}\,{}_r\phi_s\left[\begin{matrix} (qA) \\ bq^2,(qB) \end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C20[n1].`
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C21

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} (A) \\ b/q,(B) \end{matrix};q,z\right] - \frac{(-1)^{1-r+s}\,bz}{(1-b)\left(1-\frac{b}{q}\right)q}\,\frac{\prod\limits_{i=1}^{r}(1-A_i)}{\prod\limits_{i=1}^{s-1}(1-B_i)}\,{}_r\phi_s\left[\begin{matrix} (qA) \\ bq,(qB) \end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C21[n1].`
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C22

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{3-p+q}}{bz}\frac{\prod\limits_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod\limits_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\ b/q^2,(B/q)\end{matrix};q,q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{3-p+q}}{bz}\frac{\prod\limits_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod\limits_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\ b/q,(B/q)\end{matrix};q,q^{-1+r-s}z\right]
$$

Usage: `Expr/.C22[n1].`
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C23

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ bq,(B)\end{matrix};q,qz\right] + \frac{(-1)^{1-r+s}z}{(1-b)(1-bq)}\frac{\prod\limits_{i=1}^{r}(1-A_i)}{\prod\limits_{i=1}^{s-1}(1-B_i)}{}_r\phi_s\left[\begin{matrix}(qA)\\ bq^2,(qB)\end{matrix};q,q^{1-r+s}z\right]
$$

Usage: `Expr/.C23[n1].`
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C24

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ b/q,(B)\end{matrix};q,\frac{z}{q}\right] - \frac{(-1)^{1-r+s}z}{(1-b)\left(1-\frac{b}{q}\right)q}\frac{\prod\limits_{i=1}^{r}(1-A_i)}{\prod\limits_{i=1}^{s-1}(1-B_i)}{}_r\phi_s\left[\begin{matrix}(qA)\\ bq,(qB)\end{matrix};q,q^{-r+s}z\right]
$$

Usage: `Expr/.C24[n1].`
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C25

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};q,z\right]\longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{z}\frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\ b/q^2,(B/q)\end{matrix};q,q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{z}\frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}{}_r\phi_s\left[\begin{matrix}(A/q)\\ b/q,(B/q)\end{matrix};q,q^{r-s}z\right]
$$

Usage: `Expr/.C25[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C26

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ (B)\end{matrix};q,z\right]\longrightarrow \frac{(1-b)}{a-b}{}_r\phi_s\left[\begin{matrix}a,bq,(A)\\ (B)\end{matrix};q,\frac{z}{q}\right]+\frac{(1-a)}{-a+b}{}_r\phi_s\left[\begin{matrix}aq,b,(A)\\ (B)\end{matrix};q,\frac{z}{q}\right]
$$

Usage: `Expr/.C26[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C27

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ (B)\end{matrix};q,z\right]\longrightarrow \frac{\left(b-\frac{a}{q}\right)}{1-\frac{a}{q}}{}_r\phi_s\left[\begin{matrix}a/q,b,(A)\\ (B)\end{matrix};q,qz\right]+\frac{(1-b)}{1-\frac{a}{q}}{}_r\phi_s\left[\begin{matrix}a/q,bq,(A)\\ (B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C27[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C28

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ (B)\end{matrix};q,z\right]\longrightarrow \frac{a\,(1-b)}{a-b}{}_r\phi_s\left[\begin{matrix}a,bq,(A)\\ (B)\end{matrix};q,z\right]+\frac{(1-a)\,b}{-a+b}{}_r\phi_s\left[\begin{matrix}aq,b,(A)\\ (B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C28[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C29

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix} ; q, z\right] \longrightarrow \frac{\left(b - \frac{a}{q}\right)}{b\left(1 - \frac{a}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, b, (A) \\ (B) \end{matrix} ; q, z\right] + \frac{a\,(1-b)}{b\left(1 - \frac{a}{q}\right) q} {}_r\phi_s\left[\begin{matrix} a/q, bq, (A) \\ (B) \end{matrix} ; q, z\right] $$

Usage: `Expr/.C29[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C30

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix} ; q, z\right] \longrightarrow (-1)^{1-r+s}\left(-b + \frac{a}{q}\right) z \frac{\prod_{i=1}^{r-2}(1 - A_i)}{\prod_{i=1}^{s}(1 - B_i)} {}_r\phi_s\left[\begin{matrix} a, bq, (qA) \\ (qB) \end{matrix} ; q, q^{1-r+s} z\right] + {}_r\phi_s\left[\begin{matrix} a/q, bq, (A) \\ (B) \end{matrix} ; q, z\right] $$

Usage: `Expr/.C30[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C31

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix} ; q, z\right] \longrightarrow \frac{(-1)^{1-r+s} q^{1-r+s}}{\left(\frac{a}{q} - \frac{b}{q}\right) z} \frac{\prod_{i=1}^{s}(1 - \frac{B_i}{q})}{\prod_{i=1}^{r-2}(1 - \frac{A_i}{q})} {}_r\phi_s\left[\begin{matrix} a, b/q, (A/q) \\ (B/q) \end{matrix} ; q, q^{-1+r-s} z\right] $$
$$ - \frac{(-1)^{1-r+s} q^{1-r+s}}{\left(\frac{a}{q} - \frac{b}{q}\right) z} \frac{\prod_{i=1}^{s}(1 - \frac{B_i}{q})}{\prod_{i=1}^{r-2}(1 - \frac{A_i}{q})} {}_r\phi_s\left[\begin{matrix} a/q, b, (A/q) \\ (B/q) \end{matrix} ; q, q^{-1+r-s} z\right] $$

Usage: `Expr/.C31[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C32

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} a/q, b/q, (A) \\ (B) \end{matrix}; q, qz\right]
$$

$$
+ (-1)^{1-r+s}\left(1-\frac{ab}{q}\right) z \frac{\prod_{i=1}^{r-2}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)} {}_{r+1}\phi_{s+1}\left[\begin{matrix} a, b, ab, (qA) \\ ab/q, (qB) \end{matrix}; q, q^{1-r+s} z\right]
$$

Usage: `Expr/.C32[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe`.

C33

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq, bq, (A) \\ (B) \end{matrix}; q, \frac{z}{q}\right]
$$

$$
- \frac{(-1)^{1-r+s}(1-abq)\, z}{q} \frac{\prod_{i=1}^{r-2}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)} {}_{r+1}\phi_{s+1}\left[\begin{matrix} aq, bq, abq^2, (qA) \\ abq, (qB) \end{matrix}; q, q^{-r+s} z\right]
$$

Usage: `Expr/.C33[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe`.

C34

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{b}{q}\right)}{a-\frac{b}{q}} {}_r\phi_s\left[\begin{matrix} a, (A) \\ b/q, (B) \end{matrix}; q, \frac{z}{q}\right] + \frac{(1-a)}{-a+\frac{b}{q}} {}_r\phi_s\left[\begin{matrix} aq, (A) \\ b, (B) \end{matrix}; q, \frac{z}{q}\right]
$$

Usage: `Expr/.C34[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe`.

C35

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(-a+b)}{\left(1-\frac{a}{q}\right) q} {}_r\phi_s\left[\begin{matrix} a/q, (A) \\ b, (B) \end{matrix}; q, qz\right] + \frac{\left(1-\frac{b}{q}\right)}{1-\frac{a}{q}} {}_r\phi_s\left[\begin{matrix} a/q, (A) \\ b/q, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C35[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe`.

C36

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{(a-b)}{1-b}\,{}_r\phi_s\left[\begin{matrix} a,(A) \\ bq,(B) \end{matrix};q,qz\right] + \frac{(1-a)}{1-b}\,{}_r\phi_s\left[\begin{matrix} aq,(A) \\ bq,(B) \end{matrix};q,z\right]$$

Usage: `Expr/.C36[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C37

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{a\left(1-\frac{b}{q}\right)}{a-\frac{b}{q}}\,{}_r\phi_s\left[\begin{matrix} a,(A) \\ b/q,(B) \end{matrix};q,z\right] + \frac{(1-a)\,b}{\left(-a+\frac{b}{q}\right)q}\,{}_r\phi_s\left[\begin{matrix} aq,(A) \\ b,(B) \end{matrix};q,z\right]$$

Usage: `Expr/.C37[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C38

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{(-a+b)}{b\left(1-\frac{a}{q}\right)}\,{}_r\phi_s\left[\begin{matrix} a/q,(A) \\ b,(B) \end{matrix};q,z\right] + \frac{a\left(1-\frac{b}{q}\right)}{b\left(1-\frac{a}{q}\right)}\,{}_r\phi_s\left[\begin{matrix} a/q,(A) \\ b/q,(B) \end{matrix};q,z\right]$$

Usage: `Expr/.C38[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C39

Description: Contiguous relation in form of a rule.

$${}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{(a-b)}{a\,(1-b)}\,{}_r\phi_s\left[\begin{matrix} a,(A) \\ bq,(B) \end{matrix};q,z\right] + \frac{(1-a)\,b}{a\,(1-b)}\,{}_r\phi_s\left[\begin{matrix} aq,(A) \\ bq,(B) \end{matrix};q,z\right]$$

Usage: `Expr/.C39[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C40

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}\left(\frac{a}{q}-\frac{b}{q}\right)z}{(1-b)\left(1-\frac{b}{q}\right)} \frac{\prod_{i=1}^{s-1}(1-B_i)}{\prod_{i=1}^{r-1}(1-A_i)} {}_r\phi_s\left[\begin{matrix} a,(qA) \\ bq,(qB) \end{matrix};q,q^{1-r+s}z\right] + {}_r\phi_s\left[\begin{matrix} a/q,(A) \\ b/q,(B) \end{matrix};q,z\right] $$

Usage: `Expr/.C40[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C41

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq,(A) \\ bq,(B) \end{matrix};q,z\right] - \frac{(-1)^{1-r+s}(a-b)z}{(1-b)(1-bq)} \frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s-1}(1-B_i)} {}_r\phi_s\left[\begin{matrix} aq,(qA) \\ bq^2,(qB) \end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C41[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C42

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(-\frac{b}{q^2}+\frac{a}{q}\right)z} \frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-1}(1-\frac{A_i}{q})} {}_r\phi_s\left[\begin{matrix} a,(A/q) \\ b/q,(B/q) \end{matrix};q,q^{-1+r-s}z\right] $$
$$ - \frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(-\frac{b}{q^2}+\frac{a}{q}\right)z} \frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-1}(1-\frac{A_i}{q})} {}_r\phi_s\left[\begin{matrix} a/q,(A/q) \\ b/q^2,(B/q) \end{matrix};q,q^{-1+r-s}z\right] $$

Usage: `Expr/.C42[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C43

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} a/q,(A) \\ bq,(B) \end{matrix};q,qz\right] + \frac{(-1)^{1-r+s}(1-ab)z}{(1-b)(1-bq)} \frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s-1}(1-B_i)} {}_{r+1}\phi_{s+1}\left[\begin{matrix} a,abq,(qA) \\ bq^2,ab,(qB) \end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C43[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C44

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq, (A) \\ b/q, (B) \end{matrix}; q, \frac{z}{q}\right] - \frac{(-1)^{1-r+s}\,(1-ab)\,z}{(1-b)\left(1-\frac{b}{q}\right)q}\,\frac{\prod_{i=1}^{r-1}(1-A_i)}{\prod_{i=1}^{s-1}(1-B_i)}\,{}_{r+1}\phi_{s+1}\left[\begin{matrix} aq, abq, (qA) \\ bq, ab, (qB) \end{matrix}; q, q^{-p+q} z\right] $$

Usage: `Expr/.C44[m1,n1].`
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C45

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ a, b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{b}{q}\right)q}{a-b}\,{}_r\phi_s\left[\begin{matrix} (A) \\ a, b/q, (B) \end{matrix}; q, \frac{z}{q}\right] + \frac{\left(1-\frac{a}{q}\right)q}{-a+b}\,{}_r\phi_s\left[\begin{matrix} (A) \\ a/q, b, (B) \end{matrix}; q, \frac{z}{q}\right] $$

Usage: `Expr/.C45[n1,n2].`
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C46

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ a, b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(-a+\frac{b}{q}\right)}{1-a}\,{}_r\phi_s\left[\begin{matrix} (A) \\ aq, b, (B) \end{matrix}; q, qz\right] + \frac{\left(1-\frac{b}{q}\right)}{1-a}\,{}_r\phi_s\left[\begin{matrix} (A) \\ aq, b/q, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C46[n1,n2].`
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C47

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ a, b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{a\left(1-\frac{b}{q}\right)}{a-b}\,{}_r\phi_s\left[\begin{matrix} (A) \\ a, b/q, (B) \end{matrix}; q, z\right] + \frac{b\left(1-\frac{a}{q}\right)}{-a+b}\,{}_r\phi_s\left[\begin{matrix} (A) \\ a/q, b, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C47[n1,n2].`
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C48

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};q,z\right] \longrightarrow \frac{\left(-a+\frac{b}{q}\right)q}{(1-a)\,b}\,{}_r\phi_s\left[\begin{matrix}(A)\\ aq,b,(B)\end{matrix};q,z\right] + \frac{a\left(1-\frac{b}{q}\right)q}{(1-a)\,b}\,{}_r\phi_s\left[\begin{matrix}(A)\\ aq,b/q,(B)\end{matrix};q,z\right] $$

Usage: `Expr/.C48[n1,n2]`.
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C49

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ aq,b/q,(B)\end{matrix};q,z\right] + \frac{(-1)^{1-r+s}\left(a-\frac{b}{q}\right)z}{(1-a)\,(1-b)\left(1-\frac{b}{q}\right)(1-aq)}\,\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)}\,{}_r\phi_s\left[\begin{matrix}(qA)\\ aq^2,bq,(qB)\end{matrix};q,q^{1-r+s}z\right] $$

Usage: `Expr/.C49[n1,n2]`.
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C50

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(\frac{a}{q^2}-\frac{b}{q^2}\right)z}\,\frac{\prod_{i=1}^{s-2}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}\,{}_r\phi_s\left[\begin{matrix}(A/q)\\ a/q^2,b/q,(B/q)\end{matrix};q,q^{-1+r-s}z\right] - \frac{(-1)^{1-r+s}\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(\frac{a}{q^2}-\frac{b}{q^2}\right)z}\,\frac{\prod_{i=1}^{s-2}(1-\frac{B_i}{q})}{\prod_{i=1}^{r}(1-\frac{A_i}{q})}\,{}_r\phi_s\left[\begin{matrix}(A/q)\\ a/q,b/q^2,(B/q)\end{matrix};q,q^{-1+r-s}z\right] $$

Usage: `Expr/.C50[n1,n2]`.
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C51

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};q,z\right]\longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ aq,bq,(B)\end{matrix};q,qz\right]
+\frac{(-1)^{1-r+s}\,(1-abq)\,z}{(1-a)\,(1-b)\,(1-aq)\,(1-bq)}\,\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)}\,{}_{r+1}\phi_{s+1}\left[\begin{matrix}abq^2,(qA)\\ aq^2,bq^2,abq,(qB)\end{matrix};q,q^{1-r+s}z\right]
$$

Usage: `Expr/.C51[n1,n2]`.
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C52

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};q,z\right]\longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ a/q,b/q,(B)\end{matrix};q,\frac{z}{q}\right]
-\frac{(-1)^{1-r+s}\left(1-\frac{ab}{q}\right)z}{(1-a)\,(1-b)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q}\,\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)}\,{}_{r+1}\phi_{s+1}\left[\begin{matrix}ab,(qA)\\ aq,bq,ab/q,(qB)\end{matrix};q,q^{-r+s}z\right]
$$

Usage: `Expr/.C52[n1,n2]`.
`m1`, `m2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C53

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,c,(A)\\ (B)\end{matrix};q,z\right]\longrightarrow \frac{(1-b)\left(a-\frac{c}{q}\right)}{(a-b)\left(1-\frac{c}{q}\right)}{}_r\phi_s\left[\begin{matrix}a,bq,c/q,(A)\\ (B)\end{matrix};q,z\right]+\frac{(1-a)\left(b-\frac{c}{q}\right)}{(-a+b)\left(1-\frac{c}{q}\right)}{}_r\phi_s\left[\begin{matrix}aq,b,c/q,(A)\\ (B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C53[m1,m2,m3]`.
`m1`, `m2`, `m3` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C54

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ c,(B)\end{matrix};q,z\right]\longrightarrow \frac{(1-b)\,(a-c)}{(a-b)\,(1-c)}{}_r\phi_s\left[\begin{matrix}a,bq,(A)\\ cq,(B)\end{matrix};q,z\right]+\frac{(1-a)\,(b-c)}{(-a+b)\,(1-c)}{}_r\phi_s\left[\begin{matrix}aq,b,(A)\\ cq,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C54[m1,m2,n1]`.
`m1`, `m2` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C55

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, b, (A) \\ c, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(-b+\frac{a}{q}\right)\left(1-\frac{c}{q}\right)}{\left(1-\frac{a}{q}\right)\left(-b+\frac{c}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, b, (A) \\ c/q, (B) \end{matrix}; q, z\right] + \frac{(1-b)\left(\frac{a}{q}-\frac{c}{q}\right)}{\left(1-\frac{a}{q}\right)\left(b-\frac{c}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, bq, (A) \\ c, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C55[m1,m2,n1]`.
`m1`, `m2` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C56

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, (A) \\ b, c, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(a-c)\left(1-\frac{b}{q}\right)}{(1-c)\left(a-\frac{b}{q}\right)} {}_r\phi_s\left[\begin{matrix} a, (A) \\ b/q, cq, (B) \end{matrix}; q, z\right] + \frac{(1-a)\left(-c+\frac{b}{q}\right)}{(1-c)\left(-a+\frac{b}{q}\right)} {}_r\phi_s\left[\begin{matrix} aq, (A) \\ b, cq, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C56[m1,n1,n2]`.
`m1` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C57

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, (A) \\ b, c, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(\frac{a}{q}-\frac{b}{q}\right)\left(1-\frac{c}{q}\right)}{\left(1-\frac{a}{q}\right)\left(-\frac{b}{q}+\frac{c}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, (A) \\ b, c/q, (B) \end{matrix}; q, z\right] + \frac{\left(1-\frac{b}{q}\right)\left(\frac{a}{q}-\frac{c}{q}\right)}{\left(1-\frac{a}{q}\right)\left(\frac{b}{q}-\frac{c}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, (A) \\ b/q, c, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C57[m1,n1,n2]`.
`m1` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C58

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} (A) \\ a, b, c, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(-c+\frac{a}{q}\right)\left(1-\frac{b}{q}\right)}{(1-c)\left(\frac{a}{q}-\frac{b}{q}\right)} {}_r\phi_s\left[\begin{matrix} (A) \\ a, b/q, cq, (B) \end{matrix}; q, z\right] + \frac{\left(1-\frac{a}{q}\right)\left(-c+\frac{b}{q}\right)}{(1-c)\left(-\frac{a}{q}+\frac{b}{q}\right)} {}_r\phi_s\left[\begin{matrix} (A) \\ a/q, b, cq, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C58[n1,n2,n3]`.
`n1`, `n2`, `n3` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C59

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, cq^2/a, b, c/b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(b-\frac{a}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{a}{q}\right)\left(1-\frac{cq}{a}\right)}{}_r\phi_s\left[\begin{matrix} a/q, cq/a, b, c/b, (A) \\ (B) \end{matrix}; q, qz\right]
+\frac{(1-b)\left(1-\frac{c}{b}\right)}{\left(1-\frac{a}{q}\right)\left(1-\frac{cq}{a}\right)}{}_r\phi_s\left[\begin{matrix} a/q, cq/a, bq, cq/b, (A) \\ (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C59[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C60

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, b, c/b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{(b-a)\left(1-\frac{c}{ab}\right)}{}_r\phi_s\left[\begin{matrix} aq, cq/a, b, c/b, (A) \\ (B) \end{matrix}; q, \frac{z}{q}\right]
-\frac{(1-b)\left(1-\frac{c}{b}\right)}{(b-a)\left(1-\frac{c}{ab}\right)}{}_r\phi_s\left[\begin{matrix} a, c/a, bq, cq/b, (A) \\ (B) \end{matrix}; q, \frac{z}{q}\right]
$$

Usage: `Expr/.C60[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C61

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, b, c/b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)(1-b)\,c}{ab\left(1-\frac{c}{aq}\right)\left(1-\frac{c}{bq}\right)q}{}_r\phi_s\left[\begin{matrix} aq, c/aq, bq, c/bq, (A) \\ (B) \end{matrix}; q, z\right]
+\frac{\left(1-\frac{c}{q}\right)\left(1-\frac{c}{abq}\right)}{\left(1-\frac{c}{aq}\right)\left(1-\frac{c}{bq}\right)}{}_{2+r}\phi_{2+s}\left[\begin{matrix} \sqrt{c}\sqrt{q}, -\sqrt{c}\sqrt{q}, a, c/aq, b, c/bq, (A) \\ \sqrt{c}/\sqrt{q}, -\sqrt{c}/\sqrt{q}, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C61[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C62

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, b, cq^2/b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq, cq/a, b/q, c*q/b, (A) \\ (B) \end{matrix}; q, z\right]
$$
$$
+ (-1)^{1-r+s}\left(\frac{b}{q}-a\right)\left(1-\frac{c*q}{ab}\right)(1-cq)\, z\, \frac{\prod_{i=1}^{r-4}(1-A_i)}{\prod_{i=1}^{s}(1-B_i)}\, {}_{1+r}\phi_{1+s}\left[\begin{matrix} aq, cq/a, b, cq^2/b, cq^2, (qA) \\ cq, (qB) \end{matrix}; q, q^{1-r+s}z\right]
$$

Usage: `Expr/.C62[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C63

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, cq^2/a, b, c/b, (A) \\ (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{dq}\right)\left(1-\frac{cq}{ad}\right)}{\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{cq}{a}\right)}\, {}_r\phi_s\left[\begin{matrix} a/q, cq/a, bq, cq/b, (A) \\ (B) \end{matrix}; q, z\right]
$$
$$
+ \frac{\left(1-\frac{c}{d}\right)(1-d)\left(-b+\frac{a}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)d\left(1-\frac{a}{q}\right)\left(1-\frac{cq}{a}\right)}\, {}_{2+r}\phi_{2+s}\left[\begin{matrix} a/q, cq/a, b, c/b, dq, cq/d, (A) \\ d, c/d, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C63[m1,m2,m3,m4,`d`]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C64

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, cq/a, b, (A) \\ c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{c}{ab}\right)\left(b-\frac{a}{q}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{a}{q}\right)}\, {}_r\phi_s\left[\begin{matrix} a/q, c/a, b, (A) \\ c/b, (B) \end{matrix}; q, qz\right]
$$
$$
+ \frac{(1-b)\left(1-\frac{c}{bq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{a}{q}\right)}\, {}_r\phi_s\left[\begin{matrix} a/q, c/a, bq, (A) \\ c/bq, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C64[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

Example(s):

```
In[1]:= ph[{A,B,q*C,D},{E,A*C/D,F},q,z]
```

```
           [ A, B, C q, D                   ]
           |                                |
           |                                |
Out[1]=  φ |    A C          ; q, z         |
           |                                |
        4 3|  E, ---, F                     |
           [      D                         ]
```

```
In[2]:= %/.C64[3,1,4,2]
```

```
                  A
              [ C, -, D, B                  ]
              |    q                        |  C         A
        D   φ |                ; q, q z     | (-; q)  (---; q)
          4 3 | A C                         |  D     1  D q     1
              [ ---, E, F                   ]
                D
Out[2]= --------------------------------------------- +
                                        A
                        (C; q)  (-; q)
                              1  q     1

             A
        [ C, -, D q, B          ]
        |    q                  |           A C
      φ |              ; q, z   | (D; q)  (---; q)
    4 3 |  A C                  |      1  D q     1
        [  ---, E, F            ]
           D q
>    -------------------------------------------
                            A
                   (C; q)  (-; q)
                         1  q     1
```

```
In[3]:= %/.pqaufl
```

```
                                             A
                                         C, -, D, B
             C          A                    q
      (1 - -) D (1 - ---)  phi                      ; q, q z
             D          D q   4 3        A C
                                         ---, E, F
                                          D
Out[3]= ---------------------------------------------- +
                                             A
                                 (1 - C) (1 - -)
                                             q

                                        A
                                    C, -, D q, B
                        A C             q
   (1 - D) (1 - ---)  phi                          ; q, z
                        D q   4 3    A C
                                     ---, E, F
                                     D q
〉      ------------------------------------------
                                        A
                            (1 - C) (1 - -)
                                        q
```

The third, first, and fourth upper parameters in `Out[1]` are `q C`, `A`, and `D`, respectively, the second lower parameter in `Out[1]` is $\frac{\texttt{A C}}{\texttt{D}}$. Hence `C64` can be applied with the replacements $a \to$ `q C`, $b \to$ `D` and $c \to$ `A C`.

See also: `ContigListe, Ers, PosListe.`

C65

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, c/a, b, (A) \\ cq/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)} {}_r\phi_s\left[\begin{matrix} aq, cq/a, b/q, (A) \\ cq^2/b, (B) \end{matrix}; q, z\right] $$

$$ - \frac{\left(\frac{b}{q}-a\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)} {}_r\phi_s\left[\begin{matrix} a, c/a, b/q, (A) \\ cq^2/b, (B) \end{matrix}; q, qz\right] $$

Usage: `Expr/.C65[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C66

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, c/a, b, (A) \\ cq/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{(b-a)\left(1-\frac{c}{ab}\right)} {}_r\phi_s\left[\begin{matrix} aq, cq/a, b, (A) \\ cq/b, (B) \end{matrix}; q, \frac{z}{q}\right] $$

$$ - \frac{(1-b)\left(1-\frac{c}{b}\right)}{(b-a)\left(1-\frac{c}{ab}\right)} {}_r\phi_s\left[\begin{matrix} a, c/a, bq, (A) \\ c/b, (B) \end{matrix}; q, \frac{z}{q}\right] $$

Usage: `Expr/.C66[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C67

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, cq/a, b, (A) \\ c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\,(1-b)\,c}{ab\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}\,{}_r\phi_s\left[\begin{matrix} aq, c/a, bq, (A) \\ cq/b, (B) \end{matrix}; q, z\right]
+ \frac{(1-c)\left(1-\frac{c}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix} \sqrt{c}q, -\sqrt{c}q, a, c/a, b, (A) \\ \sqrt{c}, -\sqrt{c}, cq/b, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C67[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C68

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, b, (A) \\ cq/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{ab\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q}\,{}_r\phi_s\left[\begin{matrix} a/q, cq/a, b/q, (A) \\ c/b, (B) \end{matrix}; q, z\right]
- \frac{ab\left(1-\frac{c}{q}\right)\left(1-\frac{cq}{ab}\right)}{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix} \sqrt{c}\sqrt{q}, -\sqrt{c}\sqrt{q}, a/q, c/a, b/q, (A) \\ \sqrt{c}/\sqrt{q}, -\sqrt{c}/\sqrt{q}, cq/b, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C68[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C69

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, b, (A) \\ cq/b, (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq, cq/a, b/q, (A) \\ cq^2/b, (B) \end{matrix}; q, z\right]
+ \frac{(-1)^{1-r+s}\left(\frac{b}{q}-a\right)(1-cq)\left(1-\frac{cq}{ab}\right)z\,\prod_{i=1}^{r-3}(1-A_i)}{\left(1-\frac{cq}{b}\right)\left(1-\frac{cq^2}{b}\right)\prod_{i=1}^{s-1}(1-B_i)}\,{}_{1+r}\phi_{1+s}\left[\begin{matrix} aq, cq/a, b, cq^2, (qA) \\ cq^3/b, cq, (qB) \end{matrix}; q, q^{1-r+s}z\right]
$$

Usage: `Expr/.C69[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C70

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, cq/a, b, (A) \\ c/b, (B) \end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} a/q, c/a, bq, (A) \\ c/bq, (B) \end{matrix}; q, z\right] $$
$$ - \frac{(-1)^{1-r+s}\,(1-c)\left(1-\frac{c}{ab}\right)\left(b-\frac{a}{q}\right) z}{\left(1-\frac{c}{b}\right)\left(1-\frac{c}{bq}\right)} \frac{\prod_{i=1}^{r-3}(1-A_i)}{\prod_{i=1}^{s-1}(1-B_i)} {}_{1+r}\phi_{1+s}\left[\begin{matrix} a, cq/a, bq, cq, (qA) \\ cq/b, c, (qB) \end{matrix}; q, q^{1-r+s} z\right] $$

Usage: `Expr/.C70[m1,m2,m3,n1].`
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C71

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, cq/a, b, (A) \\ c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-b)\left(1-\frac{c}{ad}\right)\left(1-\frac{c}{bq}\right)\left(1-\frac{a}{dq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{c}{bdq}\right)} {}_r\phi_s\left[\begin{matrix} a/q, c/a, bq, (A) \\ c/bq, (B) \end{matrix}; q, z\right] $$
$$ + \frac{\left(1-\frac{c}{ab}\right)(1-d)\left(-b+\frac{a}{q}\right)\left(1-\frac{c}{dq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right) d\left(1-\frac{a}{q}\right)\left(1-\frac{c}{bdq}\right)} {}_{2+r}\phi_{2+s}\left[\begin{matrix} a/q, c/a, b, dq, c/d, (A) \\ c/b, d, c/dq, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C71[m1,m2,m3,n1,`d`].`
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C72

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, c/a, b, (A) \\ cq/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{bd}\right)}{\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)} {}_r\phi_s\left[\begin{matrix} aq, cq/a, b/q, (A) \\ cq^2/b, (B) \end{matrix}; q, z\right] $$
$$ - \frac{\left(1-\frac{c}{d}\right)(1-d)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right) d\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)} {}_{2+r}\phi_{2+s}\left[\begin{matrix} a, c/a, b/q, dq, cq/d, (A) \\ cq^2/b, d, c/d, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C72[m1,m2,m3,n1,`d`].`
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C73

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, ab, (A) \\ ab/q, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(-1)^{1-r+s} q^{1-r+s}}{\left(1-\frac{ab}{q}\right) z} \frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-3}(1-\frac{A_i}{q})} {}_{r-1}\phi_{s-1}\left[\begin{matrix} a, b, (A/q) \\ (B/q) \end{matrix}; q, q^{-1+r-s} z\right]
$$

$$
- \frac{(-1)^{1-r+s} q^{1-r+s}}{\left(1-\frac{ab}{q}\right) z} \frac{\prod_{i=1}^{s-1}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-3}(1-\frac{A_i}{q})} {}_{r-1}\phi_{s-1}\left[\begin{matrix} a/q, b/q, (A/q) \\ (B/q) \end{matrix}; q, q^{r-s} z\right]
$$

Usage: `Expr/.C73[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C74

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, (A) \\ c/a, c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-\frac{ab}{c})\, c \left(-b+\frac{a}{q}\right)}{ab\,(1-\frac{c}{a})\left(1-\frac{a}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, b, (A) \\ cq/a, c/b, (B) \end{matrix}; q, qz\right]
$$

$$
+ \frac{(1-b)\left(1-\frac{c}{bq}\right)}{(1-\frac{c}{a})\left(1-\frac{a}{q}\right)} {}_r\phi_s\left[\begin{matrix} a/q, bq, (A) \\ cq/a, c/bq, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C74[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C75

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, (A) \\ c/a, c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\, ab \left(1-\frac{c}{aq}\right) q}{(a-b)\, c \left(1-\frac{abq}{c}\right)} {}_r\phi_s\left[\begin{matrix} aq, b, (A) \\ c/aq, c/b, (B) \end{matrix}; q, \frac{z}{q}\right]
$$

$$
- \frac{a\,(1-b)\, b \left(1-\frac{c}{bq}\right) q}{(a-b)\, c \left(1-\frac{abq}{c}\right)} {}_r\phi_s\left[\begin{matrix} a, bq, (A) \\ c/a, c/bq, (B) \end{matrix}; q, \frac{z}{q}\right]
$$

Usage: `Expr/.C75[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C76

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ c/a,c/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{(1-a)\,(1-b)\,c}{ab\,\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{}_r\phi_s\left[\begin{matrix}aq,bq,(A)\\ cq/a,cq/b,(B)\end{matrix};q,z\right]
+\frac{(1-c)\left(1-\frac{c}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{}_{2+r}\phi_{2+s}\left[\begin{matrix}\sqrt{c}q,-\sqrt{c}q,a,b,(A)\\ \sqrt{c},-\sqrt{c},cq/a,cq/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C76[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C77

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ c/a,c/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{ab\left(1-\frac{c}{aq}\right)\left(1-\frac{c}{bq}\right)}{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)}{}_r\phi_s\left[\begin{matrix}a/q,b/q,(A)\\ c/aq,c/bq,(B)\end{matrix};q,z\right]
-\frac{ab\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{q^2}\right)}{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)}{}_{2+r}\phi_{2+s}\left[\begin{matrix}\sqrt{c},-\sqrt{c},a/q,b/q,(A)\\ \sqrt{c}/q,-\sqrt{c}/q,c/a,c/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C77[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C78

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,(A)\\ c/a,c/b,(B)\end{matrix};q,z\right]\longrightarrow{}_r\phi_s\left[\begin{matrix}aq,b/q,(A)\\ c/aq,cq/b,(B)\end{matrix};q,z\right]
+\frac{(-1)^{1-r+s}\,(1-c)\left(1-\frac{c}{ab}\right)\left(-a+\frac{b}{q}\right)z}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{c}{aq}\right)\left(1-\frac{cq}{b}\right)}\frac{\prod_{i=1}^{r-2}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)}{}_{r+1}\phi_{s+1}\left[\begin{matrix}aq,b,cq,(qA)\\ cq/a,cq^2/b,c,(qB)\end{matrix};q,q^{1-r+s}z\right]
$$

Usage: `Expr/.C78[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C79

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a,b,(A) \\ c/a,c/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(1-b)\left(1-\frac{c}{ad}\right)\left(1-\frac{c}{bq}\right)\left(1-\frac{a}{dq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{c}{bdq}\right)}{}_r\phi_s\left[\begin{matrix} a/q,bq,(A) \\ cq/a,c/bq,(B)\end{matrix};q,z\right]
$$
$$
+\frac{\left(1-\frac{c}{ab}\right)(1-d)\left(-b+\frac{a}{q}\right)\left(1-\frac{c}{dq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)d\left(1-\frac{a}{q}\right)\left(1-\frac{c}{bdq}\right)}{}_{2+r}\phi_{2+s}\left[\begin{matrix} a/q,b,dq,c/d,(A) \\ cq/a,c/b,d,c/dq,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C79[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

Example(s):

```
In[1]:= ph[{A,B,B*F/E,D},{E,F},q,z]

                  B F
            [ A, B, ---, D              ]
Out[1]=  phi[       E      ; q, z      ]
        4 2 [                           ]
            [     E, F                  ]

In[2]:= %/.C79[2,3,2,1,x]

                                                      B   B F q
                                                    [ -, -----, A, D          ]
         B F        E        F         B            [ q    E                  ]
      (1 - ---) (1 - -) (1 - -) (1 - ---)  phi      [                ; q, z   ]
          E         q        x        q x  4 2      [         E               ]
                                                    [    F q, -               ]
                                                              q
Out[2]= ------------------------------------------------------------------ +
                                 B        B F        E
                    (1 - F) (1 - -) (1 - ---) (1 - ---)
                                 q        E x       q x
```

```
〉
                                                                  B   B F       B F
                                                                 -, ---, q x, ---, A, D
           E      B F    B       B F                              q   E          x
    (1 - -) (-(---) + -) (1 - ---) (1 - x)  phi                                           ; q, z
           B      E      q       q x        6 4                                 B F
                                                                    F q, E, x, ---
                                                                                 q x
〉   ---------------------------------------------------------------------------------
                                    B        B F         E
                         (1 - F) (1 - -) (1 - ---) (1 - ---) x
                                    q        E x        q x
```

The second and third upper parameters in `Out[1]` are `B`, and $\frac{\texttt{B F}}{\texttt{E}}$, respectively, the second and first lower parameters in `Out[1]` are `F` and `E`, respectively. Hence `C79` can be applied with the replacements $a \to$ `B`, $b \to \frac{\texttt{B F}}{\texttt{E}}$ and $c \to$ `B F`. The `x` replaces d in the right-hand side expression.

See also: `ContigListe, Ers, PosListe.`

C80

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, c/a, (A) \\ b, c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)}{\left(1-\frac{a}{q}\right)\left(1-\frac{c}{aq}\right)} {}_r\phi_s\left[\begin{matrix} a/q, c/aq, (A) \\ b, c/b, (B) \end{matrix}; q, qz\right] + \frac{\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)}{\left(1-\frac{a}{q}\right)\left(1-\frac{c}{aq}\right)} {}_r\phi_s\left[\begin{matrix} a/q, c/aq, (A) \\ b/q, c/bq, (B) \end{matrix}; q, z\right] $$

Usage: `Expr/.C80[m1,m2,n1,n2].`
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C81

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a, c/a, (A) \\ b, c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{(1-b)\left(1-\frac{c}{b}\right)} {}_r\phi_s\left[\begin{matrix} aq, cq/a, (A) \\ bq, cq/b, (B) \end{matrix}; q, z\right] - \frac{(b-a)\left(1-\frac{c}{ab}\right)}{(1-b)\left(1-\frac{c}{b}\right)} {}_r\phi_s\left[\begin{matrix} a, c/a, (A) \\ bq, cq/b, (B) \end{matrix}; q, qz\right] $$

Usage: `Expr/.C81[m1,m2,n1,n2].`
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C82

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, cq^2/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{\left(\frac{b}{q}-a\right)\left(1-\frac{cq}{ab}\right)}\,{}_r\phi_s\left[\begin{matrix}aq, cq/a, (A)\\ b, cq^2/b, (B)\end{matrix}; q, \frac{z}{q}\right] - \frac{\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)}{\left(\frac{b}{q}-a\right)\left(1-\frac{cq}{ab}\right)}\,{}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b/q, cq/b, (B)\end{matrix}; q, \frac{z}{q}\right] $$

Usage: `Expr/.C82[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C83

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\,c\left(1-\frac{b}{q}\right)}{ab\left(1-\frac{c}{b}\right)\left(1-\frac{c}{aq}\right)}\,{}_r\phi_s\left[\begin{matrix}aq, c/aq, (A)\\ b/q, cq/b, (B)\end{matrix}; q, z\right] + \frac{\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{q}\right)}{\left(1-\frac{c}{b}\right)\left(1-\frac{c}{aq}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix}\sqrt{c}\sqrt{q}, -\sqrt{c}\sqrt{q}, a, c/aq, (A)\\ \sqrt{c}/\sqrt{q}, -\sqrt{c}/\sqrt{q}, b, cq/b, (B)\end{matrix}; q, z\right] $$

Usage: `Expr/.C83[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C84

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}aq, cq/a, (A)\\ bq, cq/b, (B)\end{matrix}; q, z\right] + \frac{(-1)^{1-r+s}(b-a)\left(1-\frac{c}{ab}\right)(1-cq)\,z}{(1-b)\left(1-\frac{c}{b}\right)(1-bq)\left(1-\frac{cq}{b}\right)}\,\frac{\prod_{i=1}^{r-2}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)}\,{}_{1+r}\phi_{1+s}\left[\begin{matrix}aq, cq/a, cq^2, (qA)\\ bq^2, cq^2/b, cq, (qB)\end{matrix}; q, q^{1-r+s}z\right] $$

Usage: `Expr/.C84[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C85

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}a/q, c/aq, (A)\\ b/q, c/bq, (B)\end{matrix}; q, z\right]
$$
$$
- \frac{(-1)^{1-r+s}\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right) z}{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)} \frac{\prod_{i=1}^{r-2}(1-A_i)}{\prod_{i=1}^{s-2}(1-B_i)} {}_{1+r}\phi_{1+s}\left[\begin{matrix}a, c/a, c, (qA)\\ bq, cq/b, c/q, (qB)\end{matrix}; q, q^{1-r+s} z\right]
$$

Usage: `Expr/.C85[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C86

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)\left(1-\frac{a}{dq}\right)\left(1-\frac{c}{adq}\right)}{\left(1-\frac{a}{q}\right)\left(1-\frac{c}{aq}\right)\left(1-\frac{b}{dq}\right)\left(1-\frac{c}{bdq}\right)} {}_r\phi_s\left[\begin{matrix}a/q, c/aq, (A)\\ b/q, c/bq, (B)\end{matrix}; q, z\right]
$$
$$
+ \frac{\left(1-\frac{c}{ab}\right)(1-d)\left(1-\frac{c}{dq^2}\right)\left(\frac{a}{q}-\frac{b}{q}\right)}{d\left(1-\frac{a}{q}\right)\left(1-\frac{c}{aq}\right)\left(1-\frac{b}{dq}\right)\left(1-\frac{c}{bdq}\right)} {}_{2+r}\phi_{2+s}\left[\begin{matrix}a/q, c/aq, dq, c/dq, (A)\\ b, c/b, d, c/dq^2, (B)\end{matrix}; q, z\right]
$$

Usage: `Expr/.C86[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C87

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a, c/a, (A)\\ b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)}{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)} {}_r\phi_s\left[\begin{matrix}aq, cq/a, (A)\\ bq, cq/b, (B)\end{matrix}; q, z\right]
$$
$$
- \frac{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d} {}_{2+r}\phi_{2+s}\left[\begin{matrix}a, c/a, dq, cq/d, (A)\\ bq, cq/b, d, c/d, (B)\end{matrix}; q, z\right]
$$

Usage: `Expr/.C87[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C88

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, ab/q, (A) \\ b, ab/q^2, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(1-\frac{ab}{q^2}\right)z}\frac{\prod_{i=1}^{s-2}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-2}(1-\frac{A_i}{q})}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a, (A/q) \\ b/q^2, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(1-\frac{ab}{q^2}\right)z}\frac{\prod_{i=1}^{s-2}(1-\frac{B_i}{q})}{\prod_{i=1}^{r-2}(1-\frac{A_i}{q})}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a/q, (A/q) \\ b/q, (B/q) \end{matrix}; q, q^{r-s}z\right]
$$

Usage: `Expr/.C88[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C89

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, (A) \\ c/a, b, cq/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{a}{q}\right)}{}_r\phi_s\left[\begin{matrix} a/q, (A) \\ cq/a, b, cq/b, (B) \end{matrix}; q, qz\right]
$$

$$
+\frac{\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{a}{q}\right)}{}_r\phi_s\left[\begin{matrix} a/q, (A) \\ cq/a, b/q, c/b, (B) \end{matrix}; q, z\right]
$$

Usage: `Expr/.C89[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C90

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, (A) \\ cq/a, b, c/b, (B) \end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)}{(1-b)\left(1-\frac{c}{b}\right)}{}_r\phi_s\left[\begin{matrix} aq, (A) \\ c/a, bq, cq/b, (B) \end{matrix}; q, z\right]
$$

$$
-\frac{(b-a)\left(1-\frac{c}{ab}\right)}{(1-b)\left(1-\frac{c}{b}\right)}{}_r\phi_s\left[\begin{matrix} a, (A) \\ cq/a, bq, cq/b, (B) \end{matrix}; q, qz\right]
$$

Usage: `Expr/.C90[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C91

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ c/a,b,cq/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{aq}\right)}{\left(1-\frac{c}{ab}\right)\left(\frac{b}{q}-a\right)}\,{}_r\phi_s\left[\begin{matrix} aq,(A) \\ c/aq,b,cq/b,(B)\end{matrix};q,\frac{z}{q}\right] - \frac{\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)}{\left(1-\frac{c}{ab}\right)\left(\frac{b}{q}-a\right)}\,{}_r\phi_s\left[\begin{matrix} a,(A) \\ c/a,b/q,c/b,(B)\end{matrix};q,\frac{z}{q}\right] $$

Usage: `Expr/.C91[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C92

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ c/a,b,cq/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(1-a)\,c\left(1-\frac{b}{q}\right)q}{ab\left(1-\frac{c}{a}\right)\left(1-\frac{cq}{b}\right)}\,{}_r\phi_s\left[\begin{matrix} aq,(A) \\ cq/a,b/q,cq^2/b,(B)\end{matrix};q,z\right] + \frac{(1-c)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{cq}{b}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix} \sqrt{c}q,-\sqrt{c}q,a,(A) \\ \sqrt{c},-\sqrt{c},cq/a,b,cq^2/b,(B)\end{matrix};q,z\right] $$

Usage: `Expr/.C92[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C93

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix} a,(A) \\ cq/a,b,c/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{ab\left(1-\frac{c}{a}\right)\left(1-\frac{c}{bq}\right)}{(1-b)\,c\left(1-\frac{a}{q}\right)}\,{}_r\phi_s\left[\begin{matrix} a/q,(A) \\ c/a,bq,c/bq,(B)\end{matrix};q,z\right] - \frac{ab\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{q}\right)}{(1-b)\,c\left(1-\frac{a}{q}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix} \sqrt{c}\sqrt{q},-\sqrt{c}\sqrt{q},a/q,(A) \\ \sqrt{c}/\sqrt{q},-\sqrt{c}/\sqrt{q},cq/a,bq,c/b,(B)\end{matrix};q,z\right] $$

Usage: `Expr/.C93[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C94

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a,(A) \\ cq/a,b,c/b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} aq,(A) \\ c/a,bq,cq/b,(B) \end{matrix};q,z\right]
$$
$$
+\frac{(-1)^{1-r+s}\,(b-a)\left(1-\frac{c}{ab}\right)(1-cq)\,z}{(1-b)\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)(1-bq)\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)}\frac{\prod\limits_{i=1}^{r-1}(1-A_i)}{\prod\limits_{i=1}^{s-3}(1-B_i)}\,{}_{1+r}\phi_{1+s}\left[\begin{matrix} aq,cq^2,(qA) \\ cq^2/a,bq^2,cq^2/b,cq,(qB) \end{matrix};q,q^{1-r+s}z\right]
$$

Usage: `Expr/.C94[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C95

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a,(A) \\ c/a,b,cq/b,(B) \end{matrix};q,z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix} a/q,(A) \\ cq/a,b/q,c/b,(B) \end{matrix};q,z\right]
$$
$$
-\frac{(-1)^{1-r+s}\,(1-c)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)z}{(1-b)\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)}\frac{\prod\limits_{i=1}^{r-1}(1-A_i)}{\prod\limits_{i=1}^{s-3}(1-B_i)}\,{}_{1+r}\phi_{1+s}\left[\begin{matrix} a,cq,(qA) \\ cq^2/a,bq,cq^2/b,c,(qB) \end{matrix};q,q^{1-r+s}z\right]
$$

Usage: `Expr/.C95[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C96

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a,(A) \\ c/a,b,cq/b,(B) \end{matrix};q,z\right] \longrightarrow \frac{\left(1-\frac{c}{b}\right)\left(1-\frac{c}{ad}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{a}{dq}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{bd}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{dq}\right)}\,{}_r\phi_s\left[\begin{matrix} a/q,(A) \\ cq/a,b/q,c/b,(B) \end{matrix};q,z\right]
$$
$$
+\frac{(1-d)\left(\frac{a}{q}-\frac{b}{q}\right)\left(1-\frac{c}{dq}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{bd}\right)d\left(1-\frac{a}{q}\right)\left(1-\frac{b}{dq}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix} a/q,dq,c/d,(A) \\ cq/a,b,cq/b,d,c/dq,(B) \end{matrix};q,z\right]
$$

Usage: `Expr/.C96[m1,n1,n2,n3,`d`]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C97

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a,(A) \\ cq/a,b,c/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)}{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)}{}_r\phi_s\left[\begin{matrix} aq,(A) \\ c/a,bq,cq/b,(B)\end{matrix};q,z\right]
$$
$$
-\frac{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d}{}_{2+r}\phi_{2+s}\left[\begin{matrix} a,dq,cq/d,(A) \\ cq/a,bq,cq/b,d,c/d,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C97[m1,n1,n2,n3,`d`]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C98

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} ab/q^2,(A) \\ a,b,ab/q^3,(B)\end{matrix};q,z\right]
$$
$$
\longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(1-\frac{ab}{q^3}\right)z}\frac{\prod_{i=1}^{s-3}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-1}\left(1-\frac{A_i}{q}\right)}{}_{r-1}\phi_{s-1}\left[\begin{matrix} (A/q) \\ a/q^2,b/q^2,(B/q)\end{matrix};q,q^{-1+r-s}z\right]
$$
$$
-\frac{(-1)^{1-r+s}\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}}{\left(1-\frac{ab}{q^3}\right)z}\frac{\prod_{i=1}^{s-3}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-1}\left(1-\frac{A_i}{q}\right)}{}_{r-1}\phi_{s-1}\left[\begin{matrix} (A/q) \\ a/q,b/q,(B/q)\end{matrix};q,q^{r-s}z\right]
$$

Usage: `Expr/.C98[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C99

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} (A) \\ a,c/a,b,cq^2/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{\left(\frac{b}{q}-a\right)\left(1-\frac{cq}{ab}\right)}{(1-a)\left(1-\frac{c}{a}\right)}{}_r\phi_s\left[\begin{matrix} (A) \\ aq,cq/a,b,cq^2/b,(B)\end{matrix};q,qz\right]
$$
$$
+\frac{\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)}{(1-a)\left(1-\frac{c}{a}\right)}{}_r\phi_s\left[\begin{matrix} (A) \\ aq,cq/a,b/q,cq/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C99[n1,n2,n3,n4]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C100

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ a, c/a, b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{a}{q}\right)\left(1-\frac{c}{aq}\right)}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)}\,{}_r\phi_s\left[\begin{matrix}(A)\\ a/q, c/aq, b, c/b, (B)\end{matrix}; q, \frac{z}{q}\right] - \frac{\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)}\,{}_r\phi_s\left[\begin{matrix}(A)\\ a, c/a, b/q, c/bq, (B)\end{matrix}; q, \frac{z}{q}\right]
$$

Usage: `Expr/.C100[n1,n2,n3,n4].`
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C101

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ a, c/a, b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q}{ab\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}\,{}_r\phi_s\left[\begin{matrix}(A)\\ a/q, cq/a, b/q, cq/b, (B)\end{matrix}; q, z\right] + \frac{\left(1-\frac{c}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix}\sqrt{c}\sqrt{q}, -\sqrt{c}\sqrt{q}, (A)\\ \sqrt{c}/\sqrt{q}, -\sqrt{c}/\sqrt{q}, a, cq/a, b, cq/b, (B)\end{matrix}; q, z\right]
$$

Usage: `Expr/.C101[n1,n2,n3,n4].`
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C102

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}(A)\\ a, cq^2/a, b, c/b, (B)\end{matrix}; q, z\right] \longrightarrow {}_r\phi_s\left[\begin{matrix}(A)\\ a/q, cq/a, bq, cq/b, (B)\end{matrix}; q, z\right] + \frac{(-1)^{1-r+s}\left(b-\frac{a}{q}\right)(1-cq)\left(1-\frac{cq}{ab}\right)z}{(1-a)(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{q}\right)(1-bq)\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)\left(1-\frac{cq^2}{a}\right)}\,\frac{\prod_{i=1}^{r}(1-A_i)}{\prod_{i=1}^{s-4}(1-B_i)}\,{}_{1+r}\phi_{1+s}\left[\begin{matrix}cq^2, (qA)\\ aq, cq^3/a, bq^2, cq^2/b, cq, (qB)\end{matrix}; q, q^{1-r+s}z\right]
$$

Usage: `Expr/.C102[n1,n2,n3,n4].`
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C103

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}(A)\\ a,c/a,b,cq^2/b,(B)\end{matrix};q,z\right] \longrightarrow \frac{\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)}{\left(1-a\right)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{bd}\right)}\,{}_r\phi_s\left[\begin{matrix}(A)\\ aq,cq/a,b/q,cq/b,(B)\end{matrix};q,z\right] $$
$$ +\frac{\left(1-\frac{c}{d}\right)\left(1-d\right)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{\left(1-a\right)\left(1-\frac{c}{a}\right)d\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{bd}\right)}\,{}_{2+r}\phi_{2+s}\left[\begin{matrix}dq,cq/d,(A)\\ aq,cq/a,b,cq^2/b,d,c/d,(B)\end{matrix};q,z\right] $$

Usage: `Expr/.C103[n1,n2,n3,n4,`d`]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C104

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a,c/a,b,c/b,c,(A)\\ c/q,(B)\end{matrix};q,z\right] $$
$$ \longrightarrow \frac{(-1)^{1-r+s}q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-1}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-5}\left(1-\frac{A_i}{q}\right)}\,{}_{r-1}\phi_{s-1}\left[\begin{matrix}a/q,c/aq,b,c/b,(A/q)\\ (B/q)\end{matrix};q,q^{-1+r-s}z\right] $$
$$ -\frac{(-1)^{1-r+s}q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-1}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-5}\left(1-\frac{A_i}{q}\right)}\,{}_{r-1}\phi_{s-1}\left[\begin{matrix}a,c/a,b/q,c/bq,(A/q)\\ (B/q)\end{matrix};q,q^{-1+r-s}z\right] $$

Usage: `Expr/.C104[m1,m2,m3,m4,m5,n1]`.
`m1`, `m2`, `m3`, `m4`, `m5` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C105

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a,c/a,b,c,(A)\\ cq/b,c/q,(B)\end{matrix};q,z\right] $$
$$ \longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{c}{b}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-2}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-4}\left(1-\frac{A_i}{q}\right)}\,{}_{r-1}\phi_{s-1}\left[\begin{matrix}a/q,c/aq,b,(A/q)\\ c/bq,(B/q)\end{matrix};q,q^{-1+r-s}z\right] $$
$$ -\frac{(-1)^{1-r+s}\left(1-\frac{c}{b}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-2}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-4}\left(1-\frac{A_i}{q}\right)}\,{}_{r-1}\phi_{s-1}\left[\begin{matrix}a,c/a,b/q,(A/q)\\ c/b,(B/q)\end{matrix};q,q^{-1+r-s}z\right] $$

Usage: `Expr/.C105[m1,m2,m3,m4,n1,n2]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C106

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, b, c, (A) \\ cq/a, cq/b, c/q, (B) \end{matrix}; q, z\right] \longrightarrow
$$

$$
\frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{c}{aq}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-3} 1-\frac{B_i}{q}}{\prod_{i=1}^{r-3} 1-\frac{A_i}{q}}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a/q, b, (A/q) \\ c/a, c/bq, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{c}{aq}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-3} 1-\frac{B_i}{q}}{\prod_{i=1}^{r-3} 1-\frac{A_i}{q}}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a, b/q, (A/q) \\ c/aq, c/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

Usage: `Expr/.C106[m1,m2,m3,n1,n2,n3].` `m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C107

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c/a, c, (A) \\ bq, cq/b, c/q, (B) \end{matrix}; q, z\right]
$$

$$
\longrightarrow \frac{(-1)^{1-r+s}(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-3}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-3}\left(1-\frac{A_i}{q}\right)}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a/q, c/aq, (A/q) \\ b/q, c/bq, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{c}{bq}\right)q^{1-r+s}}{\left(1-\frac{c}{ab}\right)\left(-\frac{a}{q}+\frac{b}{q}\right)\left(1-\frac{c}{q}\right)z}\frac{\prod_{i=1}^{s-3}\left(1-\frac{B_i}{q}\right)}{\prod_{i=1}^{r-3}\left(1-\frac{A_i}{q}\right)}{}_{r-1}\phi_{s-1}\left[\begin{matrix} a, c/a, (A/q) \\ b, c/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

Usage: `Expr/.C107[m1,m2,m3,n1,n2,n3].`
`m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C108

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} a, c, (A) \\ cq/a, b, cq^2/b, c/q, (B) \end{matrix}; q, z\right]
$$

$$
\longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{c}{aq}\right)q^{1-r+s}\left(1-\frac{cq}{b}\right)\prod_{i=1}^{s-4}\left(1-\frac{B_i}{q}\right)}{\left(\frac{b}{q^2}-\frac{a}{q}\right)\left(1-\frac{c}{q}\right)\left(1-\frac{cq}{ab}\right)z\prod_{i=1}^{r-2}\left(1-\frac{A_i}{q}\right)}
$$

$$
{}_{r-1}\phi_{s-1}\left[\begin{matrix} a/q, (A/q) \\ c/a, b/q^2, c/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{b}{q}\right)\left(1-\frac{c}{aq}\right)q^{1-r+s}\left(1-\frac{cq}{b}\right)\prod_{i=1}^{s-4}\left(1-\frac{B_i}{q}\right)}{\left(\frac{b}{q^2}-\frac{a}{q}\right)\left(1-\frac{c}{q}\right)\left(1-\frac{cq}{ab}\right)z\prod_{i=1}^{r-2}\left(1-\frac{A_i}{q}\right)}
$$

$$
{}_{r-1}\phi_{s-1}\left[\begin{matrix} a, (A/q) \\ c/aq, b/q, cq/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

Usage: `Expr/.C108[m1,m2,n1,n2,n3,n4]`.
`m1`, `m2` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C109

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix} c, (A) \\ a, cq^2/a, b, cq^2/b, c/q, (B) \end{matrix}; q, z\right]
$$

$$
\longrightarrow \frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)\prod_{i=1}^{s-5}\left(1-\frac{B_i}{q}\right)}{\left(-\frac{a}{q^2}+\frac{b}{q^2}\right)\left(1-\frac{c}{q}\right)\left(1-\frac{cq^2}{ab}\right)z\prod_{i=1}^{r-1}\left(1-\frac{A_i}{q}\right)}
$$

$$
{}_{r-1}\phi_{s-1}\left[\begin{matrix} (A/q) \\ a/q, cq/a, b/q^2, c/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

$$
-\frac{(-1)^{1-r+s}\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{q^2}\right)\left(1-\frac{b}{q^2}\right)\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^{1-r+s}\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)\prod_{i=1}^{s-5}\left(1-\frac{B_i}{q}\right)}{\left(-\frac{a}{q^2}+\frac{b}{q^2}\right)\left(1-\frac{c}{q}\right)\left(1-\frac{cq^2}{ab}\right)z\prod_{i=1}^{r-1}\left(1-\frac{A_i}{q}\right)}
$$

$$
{}_{r-1}\phi_{s-1}\left[\begin{matrix} (A/q) \\ a/q^2, c/a, b/q, cq/b, (B/q) \end{matrix}; q, q^{-1+r-s}z\right]
$$

Usage: `Expr/.C109[m1,n1,n2,n3,n4,n5]`.
`m1` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C110

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}\sqrt{c}q, -\sqrt{c}q, a, c/a, b, c/b, (A)\\ \sqrt{c}, -\sqrt{c}, (B)\end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{(1-c)\left(1-\frac{c}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}a, cq/a, b, cq/b, (A)\\ (B)\end{matrix}; q, z\right] - \frac{(1-a)(1-b)\,c}{ab\,(1-c)\left(1-\frac{c}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq, c/a, bq, c/b, (A)\\ (B)\end{matrix}; q, z\right] $$

Usage: `Expr/.C110[m1,m2,m3,m4,m5,m6,n1,n2].`
`m1`, `m2`, `m3`, `m4`, `m5`, `m6` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C111

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a, c/a, b, c/b, dq, cq/d, (A)\\ d, c/d, (B)\end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq, cq/a, b, c/b, (A)\\ (B)\end{matrix}; q, z\right] - \frac{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}a, c/a, bq, cq/b, (A)\\ (B)\end{matrix}; q, z\right] $$

Usage: `Expr/.C111[m1,m2,m3,m4,m5,m6,n1,n2].`
`m1`, `m2`, `m3`, `m4`, `m5`, `m6` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C112

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}\sqrt{c}q, -\sqrt{c}q, a, c/a, b, (A)\\ \sqrt{c}, -\sqrt{c}, cq/b, (B)\end{matrix}; q, z\right] \longrightarrow \frac{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{(1-c)\left(1-\frac{c}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}a, cq/a, b, (A)\\ c/b, (B)\end{matrix}; q, z\right] - \frac{(1-a)(1-b)\,c}{ab\,(1-c)\left(1-\frac{c}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq, c/a, bq, (A)\\ cq/b, (B)\end{matrix}; q, z\right] $$

Usage: `Expr/.C112[m1,m2,m3,m4,m5,n1,n2,n3].`
`m1`, `m2`, `m3`, `m4`, `m5` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C113

Description: Contiguous relation in form of a rule.

$$ {}_r\phi_s\left[\begin{matrix}a, c/a, b, dq, cq/d, (A)\\ cq/b, d, c/d, (B)\end{matrix}; q, z\right] \longrightarrow \frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq, cq/a, b, (A)\\ cq/b, (B)\end{matrix}; q, z\right] - \frac{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix}a, c/a, bq, (A)\\ c/b, (B)\end{matrix}; q, z\right] $$

Usage: `Expr/.C113[m1,m2,m3,m4,m5,n1,n2,n3].`
`m1`, `m2`, `m3`, `m4`, `m5` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C114

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}\sqrt{c}q,-\sqrt{c}q,a,b,(A)\\ \sqrt{c},-\sqrt{c},cq/a,cq/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{\left(1-\frac{c}{a}\right)\left(1-\frac{c}{b}\right)}{(1-c)\left(1-\frac{c}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,b,(A)\\ c/a,c/b,(B)\end{matrix};q,z\right]
-\frac{(1-a)(1-b)\,c}{ab\,(1-c)\left(1-\frac{c}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,bq,(A)\\ cq/a,cq/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C114[m1,m2,m3,m4,n1,n2,n3,n4]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C115

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}\sqrt{c}q,-\sqrt{c}q,a,c/a,(A)\\ \sqrt{c},-\sqrt{c},b,cq^2/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{\left(1-\frac{c}{a}\right)\left(1-\frac{cq}{b}\right)}{(1-c)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,cq/a,(A)\\ b,cq/b,(B)\end{matrix};q,z\right]
-\frac{(1-a)\,c\left(1-\frac{b}{q}\right)q}{ab\,(1-c)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,c/a,(A)\\ b/q,cq^2/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C115[m1,m2,m3,m4,n1,n2,n3,n4]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C116

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,b,dq,cq/d,(A)\\ cq/a,cq/b,d,c/d,(B)\end{matrix};q,z\right]\longrightarrow\frac{(1-a)\left(1-\frac{c}{a}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{bd}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,b,(A)\\ c/a,cq/b,(B)\end{matrix};q,z\right]
-\frac{(1-b)\left(1-\frac{c}{b}\right)\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d}{(a-b)\left(1-\frac{c}{ab}\right)\left(1-\frac{c}{d}\right)(1-d)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,bq,(A)\\ cq/a,c/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C116[m1,m2,m3,m4,n1,n2,n3,n4]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C117

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,c/a,dq,cq/d,(A)\\ b,cq^2/b,d,c/d,(B)\end{matrix};q,z\right]\longrightarrow\frac{(1-a)\left(1-\frac{c}{a}\right)d\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{bd}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,cq/a,(A)\\ b,cq^2/b,(B)\end{matrix};q,z\right]
-\frac{\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,c/a,(A)\\ b/q,cq/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C117[m1,m2,m3,m4,n1,n2,n3,n4]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C118

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}\sqrt{c}q,-\sqrt{c}q,a,(A)\\ \sqrt{c},-\sqrt{c},cq/a,b,cq^2/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{\left(1-\frac{c}{a}\right)\left(1-\frac{cq}{b}\right)}{(1-c)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,(A)\\ c/a,b,cq/b,(B)\end{matrix};q,z\right]
-\frac{(1-a)\,c\left(1-\frac{b}{q}\right)q}{ab\,(1-c)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,(A)\\ cq/a,b/q,cq^2/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C118[m1,m2,m3,n1,n2,n3,n4,n5].`
`m1`, `m2`, `m3` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C119

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}a,dq,cq/d,(A)\\ cq/a,b,cq^2/b,d,c/d,(B)\end{matrix};q,z\right]\longrightarrow\frac{(1-a)\left(1-\frac{c}{a}\right)d\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{bd}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}aq,(A)\\ c/a,b,cq^2/b,(B)\end{matrix};q,z\right]
-\frac{\left(1-\frac{a}{d}\right)\left(1-\frac{c}{ad}\right)d\left(1-\frac{b}{q}\right)\left(1-\frac{cq}{b}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(a-\frac{b}{q}\right)\left(1-\frac{cq}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}a,(A)\\ cq/a,b/q,cq/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C119[m1,m2,m3,n1,n2,n3,n4,n5].`
`m1`, `m2`, `m3` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C120

Description: Contiguous relation in form of a rule.

$$
{}_r\phi_s\left[\begin{matrix}\sqrt{c}q,-\sqrt{c}q,(A)\\ \sqrt{c},-\sqrt{c},a,cq^2/a,b,cq^2/b,(B)\end{matrix};q,z\right]\longrightarrow\frac{\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{b}\right)}{(1-c)\left(1-\frac{cq^2}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}(A)\\ a,cq/a,b,cq/b,(B)\end{matrix};q,z\right]
-\frac{c\left(1-\frac{a}{q}\right)\left(1-\frac{b}{q}\right)q^2}{ab\,(1-c)\left(1-\frac{cq^2}{ab}\right)}{}_{r-2}\phi_{s-2}\left[\begin{matrix}(A)\\ a/q,cq^2/a,b/q,cq^2/b,(B)\end{matrix};q,z\right]
$$

Usage: `Expr/.C120[m1,m2,n1,n2,n3,n4,n5,n6].`
`m1`, `m2` and `n1`, `n2`, `n3`, `n4`, `n5`, `n6` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C121

Description: Contiguous relation in form of a rule.

$$
\begin{aligned}
&{}_r\phi_s\left[\begin{matrix} dq, cq/d, (A) \\ a, cq^2/a, b, cq^2/b, d, c/d, (B)\end{matrix}; q, z\right] \\
&\quad\longrightarrow \frac{d\left(1-\frac{a}{q}\right)\left(1-\frac{b}{dq}\right)\left(1-\frac{cq}{a}\right)\left(1-\frac{cq}{bd}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(\frac{a}{q}-\frac{b}{q}\right)\left(1-\frac{cq^2}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix} (A) \\ a/q, cq/a, b, cq^2/b, (B)\end{matrix}; q, z\right] \\
&\qquad - \frac{d\left(1-\frac{b}{q}\right)\left(1-\frac{a}{dq}\right)\left(1-\frac{cq}{b}\right)\left(1-\frac{cq}{ad}\right)}{\left(1-\frac{c}{d}\right)(1-d)\left(\frac{a}{q}-\frac{b}{q}\right)\left(1-\frac{cq^2}{ab}\right)}\,{}_{r-2}\phi_{s-2}\left[\begin{matrix} (A) \\ a, cq^2/a, b/q, cq/b, (B)\end{matrix}; q, z\right]
\end{aligned}
$$

Usage: `Expr/.C121[m1,m2,n1,n2,n3,n4,n5,n6]`.
`m1`, `m2` and `n1`, `n2`, `n3`, `n4`, `n5`, `n6` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe`.

ContigListe

Description: List of all contiguous relations.

Usage: `ContigListe`.

Div

Description: Function that divides `Gleichung` by `Expr`.

Usage: `Div[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                    n   c
                                   a  (-; q)
                -n                      a    n
Out[1]=  phi [ a, q   ; q, q ]  == ----------
         2 1                        (c; q)
                c                         n

In[2]:= Div[a^n]

                -n
         phi [ a, q   ; q, q ]      c
         2 1                       (-; q)
                c                   a    n
Out[2]= -------------------- == -------
                  n               (c; q)
                 a                     n
```

```
In[3]:= Gleichung

            [          ]
            [    -n    ]
          φ [ a, q  ; q, q ]     c
        2 1 [          ]      (-; q)
            [    c     ]       a   n
Out[3]= ------------------- == -------
                n              (c; q)
               a                    n
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Sub, Hoch, GlTausche, Ers.`

Drucke

Description: Function that directly sends an expression `Expr` in the Form `PrintedForm` to the printer. This function only works for DOS-machines with a printer directly connected. `PrintedForm` is an optional parameter which can be any of the format types (`InputForm`, `OutputForm`, `TeXForm`, ...). The default is `OutputForm`.

Usage: `Drucke[Expr,PrintedForm].`

See also: `TeXMat, TeX, TeXphW.`

Ers

Description: Function for controlled application of rules and functions.

Usage: `Ers[Expr,Rules,PosList].`
`Rules` can be a rule, a list of rules, or a function. `PosList` must be a list of positions in `Expr` to which `Rules` should be applied. For instance, if `PosList=`$\{\{1,2\},\{4\}\}$, then `Rules` is applied to `Expr[[1,2]]` and `Expr[[4]]` in `Expr`. If `PosList=`$\{2,3\}$, then `Rules` is applied to `Expr[[2]]` and `Expr[[3]]` in `Expr`. The positions of subexpressions can be determined by the function `PosListe`. If `Ers` is applied to an equation then the new left-hand and right-hand sides are automatically stored in the variables `LS` and `RS`.

There is an exceptional usage of `Ers`, namely
`Ers[Rules]`.
In this case the `Rules` are applied to both sides of the equation that is currently stored in `Gleichung`. Again, the new left-hand and right-hand sides are automatically stored in the variables `LS` and `RS`.

Example(s):

```
In[1]:= (-1)^n*pq[a*q,n]*pq[c,k]/pq[1/b,m]/pq[q/d,l]

             n
        (-1)  (c; q)  (a q; q)
                    k         n
Out[1]= -----------------------
              1       q
             (-; q)  (-; q)
              b    m  d    l

In[2]:= PosListe[%]
```

```
                 n                  1
Out[2]= {{(-1) , {{1}}}, {-------, {{2}}}, {(c; q) , {{3}}},
                             1                          k
                           (-; q)
                            b    m

                                 1
>     {(a q; q) , {{4}}}, {-------, {{5}}}}
              n             q
                           (-; q)
                            d    l

In[3]:= Ers[%%,neg1,{5}]

                                       1 + l
              n                       q
         (-1)  (c; q)  (a q; q)  (------; q)
                     k          n    d       -l
Out[3]= ------------------------------------
                              1
                            (-; q)
                             b    m

In[4]:= Ers[%%%,neg1,{2,4}]

                              m
              n             q
         (-1)  (c; q)  (--; q)
                     k  b     -m
Out[4]= -----------------------
          q          1 + n
         (-; q)  (a q      ; q)
          d    l              -n

In[5]:= Sum[%,{k,0,Infinity}]

                                            1 + l
          ∞      n                         q
         ____(-1)  (c; q)  (a q; q)  (------; q)
          \                      k          n    d       -l
Out[5]=    >   ------------------------------------
          /                              1
         ----                          (-; q)
          k=0                           b    m

In[6]:= PosListe[%,2]
```

```
                            n
Out[6]= {{0, {{2, 2}}}, {(-1) , {{1, 1}}}, {k, {{2, 1}}}, {∞, {{2, 3}}},

          1
>     {-------, {{1, 2}}}, {(c; q) , {{1, 3}}}, {(a q; q) , {{1, 4}}},
        1                            k                    n
       (-; q)
        b    m

         1 + l
        q
>     {(------; q)  , {{1, 5}}}}
          d       -l

In[7]:= Ers[%%,neg1,{{1,3}}]

                                  1 + l
          ∞      n               q
         ----(-1)   (a q; q)   (------; q)
          \                n     d       -l
Out[7]=    >   ---------------------------
          /         1           k
         ----     (-; q)   (c q  ; q)
          k=0       b    m          -k

In[8]:= Ers[%%%,neg1,{{1,2},{1,5}}]

                                                m
          ∞      n                            q
         ----(-1)   (c; q)   (a q; q)   (--; q)
          \                k          n  b     -m
Out[8]=    >   -------------------------------
          /                  q
         ----              (-; q)
          k=0               d    l

In[9]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                           n  c
                                          a  (-; q)
                  -n                          a   n
Out[9]=  phi [ a, q   ; q, q ] == ---------
         2 1                        (c; q)
                 c                         n

In[10]:= Ers[a->q/a]
```

```
                                n  a c
               ┌ q   -n       ┐ q  (---; q)
               │ -, q         │       q    n
Out[10]=  φ    │ a     ; q, q │ == -----------
         2 1   │              │     n
               └   c          ┘    a  (c; q)
                                            n
```

```
In[11]:= Gleichung
```

```
                                n  a c
               ┌ q   -n       ┐ q  (---; q)
               │ -, q         │       q    n
Out[11]=  φ    │ a     ; q, q │ == -----------
         2 1   │              │     n
               └   c          ┘    a  (c; q)
                                            n
```

```
In[12]:= PQ
```

```
In[13]:= pq[a,4]+1/pq[b,3]
```

```
                      1                                            2          3
Out[13]= ---------------------------- + (1 - a) (1 - a q) (1 - a q ) (1 - a q )
                                    2
         (1 - b) (1 - b q) (1 - b q )
```

```
In[14]:= Ers[%,Expand,{2}]
```

```
                           2        2    2  2      3      2  3    3  3    2  4
Out[14]= 1 - a - a q + a  q - a q  + a  q  - a q  + 2 a  q  - a  q  + a  q  -

      3  4    2  5    3  5    3  6    4  6                 1
>    a  q  + a  q  - a  q  - a  q  + a  q  + ----------------------------
                                                                       2
                                             (1 - b) (1 - b q) (1 - b q )
```

See also: `PosListe`, `ManipulationsListe`, `Subst`.

---

erw1

Description: $(a;q)_n \to (a;q)_{m+n}/(aq^n;q)_m$,
$(a;q)_n \to (a;q)_\infty/(aq^n;q)_\infty$.

The parameter `m` has to be entered on request. To apply the second rule, `m` has to be `Infinity`.

Usage: `Expr/.erw1`.

Example(s):

```
In[1]:= pq[a,n,q^2]
```

```
               2
Out[1]= (a; q )
                 n

In[2]:= %/.erw1
top-extend by:  m

               2
        (a; q )
                 m + n
Out[2]= -------------
              2 n   2
        (a q    ; q )
                     m

In[3]:= pq[a,m,1/q]

              1
Out[3]= (a; -)
              q m

In[4]:= %/.erw1
top-extend by:  Infinity

             1
        (a;-)
             q ∞
Out[4]= -------
          a  1
        (--;-)
          m q ∞
         q
```

See also: `erw2, zus1, zus2, zus3, Ers, PosListe, ManipulationsListe.`

---

erw2

Description: $(a;q)_n \to (a/q^m;q)_{m+n}/(a/q^m;q)_m$,
$(a;q)_\infty \to (a/q^m;q)_\infty/(a/q^m;q)_m$.

The parameter `m` has to be entered on request.

Usage: `Expr/.erw2.`

Example(s):

```
In[1]:= pq[a,n,q^2]

               2
Out[1]= (a; q )
                 n
```

```
In[2]:= %/.erw2
bottom-extend by:  m

           a     2
         (----; q )
           2 m     m + n
          q
Out[2]= ---------------
             a     2
           (----; q )
             2 m     m
            q

In[3]:= pqinf[b,1/q]

            1
Out[3]= (b;-)
           q ∞

In[4]:= %/.erw2
bottom-extend by:  n

             n 1
         (b q ;-)
               q ∞
Out[4]= ----------
             n  1
         (b q ; -)
                q n
```

See also: `erw1, zus1, zus2, zus3, Ers, PosListe, ManipulationsListe.`

Expandq

Description: Rule that expands all the exponents in powers.

Usage: `Expr/.Expandq.`

Example(s):

```
In[1]:= pq[a*q^3,n]/.trans

                              2      -2 - n
            n  n  3 n - (n - n )/2  q
Out[1]= (-1)  a  q                 (-------; q)
                                       a       n

In[2]:= %/.Expandq
```

```
                                2      -2 - n
           n   n  (5 n)/2 + n /2  q
Out[2]= (-1)  a  q                 (-------; q)
                                      a       n
```

See also: `SimplifyPQ, MinusOne, SumExpand, Ers, PosListe.`

Factorialpq

Description: `Factorialpq[n,k,q]` is the usual q-factorial, written in terms of q-factorial symbols `pq`. The parameter `q` is optional. It will be set equal `q` if it is omitted.

Usage: `Factorialpq[n,q]`
or: `Factorialpq[n].`

Example(s):

```
In[1]:= Factorialpq[n]

        (q; q)
             n
Out[1]= --------
              n
        (1 - q)

In[2]:= Factorialpq[5]

        (q; q)
             5
Out[2]= --------
              5
        (1 - q)
```

See also: `Binomialq, Binomialpq, Factorialq.`

Factorialq

Description: `Factorialq[n,k,q]` is the usual q-factorial, expanded into a q-series, if possible. The parameter `q` is optional. It will be set equal `q` if it is omitted.

Usage: `Factorialq[n,q]`
or: `Factorialq[n].`

Example(s):

```
In[1]:= Factorialq[n]

Out[1]= [ n ] !

            q

In[2]:= Factorialq[5]
```

Out[2]= $1 + 4\,q + 9\,q^{2} + 15\,q^{3} + 20\,q^{4} + 22\,q^{5} + 20\,q^{6} + 15\,q^{7} + 9\,q^{8} + 4\,q^{9} + q^{10}$

See also: `Binomialq`, `Binomialpq`, `Factorialpq`.

Gleichung

Description: Is a variable which stores equations. The equation `Gleichung` can be manipulated using the functions `Add`, `Sub`, `Mal`, `Div`, `Hoch`, `GlTausche`, `Ers`, and `Sum[k,m,n]`, where `m` and `n` are integers or variables. The last command causes the equation to be summed over `k` from `m` to `n`. The parameter `k` is optional. It will be set `kk`, `ii`, `jj`, `ll`, `mm`, or `nn`, if it is omitted.

Usage: `Gleichung`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n
```

Out[1]= $${}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == \frac{a^{n}\left(\frac{c}{a};\ q\right)_n}{(c;\ q)_n}$$

```
In[2]:= Gleichung
```

Out[2]= $${}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == \frac{a^{n}\left(\frac{c}{a};\ q\right)_n}{(c;\ q)_n}$$

```
In[3]:= Add[1]
```

Out[3]= $$1 + {}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == 1 + \frac{a^{n}\left(\frac{c}{a};\ q\right)_n}{(c;\ q)_n}$$

```
In[4]:= Gleichung
```

Out[4]= $1 + {}_{2}\phi_{1}\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == 1 + \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$

In[5]:= `LS=5`

Out[5]= 5

In[6]:= `Gleichung`

Out[6]= $5 == 1 + \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$

In[7]:= `Sum[n,0,m]`

Out[7]= $\sum_{n=0}^{m} 5 == \sum_{n=0}^{m} 1 + \sum_{n=0}^{m} \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$

In[8]:= `Sum[0,M]`

Out[8]= $\sum_{kk=0}^{M}\sum_{n=0}^{m} 5 == \sum_{kk=0}^{M}\sum_{n=0}^{m} 1 + \sum_{kk=0}^{M}\sum_{n=0}^{m} \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$

In[9]:= `Gleichung`

```
           M    m          M    m         M    m   n  c
         ┌───┐┌───┐      ┌───┐┌───┐     ┌───┐┌───┐a  (-; q)
          \    \          \    \         \    \       a    n
Out[9]=    >    >  5 ==    >    >  1 +    >    >  ----------
          /    /          /    /         /    /    (c; q)
         └───┘└───┘      └───┘└───┘     └───┘└───┘       n
         kk=0  n=0       kk=0  n=0      kk=0  n=0
```

See also: SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Sub, Hoch, GlTausche, Ers, Subst, PQSort.

GlTausche

Description: `GlTausche` interchanges right-hand and left-hand sides in `Gleichung`.

Usage: `GlTausche`.

Example(s):

```
In[1]:= Sgl3201
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                      -n                  c        c
           ┌   a, b, q                ┐  (-; q)   (-; q)
           │                          │   a    n  b    n
Out[1]=  φ │            1 - n; q, q   │ == ----------------
        3 2│    a b q                 │             c
           └ c, ----------            ┘  (c; q)   (---; q)
                    c                          n  a b    n

In[2]:= Gleichung

                      -n                  c        c
           ┌   a, b, q                ┐  (-; q)   (-; q)
           │                          │   a    n  b    n
Out[2]=  φ │            1 - n; q, q   │ == ----------------
        3 2│    a b q                 │             c
           └ c, ----------            ┘  (c; q)   (---; q)
                    c                          n  a b    n

In[3]:= GlTausche

          c        c                          -n
        (-; q)   (-; q)          ┌   a, b, q                ┐
          a    n  b    n         │                          │
Out[3]= ---------------- ==   φ  │            1 - n; q, q   │
                   c            3 2│    a b q               │
        (c; q)   (---; q)        └ c, ----------            ┘
              n  a b    n                  c
```

In[4]:= Gleichung

$$\text{Out[4]=}\quad \frac{\left(\frac{c}{a};q\right)_n \left(\frac{c}{b};q\right)_n}{(c;q)_n \left(\frac{c}{a\,b};q\right)_n} == {}_3\phi_2\left[\begin{matrix} a,\ b,\ q^{-n} \\ c,\ \frac{a\,b\,q^{1-n}}{c} \end{matrix};\ q,\ q\right]$$

See also: Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Sub, Hoch, Ers, Subst.

Hoch

Description: Function that takes `Gleichung` to the `Expr`-th power.

Usage: `Hoch[Expr]`.

Example(s):

In[1]:= Sgl2101
Do you want to set values for the equation? [y|n]: n
Do you want to set a value for q in the equation? [y|n]: n

$$\text{Out[1]=}\quad {}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == \frac{a^n \left(\frac{c}{a};q\right)_n}{(c;q)_n}$$

In[2]:= Hoch[3]

$$\text{Out[2]=}\quad {}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right]^3 == \frac{a^{3n} \left(\frac{c}{a};q\right)_n^3}{(c;q)_n^3}$$

In[3]:= Gleichung

```
                                    3 n  c      3
                                   a    (-; q)
                -n                       a    n
Out[3]=  ϕ [ a, q   ; q, q ]3  ==  -------------
        2 1                                3
              c                     (c; q)
                                          n
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Sub, GlTausche, Ers.`

---

hypqAttributes

Description: Shows the current setup of the session. The setup can be changed by the switches `PQ`, `phCancel`, `TeX`, and `TeXphW`. The default-setup is shown in the following Example.

Usage: `hypqAttributes.`

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.
```

See also: `PQ, phCancel, TeX, TeXphW.`

---

inv1

Description: $(a;q)_n \to (1/a;1/q)_n(-a)^n q^{\binom{n}{2}}$.

Usage: `Expr/.inv1.`

Example(s):

```
In[1]:= pq[a^2,n]

           2
Out[1]= (a  ; q)
                n

In[2]:= %/.inv1
```

```
          n   2 n    -2   1
      (-1)   a     (a   ; -)
                          q n
Out[2]= --------------------
                   2
          (n - n )/2
         q
```

See also: `inv2, Ers, PosListe, ManipulationsListe.`

---

inv2

Description: $(a;q)_n \to (aq^{n-1};1/q)_n$.

Usage: `Expr/.inv2.`

Example(s):

```
In[1]:= pq[a^2,n]

         2
Out[1]= (a ; q)
               n

In[2]:= %/.inv2

         2  -1 + n  1
Out[2]= (a  q      ; -)
                    q n
```

See also: `inv1, Ers, PosListe, ManipulationsListe.`

LaTeX

Description: Switch that changes the output of TeXForm to be usable with Plain-TEX and LATEX. By default the output of TeXForm is usable with AMS-TEX.

Usage: `LaTeX.`

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AMSTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= TeXForm[ph[{a,b},{c},q,z]]

Out[2]//TeXForm=
{} _{2} \phi _{1} \!  \left [ \matrix \let \over / a, b\\ \let \over /
 c\endmatrix ;q, {\displaystyle z} \right ]
```

```
In[3]:= LaTeX

In[4]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[5]:= TeXForm[ph[{a,b},{c},q,z]]

Out[5]//TeXForm=
{} _{2} \phi _{1} \!  \left [ \matrix { \let \over / a, b\cr \let \over / c}
 ;q, {\displaystyle z} \right ]
```

See also: `AmSTeX`, `AmSLaTeX`, `TeX`, `TeXMat`, `TeXphW`.

Limes

Description: Function for doing formal limits of basic hypergeometric expressions. If required for taking the limit, you will be asked whether or not the absolute value of some variable or expression is smaller than 1. You will be offered three options, `[y|n|u]`. If the absolute value of the variable is smaller than 1 then enter `y`, if it is greater than 1 then enter `n`, if you do not want to make an explicit declaration then enter `u` (for "undetermined"). Your decision, if explicit, is stored for the rest of your MATHEMATICA session. If you want to change your decision later, use `AbsGreater`, `AbsSmaller`, or `AbsUndetermined`, respectively. By default the absolute value of `q` is defined to be smaller than 1. Also this can be changed by `AbsGreater`, `AbsSmaller`, or `AbsUndetermined`, respectively.

If you want to let a base q tend to 1, then you should also have the basic file `hyp.m` of the MATHEMATICA package HYP in your MAHEMATICA input directory, since in these situations the file `hyp.m` will be loaded automatically. The `$ContextPath` (cf. [4]) is set that objects of the package HYP do not override the respective objects of the package HYPQ. The situation is just as if you would have loaded `hyp.m` first and then `hyp.q`. Concerning a simultaneous use of the packages HYPQ and HYP confer the section *Simultaneous use of* HYP *and* HYPQ at the beginning of this handbook.

Hint: When using `Limes` you sometimes have to do a little bit of "preparation". For instance, when applying `Limes[ ... ,Regel]` to expressions of the form $(a;q)_n$, where a and n tend to ∞ under `Regel`, then the package will be unable to do it, unless at least $1/aq^n$ tends to a definite limit. Hence, you will sometimes have to use `zerl` before being able to apply `Limes` (see the last example).

Warning: This function uses primitive algebraic techniques to do the limit. There is no check if taking the limit is actually allowed. So it is left to you to check the validity of a result of `Limes`.

Usage: `Limes[Expr, x->x0]`.

Example(s):

The derivation of the identity [1, (2.7.6)] that leads to the Rogers–Ramanujan identities, starting from Watson's transformation Tgl8702.

```
In[1]:= Tgl8702
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n
```

```
                                                       -n         2  2 + n
          a, Sqrt[a] q, -(Sqrt[a] q), b, c, d, e, q              a   q
Out[1]=  phi                                                ; q, ---------
       8 7                       a q  a q  a q  a q     1 + n      b c d e
        Sqrt[a], -Sqrt[a], ---, ---, ---, ---, a q
                            b    c    d    e

           a q         -n                 a q
           ---, d, e, q            (a q, ---; q)
           b c                            d e    n
>         == phi                ; q, q  --------------
          4 3 a q  a q  de              a q  a q
           ---, ---, ----              (---, ---; q)
            b    c     n                d    e     n
                      a q
```

```
In[2]:= Limes[%,b->Infinity]
```

```
                                                   -n         2  2 + n
          a, Sqrt[a] q, -(Sqrt[a] q), c, d, e, q             a   q
Out[2]=  phi                                            ; q, ---------   ==
       7 7                         a q  a q  a q     1 + n      c d e
        Sqrt[a], -Sqrt[a], 0, ---, ---, ---, a q
                               c    d    e

                 -n
           d, e, q
                                       a q
       phi a q   d e ; q, q  (a q; q)  (---; q)
       3 2 ---, ----                n  d e    n
            c     n
                a q
>      ----------------------------------------
                  a q       a q
                 (---; q)  (---; q)
                  d     n   e     n
```

```
In[3]:= Limes[%,c->Infinity]
```

```
                                                 -n         2  2 + n
           a, Sqrt[a] q, -(Sqrt[a] q), d, e, q             a   q
Out[3]=  phi                                          ; q, ---------   ==
       6 7                              a q  a q     1 + n    d e
        Sqrt[a], -Sqrt[a], 0, 0, ---, ---, a q
                                  d    e
```

```
                -n
     ┌ d, e, q                ┐
     │                        │                    a q
  φ  │     d e   ; q, q       │ (a q; q)   (---; q)
 3 2 │  0, ----               │         n   d e     n
     └       n                ┘
          a q
〉   ------------------------------------------
               a q        a q
              (---; q)   (---; q)
                d     n   e      n
```

```
In[4]:= Limes[%,d-〉Infinity]
```

```
           ┌                                          -n            2   2 + n ┐
           │   a, Sqrt[a] q, -(Sqrt[a] q), e, q                    a    q      │
Out[4]=  φ │                                                ; q, ---------    │ ==
        5 7│                                 a q      1 + n           e       │
           └ Sqrt[a], -Sqrt[a], 0, 0, 0, ---, a q                              ┘
                                          e
```

```
                       1 + n
     ┌                       ┐
     │     -n      a q       │
  φ  │ e, q   ; q, --------  │ (a q; q)
 2 1 │                 e     │         n
     └   0                   ┘
〉   ------------------------------------
                  a q
                 (---; q)
                   e      n
```

```
In[5]:= Limes[%,e-〉Infinity]
```

```
           ┌                                      -n                        ┐
           │     a, Sqrt[a] q, -(Sqrt[a] q), q                  2   2 + n   │
Out[5]=  φ │                                               ; q, a    q      │ ==
        4 7│                                      1 + n                     │
           └ Sqrt[a], -Sqrt[a], 0, 0, 0, 0, a q                             ┘
```

```
        ┌                      ┐
        │  -n          1 + n   │
〉   φ  │ q   ; q, a q         │ (a q; q)
    1 1 │                      │         n
        └  0                   ┘
```

```
In[6]:= Limes[%,n-〉Infinity]
```

```
Out[6]=  phi [  a, Sqrt[a] q, -(Sqrt[a] q)            2  2 ]
                                              ; q, a  q    ] ==
         3 7 [ Sqrt[a], -Sqrt[a], 0, 0, 0, 0, 0            ]


>     (a q;q)    phi [ -         ]
                     [ ; q, a q  ]
              oo 0 1 [ 0         ]

In[7]:= %/.phSum/.SumRegeln

          oo
         ----                                  2
         \        kk   2 kk  -kk/2 + (5 kk )/2
Out[7]=   >  ((-1)    a     q                     (a; q)    (-(Sqrt[a] q); q)
         /                                               kk                  kk
         ----
        kk=0

>          (Sqrt[a] q; q)   ) / ((-Sqrt[a]; q)   (Sqrt[a]; q)   (q; q)   ) ==
                         kk                   kk              kk       kk

                   oo         2
                  ----  kk   kk
                  \    a    q
>     (a q;q)   (  >   --------)
             oo   /    (q; q)
                  ----       kk
                 kk=0

In[8]:= PosListe[%,3]

                                     kk                      2 kk
Out[8]= {{0, {{1, 2, 2}}}, {(-1)   , {{1, 1, 1}}}, {a     , {{1, 1, 2}}},

>     {kk, {{1, 2, 1}}}, {q, {{2, 1, 2}}}, {a q, {{2, 1, 1}}},

                     2
        -kk/2 + (5 kk )/2
>     {q                   , {{1, 1, 3}}}, {oo, {{1, 2, 3}}},

                                                1
>     {{kk, 0, oo}, {{2, 2, 2}}}, {---------------, {{1, 1, 4}}},
                                   (-Sqrt[a]; q)
                                                kk
```

```
            1
⟩    {--------------, {{1, 1, 5}}}, {(a; q)  , {{1, 1, 6}}},
      (Sqrt[a]; q)                          kk
                  kk

                                             2
                                      kk   kk
         1                           a    q
⟩    {--------, {{1, 1, 7}}}, {--------, {{2, 2, 1}}},
      (q; q)                    (q; q)
            kk                        kk

⟩    {(-(Sqrt[a] q); q)  , {{1, 1, 8}}}, {(Sqrt[a] q; q)  , {{1, 1, 9}}}}
                      kk                                 kk

In[9]:= Ers[%%,lina1,{{1,1,4},{1,1,5}}]

         ∞
        ┌───┐                              2
         \        kk  2 kk  -kk/2 + (5 kk )/2
Out[9]=   ⟩  ((-1)   a     q                   (a; q)   (-(Sqrt[a] q); q)
         /                                          kk                   kk
        └───┘
        kk=0

⟩         (Sqrt[a] q; q)  ) /
                       kk

⟩       ((1 - Sqrt[a]) (1 + Sqrt[a]) (q; q)   (-(Sqrt[a] q); q)
                                          kk                    -1 + kk

                                                   ∞         2
                                                  ┌───┐ kk  kk
                                                   \    a   q
⟩         (Sqrt[a] q; q)       ) == (a q;q)   (   ⟩   --------)
                       -1 + kk              ∞   /   (q; q)
                                                  └───┘      kk
                                                  kk=0

In[10]:= PosListe[%,3]

                                   kk
Out[10]= {{0, {{1, 2, 2}}}, {(-1)   , {{1, 1, 1}}},

           1                          1
⟩    {-----------, {{1, 1, 2}}}, {-----------, {{1, 1, 3}}},
      1 - Sqrt[a]                  1 + Sqrt[a]
```

```
       2 kk
〉    {a     , {{1, 1, 4}}}, {kk, {{1, 2, 1}}}, {q, {{2, 1, 2}}},

                                        2
                           -kk/2 + (5 kk )/2
〉    {a q, {{2, 1, 1}}}, {q                  , {{1, 1, 5}}}, {∞, {{1, 2, 3}}},

〉    {{kk, 0, ∞}, {{2, 2, 2}}}, {(a; q)  , {{1, 1, 6}}},
                                       kk

                                       2
                                kk  kk
         1                     a    q
〉    {--------, {{1, 1, 7}}}, {--------, {{2, 2, 1}}},
      (q; q)                   (q; q)
            kk                       kk

                 1
〉    {-----------------------, {{1, 1, 8}}},
      (-(Sqrt[a] q); q)
                       -1 + kk

〉    {(-(Sqrt[a] q); q)  , {{1, 1, 9}}},
                       kk

                1
〉    {--------------------, {{1, 1, 10}}}, {(Sqrt[a] q; q)  , {{1, 1, 11}}}}
      (Sqrt[a] q; q)                                       kk
                    -1 + kk

In[11]:= Ers[%%,lina2,{{1,1,9},{1,1,11}}]

          ∞
         ___                        2
          \       kk   2 kk   -kk/2 + (5 kk )/2              kk
Out[11]=   〉  ((-1)    a      q                     (1 - Sqrt[a] q   )
          /
         ‾‾‾
         kk=0

                        kk
〉         (1 + Sqrt[a] q   ) (a; q)   ) / ((1 - Sqrt[a]) (1 + Sqrt[a]) (q; q)   )
                                  kk                                         kk
```

```
                       ∞          2
                      ____ kk   kk
                       \    a    q
>        == (a q;q)  (  >   --------)
                   ∞   /    (q; q)
                      ____        kk
                      kk=0
```

```
In[12]:= Ers[%,ExpandAll,{{1,1}}]
```

```
                                          2
           ∞       kk   2 kk   (5 kk )/2
          ____(-1)    a      q            (a; q)
           \                                    kk
Out[12]=    >   -------------------------------- -
           /     kk/2                 kk/2
          ____q      (q; q)    - a q      (q; q)
          kk=0             kk                   kk
```

```
                                               2
              kk   1 + 2 kk   2 kk + (5 kk )/2
          (-1)    a          q                  (a; q)
                                                      kk
>         ------------------------------------------ ==
                kk/2                 kk/2
               q      (q; q)    - a q      (q; q)
                            kk                   kk
```

```
                  ∞          2
                 ____ kk   kk
                  \    a    q
>     (a q;q)  (  >   --------)
              ∞   /    (q; q)
                 ____        kk
                 kk=0
```

```
In[13]:= Ers[%,Factor,{{1,1}}]
```

```
                                                  2
           ∞       kk   2 kk   -kk/2 + (5 kk )/2            2 kk
          ____(-1)    a      q                    (-1 + a q     ) (a; q)
           \                                                            kk
Out[13]=    >   -------------------------------------------------------- ==
           /                          (-1 + a) (q; q)
          ____                                      kk
          kk=0
```

```
                  ∞         2
                ┌──┐ kk  kk
                 \   a   q
〉    (a q;q)  (  〉  --------)
             ∞   /   (q; q)
                └──┘      kk
                kk=0
```

Examples for letting the base q tend to 1.

```
In[1]:= Sgl3201
Do you want to set values for the equation?  [y|n]:  y
a=q^a
b=q^b
c=q^c
n=n
Do you want to set a value for q in the equation?  [y|n]:  n

                                                  -a + c         -b + c
                  a   b   -n                    (q       ; q)  (q       ; q)
                 q , q , q                                  n              n
Out[1]=  phi[                          ; q, q ] == --------------------------
         3 2   c    1 + a + b - c - n               c          -a - b + c
              q  , q                            (q ; q)   (q            ; q)
                                                       n                    n

In[2]:= Limes[%,q->1]

                                              (-a + c)  (-b + c)
                  a, b, -n                            n         n
Out[2]=  F [                        ; 1 ] == ------------------
         3 2 c, 1 + a + b - c - n            (c)  (-a - b + c)
                                                n              n

In[3]:= Sgl2103
Do you want to set values for the equation?  [y|n]:  y
a=q^a
b=q^b
c=q^c
Do you want to set a value for q in the equation?  [y|n]:  n

                                            -a + c    -b + c
              a   b                       (q       , q       ; q)
             q , q        -a - b + c                            ∞
Out[3]=  phi[        ; q, q           ] == ----------------------
         2 1    c                           c    -a - b + c
               q                          (q , q            ; q)
                                                               ∞
```

```
In[4]:= Limes[%,q->1]

            [ a, b       ]     Γ(c) Γ(-a - b + c)
Out[4]=   F [       ; 1  ] == -------------------
         2 1[  c         ]    Γ(-a + c) Γ(-b + c)
```

An example that should illustrate the hint in the Description of `Limes`.

```
In[1]:= pq[a/q^n,2*n]/a^n*(-1)^n*q^Binomial[n+1,2]

            n  (n (1 + n))/2  a
        (-1)  q              (--; q)
                               n    2 n
                              q
Out[1]= -------------------------------
                         n
                        a

In[2]:= Limes[%,n->Infinity]

The expression

Indeterminate

was obtained.

Therefore the limit n -> ∞ could not be determined.
Here is your expression:

            n  (n (1 + n))/2  a
        (-1)  q              (--; q)
                               n    2 n
                              q
Out[2]= -------------------------------
                         n
                        a

In[3]:= %/.zerl
bottom-split by:  n
```

```
                n   (n (1 + n))/2          a
        (-1)  q               (a; q)  (--; q)
                                    n   n    n
                                       q
Out[3]= -------------------------------------
                           n
                          a

In[4]:= Limes[%,n->Infinity]
Is 2 n even, odd, or neither of both?
[e|o|n]:  e

                    q
Out[4]= (a;q)   (-;q)
             ∞   a   ∞
```

See also: `AbsGreater`, `AbsSmaller`, `AbsUndetermined`, `MinusOne`.

---

lina1

Description: $(a;q)_n \to (1-a)(aq;q)_{n-1}$,
$(a;q)_\infty \to (1-a)(aq;q)_\infty$.

Usage: `Expr/.lina1`.

Example(s):

```
In[1]:= pq[a,m,q^2]

              2
Out[1]= (a; q )
                m

In[2]:= %/.lina1

                     2   2
Out[2]= (1 - a) (a q ; q )
                           -1 + m

In[3]:= 1/pqinf[a,q^2]

            1
Out[3]= -------
             2
        (a;q )
              ∞

In[4]:= %/.lina1
```

```
                  1
Out[4]= ------------------
                      2  2
        (1 - a) (a q ;q )
                         ∞
```

See also: `lina2, linz, Ers, PosListe, ManipulationsListe.`

---

lina2

Description: $(a;q)_n \to (1-aq^{n-1})(a;q)_{n-1}$.

Usage: `Expr/.lina2.`

Example(s):

```
In[1]:= pq[a,m,q^2]

               2
Out[1]= (a; q )
                 m

In[2]:= %/.lina2

                -2 + 2 m       2
Out[2]= (1 - a q        ) (a; q )
                                 -1 + m
```

See also: `lina1, linz, Ers, PosListe, ManipulationsListe.`

---

linz

Description: Rule that absorbs linear terms.

Usage: `Expr/.linz.`

Example(s):

```
In[1]:= (1-a)*pq[a*q^2,m,q^2]/(1-a*q^(2*m-2))/pq[a,m-1,q^2]

                          2   2
              (1 - a) (a q ; q )
                                m
Out[1]= -------------------------------
                -2 + 2 m       2
        (1 - a q        ) (a; q )
                                 -1 + m

In[2]:= %/.linz
```

```
                     2
                (a; q )
                       1 + m
Out[2]= ------------------------------
               -2 + 2 m        2
        (1 - a q        ) (a; q )
                                -1 + m
```

```
In[3]:= %/.linz
```

```
              2
        (a; q )
               1 + m
Out[3]= ------------
                2
          (a; q )
                 m
```

```
In[4]:= 1/(1-b/q)/pqinf[b]
```

```
               1
Out[4]= --------------
              b
        (1 - -) (b;q)
              q      ∞
```

```
In[5]:= %/.linz
```

```
          1
Out[5]= ------
         b
        (-;q)
         q   ∞
```

<u>See also</u>: `lina1`, `lina2`, `Ers`, `PosListe`, `ManipulationsListe`.

LS

<u>Description</u>: `LS` is the left-hand side in `Gleichung`.

<u>Usage</u>: `LS`.

<u>Example(s)</u>:

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n
```

Out[1]= $${}_{2}\phi_{1}\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$$

In[2]:= LS

Out[2]= $${}_{2}\phi_{1}\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right]$$

In[3]:= Add[1]

Out[3]= $$1 + {}_{2}\phi_{1}\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right] == 1 + \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$$

In[4]:= LS

Out[4]= $$1 + {}_{2}\phi_{1}\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix};\ q,\ q\right]$$

In[5]:= LS=pq[a,m]

Out[5]= $(a;\ q)_{m}$

In[6]:= Gleichung

Out[6]= $$(a;\ q)_{m} == 1 + \frac{a^{n}\left(\frac{c}{a};\ q\right)_{n}}{(c;\ q)_{n}}$$

See also: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `RS`, `Mal`, `Add`, `Div`, `Sub`, `Hoch`, `GlTausche`, `Ers`, `Subst`.

Mal

Description: Function that multiplies `Gleichung` by `Expr`.

Usage: `Mal[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                      n  c
                                     a  (-; q)
              -n                         a   n
Out[1]=  phi [ a, q  ; q, q ] == ----------
         2 1                        (c; q)
            c                            n

In[2]:= Mal[pq[c,n,q]]

              -n                              n  c
Out[2]=  phi [ a, q  ; q, q ] (c; q)  == a  (-; q)
         2 1                        n         a   n
            c

In[3]:= Gleichung

              -n                              n  c
Out[3]=  phi [ a, q  ; q, q ] (c; q)  == a  (-; q)
         2 1                        n         a   n
            c
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Add, Div, Sub, Hoch, GlTausche, Ers.`

ManipulationsListe

Description: Gives a list of all available rules for manipulating finite and infinite q-factorial symbols.

Usage: `ManipulationsListe.`

MinusOne

Description: : Rule for getting rid of expressions of the form $(-1)^N$ where N is an even or odd integer.

Usage: `Expr/.MinusOne.`

```
In[1]:= pq[a,2*n]

Out[1]= (a; q)
             2 n
```

```
In[2]:= %/.trans

                      1 - 2 n
           2 n  2 n  q
        (-1)    a    (--------; q)
                        a        2 n
Out[2]= -----------------------------
                         2
               (2 n - 4 n )/2
              q

In[3]:= %/.MinusOne
Is 2 n even, odd, or neither of both?
[e|o|n]:  e

              1 - 2 n
         2 n  q
        a    (--------; q)
                a        2 n
Out[3]= ---------------------
                     2
           (2 n - 4 n )/2
          q
```

See also: `SimplifyPQ, Expandq, SumExpand.`

---

neg1

Description: $(a;q)_n \to 1/(aq^n;q)_{-n}$.

Usage: `Expr/.neg1.`

Example(s):

```
In[1]:= pq[a^2,-n]

          2
Out[1]= (a ; q)
               -n

In[2]:= %/.neg1

            1
Out[2]= ------------
          2  -n
        (a  q   ; q)
                   n
```

See also: `neg2, Ers, PosListe, ManipulationsListe.`

---

neg2

Description: $(a;q)_n \to q^{\binom{n+1}{2}}/((-q/a)^n(q/a;q)_{-n})$.

Usage: `Expr/.neg2`.

Example(s):

```
In[1]:= pq[a^2,-n]

             2
Out[1]= (a  ; q)
                -n

In[2]:= %/.neg2

                          2
                 n/2 + n /2
            n   q
Out[2]= (-1)  --------------
               2 n  q
              a    (--; q)
                     2    n
                    a
```

See also: `neg1, Ers, PosListe, ManipulationsListe`.

---

ph

Description: `ph[List1,List2,q,z]` is the basic hypergeometric series with upper parameters `List1`, lower parameters `List2`, base `q`, and argument `z`.

Usage: `ph[List1,List2,q,z]`.

Example(s):

```
In[1]:= ph[{a,b},{c},q,z]
```

$$\text{Out[1]=}\ {}_{2}\phi_{1}\left[\begin{matrix} a,\ b \\ c \end{matrix}\ ;\ q,\ z\right]$$

```
In[2]:= ph[{a,b,c},{d,e,0},q,z]
```

$$\text{Out[2]=}\ {}_{3}\phi_{3}\left[\begin{matrix} a,\ b,\ c \\ d,\ e,\ 0 \end{matrix}\ ;\ q,\ z\right]$$

See also: `SListe, TListe, SumRegeln, Sumph, phSum, pq, pqinf, phCancel, phOrdne, phPerm, phTausche,` PQ, phFormat.

---

phCancel

Description: Switch that activates automatic cancelling of the upper and lower parameters in ph[], or makes it inactive, respectively. By default automatic cancelling is active.

Usage: phCancel.

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= ph[{a,b},{a,c},q,z]
```

$$\text{Out[2]= } {}_1\phi_1\left[\begin{matrix} b \\ c \end{matrix} ; q,\ z\right]$$

```
In[3]:= pq[{a,b},{a,c},n,q^2]
```

$$\text{Out[3]= } \frac{(b;\ q^2)_n}{(c;\ q^2)_n}$$

```
In[4]:= pqinf[{a,b},{a,c}]
```

$$\text{Out[4]= } \frac{(b;\ q)_\infty}{(c;\ q)_\infty}$$

```
In[5]:= phCancel

In[6]:= hypqAttributes
```

```
Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph inactive.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[7]:= ph[{a,b},{a,c},q,z]

            ┌              ┐
            │ a, b         │
Out[7]=  φ  │      ; q, z  │
        2 2 │ a, c         │
            └              ┘

In[8]:= pq[{a,b},{a,c},n,q^2]

              2
        (b; q )
               n
Out[8]= --------
              2
        (c; q )
               n

In[9]:= pqinf[{a,b},{a,c}]

        (b; q)
              ∞
Out[9]= -------
        (c; q)
              ∞

In[10]:= phCancel

In[11]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.
```

See also: ph, W, hypqAttributes.

---

phEinf

Description: Rule that inactivates automatic cancelling in `ph[]` and then adds a parameter which has to be entered on request to the upper and lower parameters of `ph[]`.

Usage: `Expr/.phEinf`.

Example(s):

```
In[1]:= ph[{b,c,q},{q*a/c,q*a/b},q,z]

                b, c, q

Out[1]=  phi    a q  a q; q, z
            3 2 ---, ---
                 c    b

In[2]:= %/.phEinf
Add the parameter:  a

                a, b, c, q

Out[2]=  phi       a q  a q; q, z
            4 3 a, ---, ---
                    c    b
```

See also: `phOrdne, phPerm, phTausche, PQSort, SumRegeln, SumUmkehr, Ers, PosListe.`

---

phFormat

Description: Switch that activates basic hypergeometric output, or makes it inactive, respectively. By default basic hypergeometric output is active.

Usage: `phFormat.`

Example(s):

```
In[1]:= pq[a,n]/pqinf[b,q^2]*ph[{c,d},{c*d},q,z]

                c, d
           phi        ; q, z  (a; q)
            2 1 c d                 n
Out[1]= -------------------------
                       2
                  (b;q )
                        ∞

In[2]:= Tgl5402
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n


Format::toobig:  Expression too big for output.
                 Enter "phFormat" and retry.
```

```
In[3]:= phFormat

In[4]:= %%

                              1 - n   1 - n   1 - n
          -n                 q       q       q        2  2  2    -2 + 2 n
Out[4]= ph[{q  , b, c, d, e}, {------, ------, ------, b  c  d  e q        },
                               b       c       d

                  1 - 2 n          3/2 - n
                 q                q
〉     q, q] == ph[{--------, ----------------------,
                  b c d    Sqrt[b] Sqrt[c] Sqrt[d]

                3/2 - n            1 - n   1 - n   1 - n
               q                  q       q       q        -n/2    -n/2
〉       -(----------------------), ------, ------, ------, q    , -q    ,
          Sqrt[b] Sqrt[c] Sqrt[d]    c d     b d     b c

                                        3 - 3 n
         1/2 - n/2    1/2 - n/2        q
〉       q         , -q         , e, ----------},
                                       2  2  2
                                      b  c  d  e

              1/2 - n                    1/2 - n              1 - n   1 - n
             q                          q                    q       q
〉       {----------------------, -(----------------------), ------, ------,
         Sqrt[b] Sqrt[c] Sqrt[d]    Sqrt[b] Sqrt[c] Sqrt[d]     b       c

         1 - n   2 - (3 n)/2     2 - (3 n)/2    3/2 - (3 n)/2
        q       q               q              q
〉       ------, ------------, -(------------), --------------,
          d        b c d          b c d           b c d

          3/2 - (3 n)/2    2 - 2 n
         q                q                    -1 + n
〉       -(--------------), --------, b c d e q      }, q, q]
              b c d        b c d e

          2 - 2 n   3 - 3 n     2 - 2 n    3 - 3 n
         q         q           q          q
〉      pq[{--------, --------}, {--------, ----------}, n, q]
         b c d e    2  2  2     b c d      2  2  2
                   b  c  d                b  c  d  e

In[5]:= %1
```

```
        ph[{c, d}, {c d}, q, z] pq[a, n, q]
Out[5]= -----------------------------------
                                      2
                   pqinf[b, q ]
```

```
In[6]:= phFormat
```

```
In[7]:= %1
```

```
          [                 ]
          [ c, d            ]
        φ [      ; q, z     ] (a; q)
      2 1 [ c d             ]        n
          [                 ]
Out[7]= -------------------------
                    2
                (b;q )
                     ∞
```

See also: `ph`.

---

phinv

Description: : Rule that transforms a basic hypergeometric series `ph[List1,List2,q,z]` with base `q` into a basic hypergeometric series `ph[...,1/q,..]` with base `1/q`.

Usage: `Expr/.phinv`.

Example(s):

```
In[1]:= ph[{a,b},{c},q,z]
```

```
          [                 ]
          [ a, b            ]
Out[1]= φ [      ; q, z     ]
      2 1 [  c              ]
          [                 ]
```

```
In[2]:= %/.phinv
```

```
            1   1
          [ -,  -                ]
          [ a   b   1   a b z    ]
Out[2]= φ [       ; -,  -----    ]
      2 1 [  1      q    c q     ]
          [  -                   ]
             c
```

```
In[3]:= ph[{a,b,c},{d,0},q,z]
```

```
              ┌                ┐
              │ a, b, c        │
Out[3]=   φ   │          ; q, z│
        3 2   │  d, 0          │
              └                ┘
```

```
In[4]:= %/.phinv
```

```
              1  1  1
           ┌  -, -, -                  ┐
           │  a  b  c  1   a b c z     │
Out[4]=  φ │           ; -, -------    │
       3 1 │     1       q    d q      │
           └     -                     ┘
                 d
```

See also: `SumRegeln, Ers, PosListe.`

---

phOrdne

Description: Rule that tries to order the parameters of a basic hypergeometric series in "well-poised" order. If there is an upper parameter of the form q^{-n}, where n might be a nonnegative integer, then it is put at the very last place in the upper list. If the parameters could be paired such that the product of each pair equals **A**, however **A** is missing in the upper parameters, then you have to add **A** to the upper and lower parameters by **phEinf** before applying **phOrdne**.

Usage: `Expr/.phOrdne.`

Example(s):

```
In[1]:= ph[{q^-n,b,q*Sqrt[a],a,-q*Sqrt[a]},{a*q/b,-Sqrt[a],Sqrt[a],
        a*q^(1+n)},q,z]
```

```
           ┌  -n                                               ┐
           │ q   , b, Sqrt[a] q, a, -(Sqrt[a] q)               │
Out[1]=  φ │                                         ; q, z    │
       5 4 │  a q                           1 + n              │
           └  ---, -Sqrt[a], Sqrt[a], a q                      ┘
               b
```

```
In[2]:= %/.phOrdne
```

```
           ┌                                     -n            ┐
           │ a, Sqrt[a] q, -(Sqrt[a] q), b, q                  │
Out[2]=  φ │                                         ; q, z    │
       5 4 │                           a q      1 + n          │
           └  Sqrt[a], -Sqrt[a], ---, a q                      ┘
                                    b
```

```
In[3]:= ph[{q^-n,b,c},{d,e},q^2,z]
```

Out[3]= $${}_3\phi_2\left[\begin{matrix} q^{-n}, b, c; q^{2}, z \\ d, e \end{matrix}\right]$$

```
In[4]:= %/.phOrdne
```

Out[4]= $${}_3\phi_2\left[\begin{matrix} b, c, q^{-n}; q^{2}, z \\ e, d \end{matrix}\right]$$

```
In[5]:= ph[{b,c,q},{a*q/c,a*q/b},q,z]
```

Out[5]= $${}_3\phi_2\left[\begin{matrix} b, c, q \\ \frac{a\,q}{c}, \frac{a\,q}{b}; q, z \end{matrix}\right]$$

```
In[6]:= %/.phOrdne
```

Out[6]= $${}_3\phi_2\left[\begin{matrix} b, c, q \\ \frac{a\,q}{c}, \frac{a\,q}{b}; q, z \end{matrix}\right]$$

```
In[7]:= %/.phEinf
Add the parameter:  a
```

Out[7]= $${}_4\phi_3\left[\begin{matrix} a, b, c, q \\ a, \frac{a\,q}{c}, \frac{a\,q}{b}; q, z \end{matrix}\right]$$

```
In[8]:= %/.phOrdne
```

Out[8]= $${}_4\phi_3\left[\begin{matrix} a, b, c, q \\ \frac{a\,q}{b}, \frac{a\,q}{c}, a; q, z \end{matrix}\right]$$

```
In[9]:= phCancel
```

```
In[10]:= %%/.phOrdne

           ┌ b, c, q           ┐
           │                   │
Out[10]= φ │ a q  a q; q, z    │
        3 2│ ---, ---          │
           └  b    c           ┘
```

See also: `phEinf, phPerm, phTausche, ph, W, PQSort, Ers, PosListe.`

---

phPerm

Description: Rule for permuting parameters in basic hypergeometric series.

Usage: `Expr/.phPerm[<Permutation>,x].`

`x` can be `u`, `l`, `b`. u causes a permutation of upper parameters, `l` causes a permutation of lower parameters, `b` causes a simultaneous permutation of respective upper and lower parameters. `Permutation` must be a sequence of positive numbers forming a permutation. The effect is that the new parameter at position `i` is the old parameter from position `Permutation[i]`.

Example(s):

```
In[1]:= ph[{a,b,c,d},{e,f,g},q,z]

          ┌                  ┐
          │ a, b, c, d       │
Out[1]= φ │            ; q, z│
       4 3│  e, f, g         │
          └                  ┘

In[2]:= %1/.phPerm[3,2,1,u]

          ┌                  ┐
          │ c, b, a, d       │
Out[2]= φ │            ; q, z│
       4 3│  e, f, g         │
          └                  ┘

In[3]:= %1/.phPerm[3,2,1,l]

          ┌                  ┐
          │ a, b, c, d       │
Out[3]= φ │            ; q, z│
       4 3│  g, f, e         │
          └                  ┘

In[4]:= %1/.phPerm[2,1,b]
```

```
                a, c, b, d
Out[4]=  φ                ; q, z
        4 3     f, e, g
```

See also: phTausche, phOrdne, ph, W, PQSort, Ers, PosListe.

---

phSum

Description: Rule that transforms a ph[] into a Sum[].

Usage: Expr/.phSum.

Example(s):

```
In[1]:= ph[{a,b},{c},q^2,z]

                a, b   2
Out[1]=  φ           ; q , z
        2 1     c

In[2]:= %/.phSum
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k

          oo   k      2        2
          ___ z  (a; q )  (b; q )
          \              k        k
Out[2]=    )  -------------------
          /         2      2   2
          ---  (c; q )  (q ; q )
          k=0          k          k

In[3]:= ph[{a,q^(-2*n)},{c},q^2,z]

                    -2 n   2
Out[3]=  φ      a, q     ; q , z
        2 1
                   c

In[4]:= %/.phSum
Is n a nonnegative integer?
[y|n]:    y
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  j
```

```
           n    j        2      -2 n    2
         ___ z   (a; q )   (q     ; q )
         \               j              j
Out[4]=   )   -----------------------
         /              2      2    2
         ---     (c; q )   (q ; q )
          j=0            j          j
```

```
In[5]:= ph[{a,b},{c,d},q,z]
```

```
              [ a, b            ]
Out[5]=   phi [       ; q, z    ]
          2 2 [ c, d            ]
```

```
In[6]:= %/.phSum
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k
```

```
                                  2
          oo       k   -k/2 + k /2  k
         ___ (-1)   q              z   (a; q)   (b; q)
         \                                   k        k
Out[6]=   )   -------------------------------------
         /            (c; q)   (d; q)   (q; q)
         ---                k        k        k
          k=0
```

See also: `ph, Sumph, Ers, PosListe.`

---

phTausche

Description: Rule for reordering parameters in basic hypergeometric series.

Usage: `Expr/.phTausche[n1,n2,x].`
`x` can be `u`, `l`, `b`. `u` causes a reordering of upper parameters, `l` causes a reordering of lower parameters, `b` causes a simultaneous reordering of respective upper and lower parameters. `n1` is the position of the parameter to be reordered, `n2` is the new position.

Example(s):

```
In[1]:= ph[{q^-n,b,q*Sqrt[a],a,-q*Sqrt[a]},{a*q/b,-Sqrt[a],Sqrt[a],
        a*q^(1+n)},q,z]
```

```
              [  -n                                                ]
              [ q   , b, Sqrt[a] q, a, -(Sqrt[a] q)                ]
Out[1]=   phi [                                        ; q, z      ]
          5 4 [  a q                        1 + n                  ]
              [  ---, -Sqrt[a], Sqrt[a], a q                       ]
                  b
```

```
In[2]:= %/.phTausche[1,3,u]

                          -n
          b, Sqrt[a] q, q   , a, -(Sqrt[a] q)
Out[2]=  phi                                    ; q, z
        5 4  a q                        1 + n
             ---, -Sqrt[a], Sqrt[a], a q
              b

In[3]:= %/.phTausche[4,2,1]

                          -n
          b, Sqrt[a] q, q   , a, -(Sqrt[a] q)
Out[3]=  phi                                    ; q, z
        5 4  a q     1 + n
             ---, a q     , -Sqrt[a], Sqrt[a]
              b

In[4]:= %/.phTausche[1,4,b]

               -n
          b, q   , a, -(Sqrt[a] q), Sqrt[a] q
Out[4]=  phi                                    ; q, z
        5 4    1 + n                    a q
             a q     , -Sqrt[a], Sqrt[a], ---
                                           b
```

See also: `phPerm, phOrdne, ph, W, PQSort, Ers, PosListe.`

PosListe

Description: Function that provides a list of subexpressions of `Expr` together with the respective positions in `Expr`. This helps to use controlled application of rules or functions by means of `Ers`.

Usage: `PosListe[Expr].`

Example(s):

```
In[1]:= pq[a,n]/pq[q,n]*Sum[pq[b,k]/pq[c,k+1]*q^k,{k,0,Infinity}]

           ∞   k
          ____q  (b; q)
          \            k
         ( >   ----------) (a; q)
          /    (c; q)            n
          ‾‾‾‾       1 + k
          k=0
Out[1]= --------------------------
                 (q; q)
                       n
```

```
In[2]:= PosListe[%]

           ∞   k
           ┌──┐q  (b; q)
           \            k                              1
Out[2]= {{  〉  -----------, {{1}}}, {(a; q)  , {{2}}}, {-------, {{3}}}}
           /   (c; q)                     n            (q; q)
           └──┘     1 + k                                     n
           k=0

In[3]:= PosListe[%%,2]

Out[3]= {{-1, {{3, 2}}}, {a, {{2, 1}}}, {n, {{2, 2}}}, {q, {{2, 3}}},

                               k
                              q  (b; q)
                                        k
〉    {{k, 0, ∞}, {{1, 2}}}, {-----------, {{1, 1}}}, {(q; q)  , {{3, 1}}}}
                              (c; q)                          n
                                   1 + k

In[4]:= PosListe[%%%,3]

Out[4]= {{0, {{1, 2, 2}}}, {k, {{1, 2, 1}}}, {n, {{3, 1, 2}}},

                                    k
〉    {q, {{3, 1, 1}, {3, 1, 3}}}, {q , {{1, 1, 1}}}, {∞, {{1, 2, 3}}},

                                    1
〉    {(b; q) , {{1, 1, 2}}}, {-----------, {{1, 1, 3}}}}
            k                 (c; q)
                                   1 + k
```

See also: `Ers`, `Subst`.

---

pq

Description: `pq[x,n,q]` is the q-factorial symbol $(x;q)_n$. `pq[List1,List2,n,q]` is also provided as the usual abbreviation for the quotient of q-factorial symbols (see [1, (1.2.41)]). In both cases the parameter `q` is optional. It will be set equal `q` if it is omitted.

Usage: `pq[x,n,q]`
or: `pq[x,n]`
or: `pq[List1,List2,n,q]`
or: `pq[List1,List2,n]`.

Example(s):

```
In[1]:= pq[a,n]
```

```
Out[1]= (a; q)
              n

In[2]:= pq[a,n,q^2]

             2
Out[2]= (a; q )
               n

In[3]:= pq[{a,b},{c,d},2*m]

        (a, b; q)
                2 m
Out[3]= ------------
        (c, d; q)
                2 m

In[4]:= pq[{a,b},{c,d},2*m,1/q]

               1
        (a, b; -)
               q 2 m
Out[4]= ------------
               1
        (c, d; -)
               q 2 m
```

See also: `ph, pqinf, Binomialq, Binomialpq, Factorialq, Factorialpq, PQ, pqaufl, pqzerl, pqzus, phFormat.`

PQ

Description: Is a switch that activates automatic evaluating of q-factorial symbols pq and basic hypergeometric series ph, or makes it inactive, respectively. By default automatic evaluating is inactive.

Usage: `PQ.`

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= pq[a,5]

Out[2]= (a; q)
              5
```

```
In[3]:= ph[{a,b},{c},q,z]

             [ a, b          ]
Out[3]=  φ   [       ; q, z  ]
        2 1  [  c            ]

In[4]:= ph[{q^-n,b},{c},q,z]

             [  -n             ]
Out[4]=  φ   [ q   , b; q, z   ]
        2 1  [                 ]
             [   c             ]

In[5]:= ph[{q^-3,b},{c},q,z]

             [  -3             ]
Out[5]=  φ   [ q   , b; q, z   ]
        2 1  [                 ]
             [   c             ]

In[6]:= PQ

In[7]:= hypqAttributes

Automatic evaluation of pq and ph is active.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[8]:= pq[a,5]

                                        2           3           4
Out[8]= (1 - a) (1 - a q) (1 - a q ) (1 - a q ) (1 - a q )

In[9]:= ph[{a,b},{c},q,z]
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k
```

```
            ∞   k
          ┌──┐z  (a; q)  (b; q)
           \            k       k
Out[9]=    >  -----------------
           /    (c; q)  (q; q)
          └──┘        k       k
           k=0

In[10]:= ph[{q^-n,b},{c},q,z]
Is n a nonnegative integer?
[y|n]:   y
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  j

             n   j           -n
           ┌──┐z  (b; q)  (q  ; q)
            \            j         j
Out[10]=    >  -------------------
            /     (c; q)  (q; q)
           └──┘         j       j
            j=0

In[11]:= ph[{q^-3,b},{c},q,z]
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  s

                                 -3                      -3        -2              2
               (1 - b) (1 - q   ) z   (1 - b) (1 - q   ) (1 - q   ) (1 - b q) z
Out[11]= 1 + ------------------- + ---------------------------------------- +
                (1 - c) (1 - q)                                            2
                                    (1 - c) (1 - q) (1 - c q) (1 - q )

                 -3         -2       1                    2   3
      (1 - b) (1 - q   ) (1 - q   ) (1 - -) (1 - b q) (1 - b q ) z
                                     q
>     ------------------------------------------------------------
                                        2          2         3
        (1 - c) (1 - q) (1 - c q) (1 - q ) (1 - c q ) (1 - q )

In[12]:= PQ

In[13]:= hypqAttributes
```

```
Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.
```

See also: `ph`, `pq`, `hypqAttributes`.

---

pqaufl

Description: Rule that writes `pq[x,n,q]` as the defining product `Product[1-x*q^i,{i,0,n-1}]`, if `n` is an integer.

Usage: `Expr/.pqaufl`.

Example(s):

```
In[1]:= pq[a,-3]/pq[b,2]*pq[c,1]

        (a; q)   (c; q)
              -3       1
Out[1]= ----------------
            (b; q)
                  2

In[2]:= %/.pqaufl

                             1 - c
Out[2]= ------------------------------------------
                       a        a        a
        (1 - b) (1 - --) (1 - --) (1 - -) (1 - b q)
                        3        2      q
                       q        q

In[3]:= ph[{a,b,c},{d,e,f},q,z]

           [ a, b, c          ]
Out[3]=  φ [         ; q, z   ]
        3 3[ d, e, f          ]

In[4]:= %/.C01

               [ q, a q, b q, c q             ]
           z  φ [                   ; q, q z  ] (a; q)  (b; q)  (c; q)
            4 4[  2                           ]       1       1       1
               [ q , d q, e q, f q            ]
Out[4]= 1 - ----------------------------------------------------------
                          (d; q)  (e; q)  (f; q)  (q; q)
                                1       1       1       1
```

```
In[5]:= %/.pqaufl

                                           [ q, a q, b q, c q              ]
          (1 - a) (1 - b) (1 - c) z  phi   [                    ; q, q z   ]
                                     4 4   [  2                            ]
                                           [ q , d q, e q, f q             ]
Out[5]= 1 - ---------------------------------------------------------
                         (1 - d) (1 - e) (1 - f) (1 - q)
```

See also: pqzerl, pqzus, pq, pqinfzerl, pqinfzus, Ers, PosListe.

---

pqinf

Description: pqinf[x,q] is the infinite q-factorial symbol $(x;q)_\infty$. pqinf[List1,List2,q] is also provided as the usual abbreviation for the quotient of infinite q-factorial symbols (see [1, (1.2.42)]). In both cases the parameter q is optional. If it is omitted it is set equal q.

Usage: pqinf[x,q]
or: pqinf[x]
or: pqinf[List1,List2,q]
or: pqinf[List1,List2].

Example(s):

```
In[1]:= pqinf[a^2*q]

           2
Out[1]= (a  q;q)
                ∞

In[2]:= pqinf[a^2*q,q^3]

           2    3
Out[2]= (a  q;q )
                 ∞

In[3]:= pqinf[{a,b},{c,d}]

        (a, b; q)
                 ∞
Out[3]= ----------
        (c, d; q)
                 ∞

In[4]:= pqinf[{a,b},{c,d},1/q]
```

```
               1
        (a, b; -)
               q ∞
Out[4]= ----------
               1
        (c, d; -)
               q ∞
```

See also: `pq, ph, PQ, pqinfzerl, pqinfzus, phFormat.`

---

pqinfzerl

Description: Rule that splits `pqinf[List1,List2,q]` into a quotient of products of infinite q-factorial symbols.

Usage: `Expr/.pqinfzerl.`

Example(s):

```
In[1]:= pqinf[{a,b},{c,d}]

        (a, b; q)
                 ∞
Out[1]= ----------
        (c, d; q)
                 ∞

In[2]:= %/.pqinfzerl

        (a;q)   (b;q)
             ∞       ∞
Out[2]= -------------
        (c;q)   (d;q)
             ∞       ∞
```

See also: `pqaufl, pqzerl, pqzus, pqinf, pqinfzus, Ers, PosListe.`

---

pqinfzus

Description: Rule that collects several infinite q-factorial symbols `pqinf[x_i,q]` to an expression `pqinf[List1,List2,q]`. The parameter `q` is optional. It is set equal `q` if it is omitted.

Usage: `Expr/.pqinfzus[q]`
or: `Expr/.pqinfzus[].`

Example(s):

```
In[1]:= pqinf[a]*pqinf[b]/pqinf[c]/pqinf[d]

        (a;q)   (b;q)
             ∞       ∞
Out[1]= -------------
        (c;q)   (d;q)
             ∞       ∞
```

```
In[2]:= %/.pqinfzus[]

        (a, b; q)
                 ∞
Out[2]= ----------
        (c, d; q)
                 ∞

In[3]:= pqinf[a,q]*pqinf[b,q^2]/pqinf[c,q^2]/pqinf[d,q]

                  2
        (a;q)  (b;q )
            ∞       ∞
Out[3]= --------------
             2
        (c;q )  (d;q)
             ∞      ∞

In[4]:= %/.pqinfzus[]

                (a; q)
            2        ∞
        (b;q )  -------
             ∞ (d; q)
                     ∞
Out[4]= ---------------
                2
            (c;q )
                 ∞

In[5]:= %/.pqinfzus[q^2]

                    2
        (a; q)  (b; q )
             ∞        ∞
Out[5]= ------- --------
        (d; q)      2
             ∞ (c; q )
                      ∞
```

See also: `pqaufl, pqzerl, pqzus, pqinf, pqinfzerl, Ers, PosListe.`

PQSort

Description: Rule that orders the parameters of basic hypergeometric series `ph[List1,List2,q,z]`, of "multiple" upper q-factorials `pq[List1,List2,n,q]`, and of "multiple" infinite q-factorials `pqinf[List1,List2,q]` in a standard order. For instance, this function can be used for a quick test if two expressions agree. It is recommended to apply `pqinfzus[q]` and `pqzus[n,q]` first.

Usage: `Expr/.PQSort.`

Example(s):

```
In[1]:= pq[{c,c/a,b},{a/b/c,b*q},n,q^2]*pqinf[{b/a,c},{a*b*c,1/b},q]*
        ph[{b/c,b*a,a},{b,a},1/q,z]

           b                                    c      2
          (-, c; q)        ┌ b            ┐ (c, -, b; q )
           a       ∞       │ -, a b  1    │    a        n
Out[1]= --------------  φ  │ c      ; -, z│ ---------------
                1      2 1 │         q    │  a          2
        (a b c, -; q)      └   b          ┘ (---, b q; q )
                b    ∞                       b c          n

In[2]:= %/.PQSort

           b                                       c   2
          (-, c; q)        ┌      b       ┐ (b, c, -; q )
           a       ∞       │ a b, -  1    │        a    n
Out[2]= --------------  φ  │      c; -, z │ ---------------
         1             2 1 │         q    │  a          2
        (-, a b c; q)      └   b          ┘ (---, b q; q )
         b           ∞                       b c          n
```

See also: `SimplifyPQ`, `SumExpand`, `phEinf`, `phOrdne`, `phPerm`, `phTausche`, `ph`, `W`, `pq`, `pqinf`.

---

pqzerl

Description: Rule that splits `pq[List1,List2,n,q]` into a quotient of products of q-factorial symbols.

Usage: `Expr/.pqzerl`.

Example(s):

```
In[1]:= pq[{a,b},{c,d},m,q^2]

                2
        (a, b; q )
                  m
Out[1]= -----------
                2
        (c, d; q )
                  m

In[2]:= %/.pqzerl
```

```
               2         2
         (a; q )   (b; q )
                 m         m
Out[2]= -----------------
               2         2
         (c; q )   (d; q )
                 m         m
```

See also: `pqaufl, pqzus, pq, pqinfzerl, pqinfzus, Ers, PosListe.`

---

pqzus

Description: Rule that collects several q-factorial symbols `pq[x_i,n,q]` to an expression `pq[List1,List2,n,q]`. The parameter `q` is optional. It is set equal `q` if it is omitted.

Usage: `Expr/.pqzus[n,q]`

or: `Expr/.pqzus[n].`

Example(s):

```
In[1]:= pq[a,n]*pq[b,m,q^2]/pq[c,n]/pq[d,m,q^2]

                        2
         (a; q)   (b; q )
               n         m
Out[1]= ----------------
                        2
         (c; q)   (d; q )
               n         m

In[2]:= %/.pqzus[n,q]

                  (a; q)
               2        n
         (b; q )   -------
                 m (c; q)
                          n
Out[2]= ----------------
                   2
             (d; q )
                   m

In[3]:= %/.pqzus[m,q^2]
```

```
                     2
        (a; q)  (b; q )
              n        m
Out[3]= ------- --------
        (c; q)       2
              n (d; q )
                       m
```

<u>See also</u>: `pqaufl`, `pqzerl`, `pq`, `pqinfzus`, `pqinfzerl`, `Ers`, `PosListe`.

RS

<u>Description</u>: `RS` is the right-hand side in `Gleichung`.

<u>Usage</u>: `RS`.

<u>Example(s)</u>:

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                n  c
                               a   (-; q)
                 -n                  a    n
Out[1]=  φ   [ a, q   ; q, q ] == ----------
        2 1                        (c; q)
               c                         n

In[2]:= RS

         n  c
        a   (-; q)
              a    n
Out[2]= ----------
         (c; q)
               n

In[3]:= Add[1]

                                            n  c
                                           a   (-; q)
                     -n                          a    n
Out[3]= 1 +  φ   [ a, q   ; q, q ] == 1 + ----------
            2 1                             (c; q)
                   c                              n

In[4]:= RS
```

```
              n   c
             a   (-; q)
                  a    n
Out[4]= 1 + ----------
              (c; q)
                   n

In[5]:= RS=1/pq[q,m]

           1
Out[5]= -------
         (q; q)
              m

In[6]:= Gleichung

                      -n                       1
Out[6]= 1 +   phi [ a, q   ; q, q ]  ==  -------
              2 1                         (q; q)
                     c                          m
```

See also: Gleichung, SumListe$gl, TransListe$gl, LS, Mal, Add, Div, Sub, Hoch, GlTausche, Ers, Subst.

S1001

Description: Summation formula ([1], (1.3.2); Appendix (II.3)) in form of a rule.

$$ {}_1\phi_0\left[\begin{matrix} a \\ - \end{matrix} ; q, z\right] \longrightarrow \frac{(az;q)_\infty}{(z;q)_\infty} $$

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S1101

Description: Summation formula ([1], Ex. 1.6(ii); Appendix (II.5)) in form of a rule.

$$ {}_1\phi_1\left[\begin{matrix} a \\ c \end{matrix} ; q, \frac{c}{a}\right] \longrightarrow \frac{(c/a;q)_\infty}{(c;q)_\infty} $$

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S2101

Description: Summation formula ([1], (1.5.3); Appendix (II.6)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, q^{-n} \\ c \end{matrix} ; q, q\right] \longrightarrow \frac{a^n (c/a;q)_n}{(c;q)_n} $$

where n is a nonnegative integer.

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S2102

Description: Summation formula ([1], (1.5.2); Appendix (II.7)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, q^{-n} \\ c \end{matrix}; q, \frac{cq^n}{a}\right] \longrightarrow \frac{(c/a;q)_n}{(c;q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2103

Description: Summation formula ([1], (1.5.1); Appendix (II.8)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, \frac{c}{ab}\right] \longrightarrow \frac{(c/a, c/b; q)_\infty}{(c, c/ab; q)_\infty} $$

Example(s):

```
In[1]:= ph[{a,b},{q^2*a*b},q,q^2]
```

$$ \text{Out[1]=}\ {}_2\phi_1\left[\begin{matrix} a, b \\ a\,b\,q^2 \end{matrix}; q, q^2\right] $$

```
In[2]:= %/.S2103
```

$$ \text{Out[2]=}\ \frac{(b\,q^2, a\,q^2; q)_\infty}{(a\,b\,q^2, q^2; q)_\infty} $$

See also: `S3201, SListe, SumListe, Ers, PosListe.`

S2104

Description: Summation formula ([1], (1.8.1); Appendix (II.9)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ aq/b \end{matrix}; q, -\frac{q}{b}\right] \longrightarrow \frac{(-q;q)_\infty}{(-q/b, aq/b; q)_\infty}(aq, aq^2/b^2; q^2)_\infty $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2105

Description: Summation formula ([1], Ex 1.6(i)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} q^{-n}, q^{1-n} \\ b^2 q \end{matrix}; q^2, q^2\right] \longrightarrow q^{-\binom{n}{2}} \frac{(b^2;q^2)_n}{(b^2;q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2106

Description: Summation formula ([1], Ex 1.7) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a^2, aq \\ a \end{matrix}; q, z\right] \longrightarrow \frac{(1+az)\,(a^2qz;q)_\infty}{(z;q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2107

Description: Summation formula ([1], Ex. 1.8) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a^2, a^2/b \\ b \end{matrix}; q^2, \frac{bq}{a^2}\right] \longrightarrow \frac{1}{2}\left(\frac{(-b/a;q)_\infty}{(-a;q)_\infty} + \frac{(b/a;q)_\infty}{(a;q)_\infty}\right)\frac{(a^2,q;q^2)_\infty}{(b,bq/a^2;q^2)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2161

Description: Summation formula ([1], (2.10.13); Appendix (II.23)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, q\right] \longrightarrow \frac{(q/c, abq/c;q)_\infty}{(aq/c, bq/c;q)_\infty} - \frac{(q/c,a,b;q)_\infty}{(c/q, aq/c, bq/c;q)_\infty}\,{}_2\phi_1\left[\begin{matrix} aq/c, bq/c \\ q^2/c \end{matrix}; q, q\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2201

Description: Summation formula ([1], Ex. 1.19(i); Appendix (II.10)) in form of a rule.

$$ {}_2\phi_2\left[\begin{matrix} a, q/a \\ -q, b \end{matrix}; q, -b\right] \longrightarrow \frac{(ab, bq/a;q^2)_\infty}{(b;q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2202

Description: Summation formula ([1], Ex. 1.19(ii); Appendix (II.11)) in form of a rule.

$$ {}_2\phi_2\left[\begin{matrix} a^2, b^2 \\ ab\sqrt{q}, -ab\sqrt{q} \end{matrix}; q, -q\right] \longrightarrow \frac{(a^2q, b^2q;q^2)_\infty}{(q, a^2b^2q;q^2)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3201

Description: Summation formula ([1], (1.7.2); Appendix (II.12)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, q^{-n} \\ c, abq^{1-n}/c \end{matrix}; q, q\right] \longrightarrow \frac{(c/a;q)_n(c/b;q)_n}{(c;q)_n(c/ab;q)_n} $$

where n is a nonnegative integer.

Example(s):

```
In[1]:= ph[{a,b,q^-n},{a*b/q^n,q},q,q]
```

```
                  -n
        ┌ a, b, q            ┐
        │                    │
Out[1]= φ│  a b      ; q, q   │
       3 2│  ---, q           │
        └   n                ┘
           q

In[2]:= %/.S3201
Is n a nonnegative integer?
[y|n]:    y

          a        b
         (--; q)  (--; q)
           n    n   n    n
          q        q
Out[2]= -------------------
           -n       a b
        (q   ; q)  (---; q)
                n   n     n
                   q

In[3]:= %%/.S3201
Is n a nonnegative integer?
[y|n]:    n

                  -n
        ┌ a, b, q            ┐
        │                    │
Out[3]= φ│  a b      ; q, q   │
       3 2│  ---, q           │
        └   n                ┘
           q

In[4]:= ph[{a,q^-m,q^-n},{a/q^(n+m),q},q,q]

        ┌     -m   -n            ┐
        │ a, q   , q             │
Out[4]= φ│                ; q, q  │
       3 2│     -m - n            │
        └ a q        , q         ┘

In[5]:= %/.S3201
Is n a nonnegative integer?
[y|n]:    n
Is m a nonnegative integer?
[y|n]:    y
```

```
           a          -m - n
         (--; q)  (q        ; q)
           m    m             m
          q
Out[5]= -------------------------
           -m           -m - n
        (q  ; q)  (a q       ; q)
                m                m
```

See also: S2103, SListe, SumListe, Ers, PosListe.

S3202

Description: Summation formula ([1], Ex. 2.7; Appendix (II.15)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} q^{-2n}, b, c \\ q^{1-2n}/b, q^{1-2n}/c \end{matrix}; q, \frac{q^{2-n}}{bc}\right] \longrightarrow \frac{(b;q)_n (c;q)_n (bc;q)_{2n} (q;q)_{2n}}{(b;q)_{2n} (c;q)_{2n} (bc;q)_n (q;q)_n} $$

where n is a nonnegative integer.

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S3203

Description: Summation formula ([1], Ex. 2.1) in form of a rule.

$$ {}_3W_2(a;-;q,t) \longrightarrow \left(1 - aqt^2\right) \frac{(aq^2t;q)_\infty}{(t;q)_\infty} $$

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S3204

Description: Summation formula ([1], Ex. 3.9) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, \lambda q, b \\ \lambda, \lambda^2 q/b \end{matrix}; q, \frac{\lambda^2}{ab^2}\right] \longrightarrow \frac{\left(1 - \lambda + \frac{\lambda}{b}\left(1 - \frac{\lambda}{a}\right)\right)}{(1-\lambda)\left(1 + \frac{\lambda}{b}\right)} \frac{(\lambda^2/b^2, \lambda^2 q/ab; q)_\infty}{(\lambda^2 q/b, \lambda^2/ab^2; q)_\infty} $$

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S3261

Description: Summation formula ([1], (2.10.12); Appendix (II.24)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, c \\ e, abcq/e \end{matrix}; q, q\right] $$
$$ \longrightarrow \frac{(q/e, bcq/e, acq/e, abq/e; q)_\infty}{(aq/e, bq/e, cq/e, abcq/e; q)_\infty} - \frac{(q/e, a, b, c, abcq^2/e^2; q)_\infty}{(e/q, aq/e, bq/e, cq/e, abcq/e; q)_\infty} {}_3\phi_2\left[\begin{matrix} aq/e, bq/e, cq/e \\ q^2/e, abcq^2/e^2 \end{matrix}; q, q\right] $$

See also: S2103, S3201, SListe, SumListe, Ers, PosListe.

S4301

Description: Summation formula ([1], (2.7.2); Appendix (II.13)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, -\sqrt{a}q, b, c \\ -\sqrt{a}, aq/b, aq/c \end{matrix}; q, \frac{\sqrt{a}q}{bc}\right] \longrightarrow \frac{(aq, \sqrt{a}q/b, \sqrt{a}q/c, aq/bc; q)_\infty}{(aq/b, aq/c, \sqrt{a}q, \sqrt{a}q/bc; q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4302

Description: Summation formula ([1], (2.7.2), terminating form; Appendix (II.14)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, -\sqrt{a}q, b, q^{-n} \\ -\sqrt{a}, aq/b, aq^{1+n} \end{matrix}; q, \frac{\sqrt{a}q^{1+n}}{b}\right] \longrightarrow \frac{(aq;q)_n(\sqrt{a}q/b;q)_n}{(\sqrt{a}q;q)_n(aq/b;q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4303

Description: Summation formula ([1], Ex. 2.8; Appendix (II.17)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} aq^n, c, -c, q^{-n} \\ \sqrt{a}\sqrt{q}, -\sqrt{a}\sqrt{q}, c^2 \end{matrix}; q, q\right] \longrightarrow \begin{cases} 0 & \text{if } n \text{ is odd} \\ \dfrac{c^n (q;q^2)_{\frac{n}{2}} (aq/c^2;q^2)_{\frac{n}{2}}}{(aq;q^2)_{\frac{n}{2}} (c^2q;q^2)_{\frac{n}{2}}} & \text{if } n \text{ is even} \end{cases} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4304

Description: Summation formula (Andrews [1976a] in [1], Appendix (II.19), corrected) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} q^{1+n}, c, -c, q^{-n} \\ e, c^2q/e, -q \end{matrix}; q, q\right] \longrightarrow q^{n(n+1)/2} \frac{(e/q^n, eq^{1+n}, c^2q^{1-n}/e, c^2q^{2+n}/e; q^2)_\infty}{(e, c^2q/e; q)_\infty} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4305

Description: Summation formula ([1], Ex. 2.6) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} q^{-n}, b, c, -q^{1-n}/bc \\ q^{1-n}/b, q^{1-n}/c, -bc \end{matrix}; q, q\right] \longrightarrow \begin{cases} 0 & \text{if } n \text{ is odd} \\ \dfrac{(bc;q)_n}{(b,c;q)_n} \dfrac{(q, b^2, c^2; q^2)_{\frac{n}{2}}}{(b^2c^2;q^2)_{\frac{n}{2}}} & \text{if } n \text{ is even} \end{cases} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4306

Description: Summation formula ([1], Ex. 2.14(i)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, \sqrt{a}q, b, q^{-n} \\ \sqrt{a}, aq/b, b^2q^{1-n} \end{matrix}; q, q\right] \longrightarrow \frac{(a/b^2, 1/b, -\sqrt{a}q/b; q)_n}{(b^{-2}, aq/b, -\sqrt{a}/b; q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4307

Description: Summation formula ([1], (2.7.2)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, \sqrt{a}q, b, c \\ \sqrt{a}, aq/b, aq/c \end{matrix}; q, -\frac{\sqrt{a}q}{bc}\right] \longrightarrow \frac{(aq, aq/bc, -\sqrt{a}q/b, -\sqrt{a}q/c; q)_\infty}{(aq/b, aq/c, -\sqrt{a}q, -\sqrt{a}q/bc; q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4308

Description: Summation formula ([1], (2.8.3), $c \to \sqrt{aq}$, $d \to -q\sqrt{a}$, sum the ${}_8\phi_7$ by [1], (2.6.2)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, -\sqrt{a}q, b, q^{-n} \\ -\sqrt{a}, aq/b, b^2q^{1-n} \end{matrix}; q, q\right] \longrightarrow \frac{(a/b^2, 1/b, \sqrt{a}q/b; q)_n}{(b^{-2}, aq/b, \sqrt{a}/b; q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4361

Description: Summation formula ([1], Ex. 2.9) in form of a rule.

$$ \begin{aligned} {}_4\phi_3\left[\begin{matrix} a, b, -b, aq/c^2 \\ aq/c, -aq/c, b^2 \end{matrix}; q, q\right] \longrightarrow & \frac{(q/b^2, -q; q)_\infty}{(aq/b^2, -aq; q)_\infty} \frac{(a^2q^2, aq^2/b^2, aq^2/c^2, a^2q^2/b^2c^2; q^2)_\infty}{(q^2/b^2, a^2q^2/c^2, aq^2, aq^2/b^2c^2; q^2)_\infty} \\ & - \frac{(q/b^2, a, b, -b, aq/c^2, aq^2/b^2c, -aq^2/b^2c; q)_\infty}{(b^2/q, q/b, -q/b, aq/b^2, aq/c, -aq/c, aq^2/b^2c^2; q)_\infty} {}_4\phi_3\left[\begin{matrix} q/b, -q/b, aq/b^2, aq^2/b^2c^2 \\ aq^2/b^2c, -aq^2/b^2c, q^2/b^2 \end{matrix}; q, q\right] \end{aligned} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5401

Description: Summation formula ([1], (2.7.1), $d \to \sqrt{aq}$) in form of a rule.

$$ {}_5W_4(a; b, c; q, \frac{\sqrt{a\,q}}{b\,c}) \longrightarrow \frac{(aq, aq/bc, \sqrt{a\,q}/b, \sqrt{a\,q}/c; q)_\infty}{(aq/b, aq/c, \sqrt{a\,q}, \sqrt{a\,q}/bc; q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5402

Description: Summation formula ([1], (2.8.3), $c \to q\sqrt{a}$, $d \to -q\sqrt{a}$, sum the ${}_8\phi_7$ by [1], (2.6.2)) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} a, \sqrt{a}q, -\sqrt{a}q, b, q^{-n} \\ \sqrt{a}, -\sqrt{a}, aq/b, b^2q^{2-n} \end{matrix}; q, q\right] \longrightarrow \frac{(a/b^2q, 1/bq; q)_n}{(1/b^2q, aq/b; q)_n} \frac{(aq/b^2; q^2)_n}{(a/b^2q; q^2)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5501

Description: Summation formula ([1], (2.7.1), $d \to \infty$) in form of a rule.

$$ {}_5\phi_5\left[\begin{matrix} a, \sqrt{a}q, -\sqrt{a}q, b, c \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, 0 \end{matrix}; q, \frac{aq}{bc}\right] \longrightarrow \frac{(aq, aq/bc; q)_\infty}{(aq/b, aq/c; q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S6501

Description: Summation formula ([1], (2.7.1); Appendix (II.20)) in form of a rule.

$$ {}_6W_5(a; b, c, d; q, \frac{aq}{bcd}) \longrightarrow \frac{(aq, aq/bc, aq/bd, aq/cd; q)_\infty}{(aq/b, aq/c, aq/d, aq/bcd; q)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S6502

Description: Summation formula ([1], (2.4.2); Appendix (II.21)) in form of a rule.

$$ {}_6W_5(a; b, c, q^{-n}; q, \frac{aq^{1+n}}{bc}) \longrightarrow \frac{(aq; q)_n (aq/bc; q)_n}{(aq/b; q)_n (aq/c; q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S8701

Description: Summation formula ([1], (2.6.2); Appendix (II.22)) in form of a rule.

$$ {}_8\phi_7\left[\begin{matrix} a, \sqrt{a}q, -\sqrt{a}q, b, c, d, a^2q^{1+n}/bcd, q^{-n} \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d, bcd/aq^n, aq^{1+n} \end{matrix}; q, q\right] \longrightarrow \frac{(aq; q)_n (aq/bc; q)_n (aq/bd; q)_n (aq/cd; q)_n}{(aq/b; q)_n (aq/c; q)_n (aq/d; q)_n (aq/bcd; q)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S8702

Description: Summation formula ([1], Ex. 2.17(i); Appendix (II.16)) in form of a rule.

$$ {}_8W_7(-\frac{c\sqrt{a\,b}}{\sqrt{q}}; a, b, c, -c, -\frac{\sqrt{a\,b\,q}}{c}; q, \frac{c\sqrt{q}}{\sqrt{a\,b}}) \longrightarrow \frac{(-c\sqrt{a\,b\,q}, -c\sqrt{q}/\sqrt{a\,b}; q)_\infty}{(-c\sqrt{b\,q}/\sqrt{a}, -c\sqrt{a\,q}/\sqrt{b}; q)_\infty} \frac{(aq, bq, c^2q/a, c^2q/b; q^2)_\infty}{(q, abq, c^2q, c^2q/ab; q^2)_\infty} $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S8703

Description: Summation formula ([1], Ex. 2.17(ii); Appendix (II.18)) in form of a rule.

$${}_8W_7(-c; a, \frac{q}{a}, c, -d, -\frac{q}{d}; q, c) \longrightarrow \frac{(-c, -cq; q)_\infty}{(cd, cq/d, -ac, -cq/a; q)_\infty}(acd, acq/d, cdq/a, cq^2/ad; q^2)_\infty$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S8704

Description: Summation formula ([1], Ex. 3.10) in form of a rule.

$${}_8W_7(-\lambda; \sqrt{\lambda}q, -\sqrt{\lambda}q, a, b, -b; q, \frac{\lambda}{ab^2}) \longrightarrow \frac{(\lambda q, \lambda/a, -\lambda q, \lambda/b^2, \lambda q/ab, -\lambda q/ab; q)_\infty}{(\lambda, \lambda q/a, -\lambda q/a, \lambda q/b, -\lambda q/b, \lambda/ab^2; q)_\infty}$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S8761

Description: Summation formula ([1], (2.11.7); Appendix (II.25)) in form of a rule.

$$\begin{aligned}{}_8W_7(a; b, c, d, e, \frac{a^2q}{bcde}; q, q) \longrightarrow & \frac{(aq, b/a, aq/cd, aq/ce, bde/a, aq/de, bce/a, bcd/a; q)_\infty}{(aq/c, aq/d, aq/e, bcde/a, bc/a, bd/a, be/a, aq/cde; q)_\infty} \\ & - \frac{(aq, b/a, c, d, e, a^2q/bcde, bq/c, bq/d, bq/e, b^2cde/a^2; q)_\infty}{(a/b, aq/c, aq/d, aq/e, bcde/a, bc/a, bd/a, be/a, aq/cde, b^2q/a; q)_\infty} \\ & \qquad {}_8W_7(\frac{b^2}{a}; b, \frac{bc}{a}, \frac{bd}{a}, \frac{be}{a}, \frac{aq}{cde}; q, q)\end{aligned}$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S10901

Description: Summation formula ([1], Ex. 2.12) in form of a rule.

$${}_{10}W_9(a; \sqrt{b}, -\sqrt{b}, \sqrt{bq}, -\sqrt{bq}, \frac{a}{b}, \frac{a^2q^{1+n}}{b}, q^{-n}; q, q) \longrightarrow \frac{(aq, a^2q/b^2; q)_n}{(aq/b, a^2q/b; q)_n}$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

SchreibeZahl

Description: Variable that counts the number of expressions already written by using `TeXMat`. Can be reset by defining a new value.

Usage: `SchreibeZahl=n_Integer.`

Example(s):

```
In[1]:= SchreibeZahl

Out[1]= 0
```

```
In[2]:= TeXMat[pq[a,n],file]

In[3]:= !type file.m
A[1]:=
pq[a, n, q]

In[3]:= !type file.tex
A[1]:=
({\let \over / a}; q) _{n}

In[3]:= SchreibeZahl=4

Out[3]= 4

In[4]:= TeXMat[pq[a,n],file]

In[5]:= !type file.m
A[1]:=
pq[a, n, q]
A[5]:=
pq[a, n, q]

In[5]:= !type file.tex
A[1]:=
({\let \over / a}; q) _{n}
A[5]:=
({\let \over / a}; q) _{n}
```

See also: `TeXMat`.

Sgl1001

Description: Summation formula ([1], (1.3.2); Appendix (II.3)) in form of an equation. It is the same summation as that in `S1001`.

See also: `Sgl2101, SumListe$gl, Gleichung`.

Sgl1101

Description: Summation formula ([1], Ex. 1.6(ii); Appendix (II.5)) in form of an equation. It is the same summation as that in `S1101`.

See also: `Sgl2101, SumListe$gl, Gleichung`.

Sgl2101

Description: Summation formula ([1], (1.5.3); Appendix (II.6)) in form of an equation. It is the same summation as that in `S2101`.

Example(s):

```
In[1]:= {a,c,q}
```

```
Out[1]= {a, c, q}

In[2]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n
```

$$\text{Out[2]=}\ {}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ c \end{matrix}\ ;\ q,\ q\right] == \frac{a^n \left(\frac{c}{a};\ q\right)_n}{(c;\ q)_n}$$

```
In[3]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  y
a=a^2
c=1/b
n=n
Do you want to set a value for q in the equation?  [y|n]:  y
q=q^2
```

$$\text{Out[3]=}\ {}_2\phi_1\left[\begin{matrix} a^2,\ q^{-2n} \\ \frac{1}{b} \end{matrix}\ ;\ q^2,\ q^2\right] == \frac{a^{2n} \left(\frac{1}{a^2 b};\ q^2\right)_n}{\left(\frac{1}{b};\ q^2\right)_n}$$

```
In[4]:= {a,c,q}

Out[4]= {a, c, q}

In[5]:= a=b^2
```

$$\text{Out[5]=}\ b^2$$

```
In[6]:= q=p

Out[6]= p
```

```
In[7]:= Sgl2101
Some variables have a value.  Should the variables
{a, c, n} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:  nv
a=a
c=c
n=n
q has a value.  Should q be cleared?
Do you want to set a value for q in the equation (v)?  [y|n|yv|nv]:  yv
q=p^2

                                  2 n  c    2
                                 b    (--; p )
                                        2     n
           2   -2 n   2   2            b
Out[7]= phi[ b , p     ; p , p ] == --------------
        2 1                                2
               c                      (c; p )
                                              n

In[8]:= {a,c,q}

          2
Out[8]= {b , c, q}

In[9]:= Sgl2101
Some variables have a value.  Should the variables
{a, c, n} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:  y
Do you want to set a value for q in the equation?  [y|n]:  n

                              n  c
                             a  (-; q)
              -n                 a   n
Out[9]= phi[ a, q  ; q, q ] == ----------
        2 1                    (c; q)
             c                       n

In[10]:= {a,c,q}

Out[10]= {a, c, q}
```

See also: `SumListe$gl`, `Gleichung`.

Sgl2102

Description: Summation formula ([1], (1.5.2); Appendix (II.7)) in form of an equation. It is the same summation as that in `S2102`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2103

Description: Summation formula ([1], (1.5.1); Appendix (II.8)) in form of an equation. It is the same summation as that in `S2103`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2104

Description: Summation formula ([1], (1.8.1); Appendix (II.9)) in form of an equation. It is the same summation as that in `S2104`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2105

Description: Summation formula ([1], Ex 1.6(i)) in form of an equation. It is the same summation as that in `S2105`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2106

Description: Summation formula ([1], Ex 1.7) in form of an equation. It is the same summation as that in `S2106`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2107

Description: Summation formula ([1], Ex. 1.8) in form of an equation. It is the same summation as that in `S2107`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2161

Description: Summation formula ([1], (2.10.13); Appendix (II.23)) in form of an equation. It is the same summation as that in `S2161`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2201

Description: Summation formula ([1], Ex. 1.19(i); Appendix (II.10)) in form of an equation. It is the same summation as that in `S2201`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl2202

Description: Summation formula ([1], Ex. 1.19(ii); Appendix (II.11)) in form of an equation. It is the same summation as that in `S2202`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl3201

Description: Summation formula ([1], (1.7.2); Appendix (II.12)) in form of an equation. It is the same summation as that in `S3201`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl3202

Description: Summation formula ([1], Ex. 2.7; Appendix (II.15)) in form of an equation. It is the same summation as that in `S3202`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl3203

Description: Summation formula ([1], Ex. 2.1) in form of an equation. It is the same summation as that in `S3203`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl3204

Description: Summation formula ([1], Ex. 3.9) in form of an equation. It is the same summation as that in `S3204`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl3261

Description: Summation formula ([1], (2.10.12); Appendix (II.24)) in form of an equation. It is the same summation as that in `S3261`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4301

Description: Summation formula ([1], (2.7.2); Appendix (II.13)) in form of an equation. It is the same summation as that in `S4301`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4302

Description: Summation formula ([1], (2.7.2), terminating form; Appendix (II.14)) in form of an equation. It is the same summation as that in `S4302`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4303

Description: Summation formula ([1], Ex. 2.8; Appendix (II.17)) in form of an equation. It is the same summation as that in `S4303`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4304

Description: Summation formula (Andrews [1976a] in [1], Appendix (II.19)) in form of an equation. It is the same summation as that in `S4304`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4305

Description: Summation formula ([1], Ex. 2.6) in form of an equation. It is the same summation as that in `S4305`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4306

Description: Summation formula ([1], Ex. 2.14(i)) in form of an equation. It is the same summation as that in `S4306`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4307

Description: Summation formula ([1], (2.7.2)) in form of an equation. It is the same summation as that in `S4307`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4308

Description: Summation formula ([1], (2.8.3), $c \to \sqrt{aq}$, $d \to -q\sqrt{a}$, sum the ${}_8\phi_7$ by [1], (2.6.2)) in form of an equation. It is the same summation as that in `S4308`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl4361

Description: Summation formula ([1], Ex. 2.9) in form of an equation. It is the same summation as that in `S4361`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5401

Description: Summation formula ([1], (2.7.1), $d \to \sqrt{aq}$) in form of an equation. It is the same summation as that in `S5401`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5402

Description: Summation formula ([1], (2.8.3), $c \to q\sqrt{a}$, $d \to -q\sqrt{a}$, sum the ${}_8\phi_7$ by [1], (2.6.2)) in form of an equation. It is the same summation as that in `S5402`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5501

Description: Summation formula ([1], (2.7.1), $d \to \infty$) in form of an equation. It is the same summation as that in `S5501`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl6501

Description: Summation formula ([1], (2.7.1); Appendix (II.20)) in form of an equation. It is the same summation as that in `S6501`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl6502

Description: Summation formula ([1], (2.4.2); Appendix (II.21)) in form of an equation. It is the same summation as that in `S6502`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl8701

Description: Summation formula ([1], (2.6.2); Appendix (II.22)) in form of an equation. It is the same summation as that in `S8701`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl8702

Description: Summation formula ([1], Ex. 2.17(i); Appendix (II.16)) in form of an equation. It is the same summation as that in `S8702`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl8703

Description: Summation formula ([1], Ex. 2.17(ii); Appendix (II.18)) in form of an equation. It is the same summation as that in `S8703`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl8704

Description: Summation formula ([1], Ex. 3.10) in form of an equation. It is the same summation as that in `S8704`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl8761

Description: Summation formula ([1], (2.11.7); Appendix (II.25)) in form of an equation. It is the same summation as that in `S8761`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl10901

Description: Summation formula ([1], Ex. 2.12) in form of an equation. It is the same summation as that in `S10901`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

SimplifyPQ

Description: Rule that simplifies arguments in `pq`, `pqinf`, `ph`, `Sum`, and expands exponents in powers.

Usage: `Expr/.SimplifyPQ.`

Example(s):

```
In[1]:= pq[a^2+a,(k-1)*2]/pqinf[1-f^2,p]*ph[{a^(1/2-(1-s)/2),-n},{a^(s/2)},q,
        (t-1)/(1-t)]
```

```
               1/2 + (-1 + s)/2
              a                  , -n      -1 + t          2
          φ                          ; q, ------   (a + a ; q)
          2 1            s/2                1 - t                2 (k - 1)
                        a
Out[1]= -------------------------------------------------------------
                                            2
                                    (1 - f ;p)
                                               ∞
```

```
In[2]:= %/.SimplifyPQ
```

```
              -n
          φ      ; q, -1   (a (1 + a); q)
          1 0   -                          2 k - 2 l
Out[2]= ----------------------------------------
                   ((1 - f) (1 + f);p)
                                       ∞
```

```
In[3]:= Simplify[%1]
```

```
              -n                   2
          φ      ; q, -1   (a + a ; q)
          1 0   -                       2 (k - l)
Out[3]= -------------------------------------
                                 2
                       (1 - f ;p)
                                  ∞
```

See also: `Expandq`, `MinusOne`, `SumExpand`, `PQSort`.

SListe

Description: Rule that gives for a basic hypergeometric series a list of applicable summation formulas. Each entry of this list has the format {`ArgumentPermutations`,`S`⟨`number`⟩}, where `ArgumentPermutations` is a sequence of reorderings of the parameters of the basic hypergeometric series (given in terms of `phPerm` and `phTausche`) and `S`⟨`number`⟩ is the name of the summation in form of a rule which can be applied subsequently. You should be aware that `SListe` automatically applies `phOrdne` before checking which summation could be applied.

Important Note: If the value returned by `SListe` is the empty set this does *not* mean that no summation can be applied. You always must remember that the list of summations included in this package is a list of *basic* summations. There are numerous special cases of these summations which are not contained in this list as a separate summation. The examples below should illustrate these remarks.

Usage: `Expr/.SListe`.

Example(s):

```
In[1]:= ph[{a,b},{c},q^2,c/a/b]

              [ a, b    2    c  ]
Out[1]=   phi [       ; q  , --- ]
          2 1 [  c           a b ]

In[2]:= %/.SListe

Be sure to apply "phOrdne" before using the following information!

Out[2]= {{S2103}}

In[3]:= ph[{q^-n,-c,c,q^(1+n)},{-q,e,c^2*q/e},q,q]

               -n           1 + n
              [ q   , -c, c, q                  ]
              [                                  ]
Out[3]=   phi [              2       ; q, q      ]
          4 3 [             c  q                 ]
              [   -q, e, ----                    ]
                              e

In[4]:= %/.SListe
Is n a nonnegative integer?
[y|n]:    y
Is -1 - n a nonnegative integer?
[y|n]:    n

Be sure to apply "phOrdne" before using the following information!

Out[4]= {{phPerm[3,1,2,4,u], phTausche[1,3,l], S4304}}

In[5]:= %3/.phOrdne/.phPerm[3,1,2,4,u]/.phTausche[1,3,l]/.S4304
Is n a nonnegative integer?
[y|n]:    y

                            2  1 - n    2  2 + n
              1            c  q        c  q         e       1 + n   2
Out[5]= ------------- (---------, ---------, --, e q      ; q )
           2                e           e        n                 ∞
          c  q                                   q
        (----, e; q)
          e        ∞
```

Now we consider two examples illustrating the note above. Though none of the implemented summations can be applied, both series are special cases of a q-analogue of Dixon's sum. This fact is also observed by using this package.

```
In[6]:= ph[{a,-q*Sqrt[a],q^-n},{-Sqrt[a],q*a*q^n},q,-Sqrt[q]*q^n]

                                -n
            [ a, -(Sqrt[a] q), q          1/2 + n ]
Out[6]=  φ  [                      ; q, -q        ]
        3 2 [                1 + n                ]
            [  -Sqrt[a], a q                      ]

In[7]:= %/.SListe

Out[7]= {}

In[8]:= Sgl4301
Do you want to set values for the equation?  [y|n]:  y
a=a
b=q^-n
c=-Sqrt[a*q]
Do you want to set a value for q in the equation?  [y|n]:  n

                                -n
            [ a, -(Sqrt[a] q), q          1/2 + n ]
Out[8]=  φ  [                      ; q, -q        ] ==
        3 2 [                1 + n                ]
            [  -Sqrt[a], a q                      ]

                     1 + n                         1/2 + n
     (a q, Sqrt[a] q      , -Sqrt[q], -(Sqrt[a] q        ); q)
                                                              ∞
〉    -----------------------------------------------------------
          1 + n                                   1/2 + n
     (a q      , -(Sqrt[a] Sqrt[q]), Sqrt[a] q, -q        ; q)
                                                              ∞

In[9]:= ph[{-Sqrt[a]*q,q,c},{-Sqrt[a],a*q/c},q,Sqrt[a]/c]

            [ -(Sqrt[a] q), q, c                   ]
            [                          Sqrt[a]     ]
Out[9]=  φ  [                 a q   ; q, -------   ]
        3 2 [   -Sqrt[a], ---              c       ]
            [                  c                   ]

In[10]:= %/.SListe

Out[10]= {}
```

```
In[11]:= %%/.phEinf
Add the parameter:  a

             ┌ a, -(Sqrt[a] q), q, c                 ┐
             │                               Sqrt[a] │
Out[11]=  φ  │                  a q    ; q,  ------- │
         4 3 │  a, -Sqrt[a], ---                c    │
             └                  c                    ┘

In[12]:= %/.SListe

Be sure to apply "phOrdne" before using the following information!

Out[12]= {{S4301}}
```

See also: `TListe`, `phPerm`, `phTausche`, `SumListe`.

Sub

Description: Function that subtracts `Expr` from `Gleichung`.

Usage: `Sub[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                 n   c
            ┌                 ┐  a  (-; q)
            │     -n          │        a    n
Out[1]=  φ  │ a, q   ; q, q   │ == ----------
        2 1 │                 │     (c; q)
            └   c             ┘            n

In[2]:= Sub[pq[a,n]/pq[c/a,n]]

                                                            n   c
            ┌                 ┐  (a; q)        (a; q)      a  (-; q)
            │     -n          │        n              n        a    n
Out[2]=  φ  │ a, q   ; q, q   │ - ------- == -(-------) + ----------
        2 1 │                 │    c              c        (c; q)
            └   c             ┘  (-; q)        (-; q)             n
                                  a    n        a    n

In[3]:= Gleichung
```

Out[3]= $${}_{2}\phi_{1}\left[\begin{matrix} a, q^{-n} \\ c \end{matrix}; q, q\right] - \frac{(a;\, q)_{n}}{(\frac{c}{a};\, q)_{n}} == -\left(\frac{(a;\, q)_{n}}{(\frac{c}{a};\, q)_{n}}\right) + \frac{a^{n} (\frac{c}{a};\, q)_{n}}{(c;\, q)_{n}}$$

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Hoch, GlTausche, Ers.`

Subst

Description: Function that substitutes `RS` instead of `LS` at position `Position` in `Expr`. The parameters `LS` and `RS` are optional. If they are omitted, the right-hand side "`RS`" of `Gleichung` is substituted instead of the left-hand side "`LS`" of `Gleichung`.

Usage: `Subst[Expr,Position,LS,RS]`.

Example(s):

```
In[1]:= Sum[pq[a,k]/pq[b,k]/pq[q,k],{k,0,Infinity}]
```

Out[1]= $$\sum_{k=0}^{\infty} \frac{(a;\, q)_{k}}{(b;\, q)_{k}\, (q;\, q)_{k}}$$

```
In[2]:= Subst[%,{1},pq[a,k]/pq[b,k],ph[{b/a,q^-k},{b},q,a*q^k]]
```

Out[2]= $$\sum_{k=0}^{\infty} \frac{{}_{2}\phi_{1}\left[\begin{matrix} \frac{b}{a}, q^{-k} \\ b \end{matrix}; q, a\, q^{k}\right]}{(q;\, q)_{k}}$$

```
In[3]:= Sgl2102
Do you want to set values for the equation?  [y|n]:  y
a=b/a
c=b
n=k
Do you want to set a value for q in the equation?  [y|n]:  n
```

```
               b    -k                   (a; q)
               -, q             k              k
Out[3]=   φ    a       ; q, a q      == -------
         2 1                            (b; q)
                 b                             k
```

In[4]:= GlTausche

```
         (a; q)           b    -k
                k         -, q             k
Out[4]= ------- ==   φ    a       ; q, a q
         (b; q)     2 1
                k           b
```

In[5]:= Gleichung

```
         (a; q)           b    -k
                k         -, q             k
Out[5]= ------- ==   φ    a       ; q, a q
         (b; q)     2 1
                k           b
```

In[6]:= Subst[%1,{1}]

```
                     b    -k
                     -, q             k
           ∞    φ    a       ; q, a q
          ____ 2 1
          \            b
Out[6]=    〉  ---------------------
          /           (q; q)
          ‾‾‾‾               k
          k=0
```

See also: Gleichung, SumListe$gl, TransListe$gl, LS, RS, GlTausche, Ers, PosListe.

SumErw1

Description: Rule that extends a Sum[] at the top.

$$\sum_{k=l}^{n} \mathrm{Expr} \to \sum_{k=l}^{n+m} \mathrm{Expr} - \sum_{k=n+1}^{n+m} \mathrm{Expr}.$$

The parameter m has to be entered on request.

Usage: Expr/.SumErw1.

Example(s):

In[1]:= Sum[a[k],{k,0,N}]

```
            N
         ┌───┐
          \
Out[1]=    〉  a[k]
          /
         └───┘
          k=0
```

```
In[2]:= %/.SumErw1
top-extend by:  3
```

```
                                              3 + N
                                             ┌───┐
                                              \
Out[2]= -a[1 + N] - a[2 + N] - a[3 + N] +     〉  a[k]
                                              /
                                             └───┘
                                              k=0
```

```
In[3]:= %%/.SumErw1
top-extend by:  M
```

```
         M + N       M + N
         ┌───┐       ┌───┐
          \           \
Out[3]=    〉  a[k] -   〉    a[k]
          /           /
         └───┘       └───┘
          k=0        k=1 + N
```

See also: `SumErw2`, `SumZerl`, `SumShift`, `SumTausche`, `SumRegeln`, `SumUmkehr`, `Ers`, `PosListe`.

SumErw2

Description: Rule that extends a `Sum[]` at the bottom.

$$\sum_{k=l}^{n} \mathrm{Expr} \rightarrow \sum_{k=l-m}^{n} \mathrm{Expr} - \sum_{k=l-m}^{l-1} \mathrm{Expr}.$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SumErw2`.

Example(s):

```
In[1]:= Sum[a[k],{k,0,N}]
```

```
          N
         ____
         \
Out[1]=   〉  a[k]
         /
         ‾‾‾‾
         k=0
```

```
In[2]:= %/.SumErw2
bottom-extend by:  3
```

```
                                          N
                                         ____
                                         \
Out[2]= -a[-3] - a[-2] - a[-1] +          〉  a[k]
                                         /
                                         ‾‾‾‾
                                        k=-3
```

```
In[3]:= %%/.SumErw2
bottom-extend by:  M
```

```
           -1           N
          ____         ____
          \            \
Out[3]= -( 〉  a[k]) +   〉  a[k]
          /            /
          ‾‾‾‾         ‾‾‾‾
          k=-M         k=-M
```

See also: `SumErw1`, `SumZerl`, `SumShift`, `SumTausche`, `SumRegeln`, `SumUmkehr`, `Ers`, `PosListe`.

SumExpand

Description: Rule that expands `Sums`.

Usage: `Expr/.SumExpand`.

Example(s):

```
In[1]:= Sum[(x[k]-y[k])^2,{k,0,(m+n)/2}]
```

```
        m + n
        -----
          2
        ┌───┐            2
Out[1]=  \    (x[k] - y[k])
          〉
         /
        └───┘
         k=0
```

```
In[2]:= %/.SumExpand
```

```
        m   n       m   n               m   n
        - + -       - + -               - + -
        2   2       2   2               2   2
        ┌───┐  2    ┌───┐               ┌───┐  2
Out[2]=  \    x[k] +  \   -2 x[k] y[k] +  \   y[k]
          〉           〉                  〉
         /           /                   /
        └───┘       └───┘               └───┘
         k=0         k=0                 k=0
```

See also: `SimplifyP`, `Expandq`, `MinusOne`, `PSort`.

SumInfinity

Description: Rule that changes the upper bound of a `Sum[]` to `Infinity`.

Usage: `Expr/.SumInfinity`.

Example(s):

```
In[1]:= Sum[pq[q^-n,k]/pq[q,k]*q^k,{k,0,n}]
```

```
          n   k   -n
        ┌───┐q  (q   ; q)
         \               k
Out[1]=   〉 ------------
         /      (q; q)
        └───┘         k
         k=0
```

```
In[2]:= %/.SumInfinity
```

```
         ∞   k   -n
        ┌──┐q  (q   ; q)
         \             k
Out[2]=   〉 ------------
         /     (q; q)
        └──┘         k
        k=0
```

See also: `Sumph, Ers, PosListe.`

SumListe

Description: List of all summation formulas.

Usage: `SumListe.`

See also: `SumListe$gl, SListe.`

SumListe$gl

Description: List of all summation formulas.

Usage: `SumListe$gl.`

See also: `SumListe.`

Sumph

Description: Rule that transforms a `Sum[]` into basic hypergeometric notation, if possible. If the upper bound is not `Infinity` you have to apply `SumInfinity` first (if allowed).

Usage: `Expr/.Sumph.`

Example(s):

```
In[1]:= Sum[pq[q^-n,k]/pq[q,k]*q^k,{k,0,Infinity}]

         ∞   k   -n
        ┌──┐q  (q   ; q)
         \             k
Out[1]=   〉 ------------
         /     (q; q)
        └──┘         k
        k=0

In[2]:= %/.Sumph

              ┌            ┐
              │  -n        │
Out[2]=   φ   │ q   ; q, q │
         1 0  │            │
              └ _          ┘
```

```
In[3]:= Sum[(1-q^(k+2))*pq[{q^-n,a},{b,c,q},k+1]*q^(k*(k+1)/2),
        {k,0,Infinity}]

          oo                                     -n
         ___                                   (q   , a; q)
         \    (k (1 + k))/2        2 + k                  1 + k
Out[3]=   )  q                (1 - q      ) -----------------
         /                                     (b, c, q; q)
         ---                                              1 + k
         k=0

In[4]:= %/.Sumph

                                      1 - n       3
                              a q, q      , q, q                          -n
          (1 - q) (1 + q)  phi                      ; q, -q   (a; q)   (q   ; q)
                          4 4                2   2                  1          1
                                b q, c q, q , q
Out[4]= ------------------------------------------------------------------
                                (b; q)   (c; q)   (q; q)
                                      1        1        1
```

See also: SumRegeln, SumInfinity, ph, phSum, Ers, PosListe.

SumRegeln

Description: Rule that transforms the expressions in a `Sum[]` into a form that could also be expressed in basic hypergeometric notation. This is useful, if you want to convert a `Sum[]` into basic hypergeometric notation but without using the `ph[]`-notation. In particular, expressions of the form $(-1)^{dk}$, where d is an integer and k is the summation index, will simplify.

Usage: `Expr/.SumRegeln.`

Example(s):

```
In[1]:= Sum[Binomialq[n,i]*Binomialq[m,k-i]*q^((n-i)*(k-i)),{i,0,Infinity}]

          oo
         ___
         \    (-i + k) (-i + n)  [   m    ] [ n ]
Out[1]=   )  q                   [        ] [   ]
         /                       [ -i + k ] [ i ]
         ---                               q     q
         i=0

In[2]:= %/.SumRegeln
```

```
                 ∞    i   -k        -n
               ┌───┐ q  (q  ; q)  (q  ; q)
         k n    \               i         i     1 - k + m
        q     (  ⟩  ------------------------) (q         ; q)
                /             1 - k + m                       k
               └───┘(q; q)  (q         ; q)
               i=0        i                i
Out[2]= -------------------------------------------------------
                                (q; q)
                                      k
```

See also: `ph`, `Sumph`, `phSum`, `MinusOne`, `Ers`, `PosListe`.

SumSammle

Description: Rule that causes all terms of an expression `Expr`, which involves a `Sum[]` to be put into the `Sum[]`.

Usage: `Expr/.SumSammle`.

Example(s):

```
In[1]:= pq[a,n]/pq[b,m]*(-1)^n*Sum[1/pq[q,k],{k,0,Infinity}]
```

```
                  ∞
                ┌───┐
            n    \      1
        (-1)   (  ⟩  -------) (a; q)
                 /   (q; q)         n
                └───┘      k
                k=0
Out[1]= -----------------------------
                  (b; q)
                        m
```

```
In[2]:= %/.SumSammle
```

```
          ∞       n
         ┌───┐ (-1)  (a; q)
          \                n
Out[2]=    ⟩  ---------------
          /   (b; q)  (q; q)
         └───┘      m       k
         k=0
```

See also: `SumRegeln`, `SumErw1`, `SumErw2`, `SumZerl`, `SumShift`, `SumTausche`, `pqzus`, `pqinfzus`, `Ers`, `PosListe`.

SumShift

Description: Rule that shifts the index in a `Sum[]`.

$$\sum_{k=l}^{n} \mathrm{Expr}(k) \to \sum_{k=l-m}^{n-m} \mathrm{Expr}(k+m).$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SumShift.`

Example(s):

```
In[1]:= Sum[a[k],{k,3,N}]

            N
          ┌───┐
           \
Out[1]=     〉  a[k]
           /
          └───┘
           k=3

In[2]:= %/.SumShift
shift summation index by:  3

         -3 + N
          ┌───┐
           \
Out[2]=     〉   a[3 + k]
           /
          └───┘
           k=0
```

See also: `SumErw1`, `SumErw2`, `SumTausche`, `SumZerl`, `SumRegeln`, `SumUmkehr`, `Ers`, `PosListe`.

SumTausche

Description: Rule that exchanges summations. You should apply `SumSammle` before applying `SumTausche`.

$$\sum_{k_1=l_1}^{n_1} \sum_{k_2=l_2}^{n_2} \mathrm{Expr} \to \sum_{k_2=l_2}^{n_2} \sum_{k_1=l_1}^{n_1} \mathrm{Expr}.$$

Usage: `Expr/.SumTausche.`

Example(s):

```
In[1]:= Sum[Sum[Binomialq[n,k+l],{k,0,n1}],{l,0,n2}]
```

$$\text{Out[1]=} \sum_{l=0}^{n2} \sum_{k=0}^{n1} \begin{bmatrix} n \\ k+1 \end{bmatrix}_q$$

```
In[2]:= %/.SumTausche
```

$$\text{Out[2]=} \sum_{k=0}^{n1} \sum_{l=0}^{n2} \begin{bmatrix} n \\ k+1 \end{bmatrix}_q$$

See also: `SumErw1`, `SumErw2`, `SumSammle`, `SumShift`, `SumZerl`, `SumRegeln`, `SumUmkehr`, `Ers`, `PosListe`.

SumUmkehr

Description: Rule that reverses the order of summation. `SumUmkehr` applies to `Sum[]` as well as `ph[]`.

Usage: `Expr/.SumUmkehr`.

Example(s):

```
In[1]:= ph[{a,q^-n},{b},q,z]
```

$$\text{Out[1]=} {}_2\phi_1\left[\begin{matrix} a,\ q^{-n} \\ b \end{matrix} ;\ q,\ z\right]$$

```
In[2]:= %/.SumUmkehr
Is n a nonnegative integer?
[y|n]:    y
```

$$\text{Out[2]= }\frac{(-1)^n z^n \,{}_2\phi_1\left[\begin{matrix} q^{-n},\ \frac{q^{1-n}}{b} \\ \frac{q^{1-n}}{a} \end{matrix};\ q,\ \frac{b\,q^{1+n}}{a\,z}\right]\frac{(a;\,q)_n}{(b;\,q)_n}}{q^{(n\,(1+n))/2}}$$

```
In[3]:= ph[{q^-m,q^-n},{b},q,z]
```

$$\text{Out[3]= }{}_2\phi_1\left[\begin{matrix} q^{-m},\ q^{-n} \\ b \end{matrix};\ q,\ z\right]$$

```
In[4]:= %/.SumUmkehr
Is m a nonnegative integer?
[y|n]:    n
Is n a nonnegative integer?
[y|n]:    y
```

$$\text{Out[4]= }\frac{(-1)^n z^n \,{}_2\phi_1\left[\begin{matrix} q^{-n},\ \frac{q^{1-n}}{b} \\ q^{1+m-n} \end{matrix};\ q,\ \frac{b\,q^{1+m+n}}{z}\right]\frac{(q^{-m};\,q)_n}{(b;\,q)_n}}{q^{(n\,(1+n))/2}}$$

```
In[5]:= Sum[pq[q^-n,k]/pq[q,k],{k,0,Infinity}]
```

$$\text{Out[5]= }\sum_{k=0}^{\infty}\frac{(q^{-n};\,q)_k}{(q;\,q)_k}$$

```
In[6]:= %/.SumUmkehr
Is n a nonnegative integer?
[y|n]:    y

           n    k + k n    -n
          ┌───┐q          (q   ; q)
          \                        k     -n
        (  〉  ------------------) (q   ; q)
          /         (q; q)                   n
          └───┘               k
          k=0
Out[6]= -----------------------------------
                        (q; q)
                              n
```

See also: `SumErw1, SumErw2, SumZerl, SumShift, SumTausche, SumRegeln, Ers, PosListe.`

SumZerl

Description: Rule that splits a `Sum[]` into two parts.

$$\sum_{k=l}^{n} \text{Expr} \to \sum_{k=l}^{l+m-1} \text{Expr} + \sum_{k=l+m}^{n} \text{Expr}.$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SumZerl.`

Example(s):

```
In[1]:= Sum[a[k],{k,0,N}]

          N
        ┌───┐
         \
Out[1]=   〉  a[k]
         /
        └───┘
         k=0

In[2]:= %/.SumZerl
bottom-split by:  3

                                        N
                                      ┌───┐
                                       \
Out[2]= a[0] + a[1] + a[2] +            〉  a[k]
                                       /
                                      └───┘
                                       k=3
```

```
In[3]:= %%/.SumZerl
bottom-split by:  M

        -1 + M         N
        ┌───┐        ┌───┐
         \            \
Out[3]=   〉   a[k] +   〉  a[k]
         /            /
        └───┘        └───┘
         k=0          k=M
```

See also: `SumErw1, SumErw2, SumShift, SumTausche, SumRegeln, SumUmkehr, Ers, PosListe.`

T2101

Description: Transformation formula ([1], (1.4.1); Appendix (III.1)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(b, az; q)_\infty}{(c, z; q)_\infty} {}_2\phi_1\left[\begin{matrix} c/b, z \\ az \end{matrix}; q, b\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2102

Description: Transformation formula ([1], (1.4.5); Appendix (III.2)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(c/b, bz; q)_\infty}{(c, z; q)_\infty} {}_2\phi_1\left[\begin{matrix} abz/c, b \\ bz \end{matrix}; q, \frac{c}{b}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2103

Description: Transformation formula ([1], (1.4.6); Appendix (III.3)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(abz/c; q)_\infty}{(z; q)_\infty} {}_2\phi_1\left[\begin{matrix} c/a, c/b \\ c \end{matrix}; q, \frac{abz}{c}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2104

Description: Transformation formula ([1], (1.5.4); Appendix (III.4)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(az; q)_\infty}{(z; q)_\infty} {}_2\phi_2\left[\begin{matrix} a, c/b \\ c, az \end{matrix}; q, bz\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2105

Description: Transformation formula ([1], (3.2.4); Appendix (III.5)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(abz/c;q)_\infty}{(bz/c;q)_\infty}\, {}_3\phi_2\left[\begin{matrix} a, c/b, 0 \\ c, cq/bz \end{matrix}; q, q\right] $$

provided the ${}_3\phi_2$ is terminating.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2106

Description: Transformation formula ([1], (1.5.6); Appendix (III.6)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} b, q^{-n} \\ c \end{matrix}; q, z\right] \longrightarrow \frac{b^n z^n}{q^n} \frac{(c/b;q)_n}{(c;q)_n}\, {}_3\phi_2\left[\begin{matrix} q^{-n}, q/z, q^{1-n}/c \\ bq^{1-n}/c, 0 \end{matrix}; q, q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2107

Description: Transformation formula ([1], Ex. 1.15(iii); Appendix (III.7)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} b, q^{-n} \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(c/b;q)_n}{(c;q)_n}\, {}_3\phi_2\left[\begin{matrix} q^{-n}, b, bz/cq^n \\ bq^{1-n}/c, 0 \end{matrix}; q, q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2108

Description: Transformation formula ([1], Ex. 1.15(ii); Appendix (III.8)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} b, q^{-n} \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(c/b;q)_n}{(c;q)_n} b^n\, {}_3\phi_1\left[\begin{matrix} q^{-n}, b, q/z \\ bq^{1-n}/c \end{matrix}; q, \frac{z}{c}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2109

Description: Transformation formula ([1], Ex. 2.2, reversed) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a, b \\ bq/a \end{matrix}; q, t\right] \longrightarrow \frac{(b, at;q)_\infty}{(t, b/a;q)_\infty}\, {}_4W_3\left(\frac{t}{q}; \frac{1}{a}; q, b\right) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2110

Description: Transformation formula ([1], (3.4.7)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a,b \\ aq/b \end{matrix}; q, \frac{qx}{b^2}\right] \longrightarrow \frac{(qx/b, aqx^2/b^2;q)_\infty}{(aqx/b, qx^2/b^2;q)_\infty} {}_8W_7(\frac{ax}{b}; x, \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}; q, \frac{qx}{b^2}) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2111

Description: Transformation formula ([1], (3.5.4)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a^2,b^2 \\ a^2q^2/b^2 \end{matrix}; q^2, \frac{q^2x^2}{b^4}\right] \longrightarrow \frac{(aq/b^2, qx^2/b^2;q)_\infty}{(a^2q/b^2, aqx^2/b^2;q)_\infty} \frac{(a^2qx^2/b^2, a^2q^2x^2/b^4;q^2)_\infty}{(qx^2/b^2, q^2x^2/b^4;q^2)_\infty} {}_8W_7(\frac{ax^2}{b^2}; a, x, -x, \frac{\sqrt{q}x}{b}, -\frac{\sqrt{q}x}{b}; q, \frac{aq}{b^2}) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2112

Description: Transformation formula ([1], Ex. 3.2(i), reversed) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a,aq \\ b^2q \end{matrix}; q^2, z^2\right] \longrightarrow \frac{(az;q)_\infty}{(z;q)_\infty} {}_3\phi_2\left[\begin{matrix} a,b,-b \\ b^2, az \end{matrix}; q, -z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2161

Description: Transformation formula ([1], (3.3.5); Appendix (III.31)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a,b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(abz/c, q/c;q)_\infty}{(az/c, q/a;q)_\infty} {}_2\phi_1\left[\begin{matrix} c/a, cq/abz \\ cq/az \end{matrix}; q, \frac{bq}{c}\right] - \frac{(b, q/c, c/a, az/q, q^2/az;q)_\infty}{(c/q, bq/c, q/a, az/c, cq/az;q)_\infty} {}_2\phi_1\left[\begin{matrix} aq/c, bq/c \\ q^2/c \end{matrix}; q, z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2162

Description: Transformation formula ([1], Appendix (III.32)) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a,b \\ c \end{matrix}; q, z\right] \longrightarrow \frac{(b, c/a, az, q/az;q)_\infty}{(c, b/a, z, q/z;q)_\infty} {}_2\phi_1\left[\begin{matrix} a, aq/c \\ aq/b \end{matrix}; q, \frac{cq}{abz}\right] + \frac{(a, c/b, bz, q/bz;q)_\infty}{(c, a/b, z, q/z;q)_\infty} {}_2\phi_1\left[\begin{matrix} b, bq/c \\ bq/a \end{matrix}; q, \frac{cq}{abz}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2163

Description: Transformation formula ([1], Ex. 3.8) in form of a rule.

$$ {}_2\phi_1\left[\begin{matrix} a,b \\ c \end{matrix};q,x\right] \longrightarrow \frac{(b,c/a,ax;q)_\infty}{(b/a,c,x;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} a,c/b,0 \\ aq/b,ax \end{matrix};q,q\right] + \frac{(a,c/b,bx;q)_\infty}{(a/b,c,x;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} b,c/a,0 \\ bq/a,bx \end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2201

Description: Transformation formula ([1], (1.5.4), reversed; Appendix (III.4), reversed) in form of a rule.

$$ {}_2\phi_2\left[\begin{matrix} a,b \\ c,d \end{matrix};q,\frac{cd}{ab}\right] \longrightarrow \frac{(d/a;q)_\infty}{(d;q)_\infty}\,{}_2\phi_1\left[\begin{matrix} a,c/b \\ c \end{matrix};q,\frac{d}{a}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2202

Description: Transformation formula ([1], Ex. 3.2(ii), reversed) in form of a rule.

$$ {}_2\phi_2\left[\begin{matrix} a^2,b^2 \\ a^2b^2q,a^2z^2 \end{matrix};q^2,a^2qz^2\right] \longrightarrow \frac{(z;q)_\infty(-a^2z;q)_\infty}{(a^2z^2;q^2)_\infty}\,{}_3\phi_2\left[\begin{matrix} a^2,ab,-ab \\ a^2b^2,-a^2z \end{matrix};q,z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3101

Description: Transformation formula ([1], Ex. 1.15(ii), reversed; Appendix (III.8), reversed) in form of a rule.

$$ {}_3\phi_1\left[\begin{matrix} b,c,q^{-n} \\ bcz/q^n \end{matrix};q,z\right] \longrightarrow b^n\frac{(q/cz;q)_n}{(q/bcz;q)_n}\,{}_2\phi_1\left[\begin{matrix} q^{-n},b \\ q/cz \end{matrix};q,\frac{q}{c}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3201

Description: Transformation formula ([1], (3.2.4), reversed; Appendix (III.5), reversed) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a,b,0 \\ c,d \end{matrix};q,q\right] \longrightarrow \frac{(q/d;q)_\infty}{(aq/d;q)_\infty}\,{}_2\phi_1\left[\begin{matrix} a,c/b \\ c \end{matrix};q,\frac{bq}{d}\right] $$

provided the ${}_3\phi_2$ is terminating.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3202

Description: Transformation formula ([1], (1.5.6), reversed; Appendix (III.6), reversed) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} b,c,q^{-n} \\ d,0 \end{matrix};q,q\right] \longrightarrow \frac{b^nc^n}{d^n}\frac{(q^{1-n}/c;q)_n}{(q^{1-n}/d;q)_n}\,{}_2\phi_1\left[\begin{matrix} q^{-n},d/c \\ q^{1-n}/c \end{matrix};q,\frac{q}{b}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3203

Description: Transformation formula ([1], Ex. 1.15(iii), reversed; Appendix (III.7), reversed) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} b, c, q^{-n} \\ d, 0 \end{matrix}; q, q\right] \longrightarrow \frac{(bq^{1-n}/d; q)_n}{(q^{1-n}/d; q)_n} {}_2\phi_1\left[\begin{matrix} q^{-n}, b \\ bq^{1-n}/d \end{matrix}; q, \frac{cq}{d}\right]$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3204

Description: Transformation formula ([1], (3.2.7); Appendix (III.9)) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, \frac{de}{abc}\right] \longrightarrow \frac{(e/a, de/bc; q)_\infty}{(e, de/abc; q)_\infty} {}_3\phi_2\left[\begin{matrix} a, d/b, d/c \\ d, de/bc \end{matrix}; q, \frac{e}{a}\right]$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3205

Description: Transformation formula ([1], (3.2.10); Appendix (III.10)) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, \frac{de}{abc}\right] \longrightarrow \frac{(b, de/ab, de/bc; q)_\infty}{(d, e, de/abc; q)_\infty} {}_3\phi_2\left[\begin{matrix} d/b, e/b, de/abc \\ de/ab, de/bc \end{matrix}; q, b\right]$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3206

Description: Transformation formula ([1], (3.2.3); Appendix (III.11)) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} b, c, q^{-n} \\ d, e \end{matrix}; q, q\right] \longrightarrow \frac{b^n c^n}{d^n} \frac{(de/bc; q)_n}{(e; q)_n} {}_3\phi_2\left[\begin{matrix} q^{-n}, d/b, d/c \\ d, de/bc \end{matrix}; q, q\right]$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3207

Description: Transformation formula ([1], (3.2.2); Appendix (III.12)) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} b, c, q^{-n} \\ d, e \end{matrix}; q, q\right] \longrightarrow c^n \frac{(e/c; q)_n}{(e; q)_n} {}_3\phi_2\left[\begin{matrix} q^{-n}, c, d/b \\ d, cq^{1-n}/e \end{matrix}; q, \frac{bq}{e}\right]$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3208

Description: Transformation formula ([1], (3.2.5); Appendix (III.13)) in form of a rule.

$${}_3\phi_2\left[\begin{matrix} b, c, q^{-n} \\ d, e \end{matrix}; q, \frac{deq^n}{bc}\right] \longrightarrow \frac{(e/c; q)_n}{(e; q)_n} {}_3\phi_2\left[\begin{matrix} q^{-n}, c, d/b \\ d, cq^{1-n}/e \end{matrix}; q, q\right]$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3209

Description: Transformation formula ([1], Ex. 2.26; Appendix (III.14)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} q^{-n}, b, c \\ q^{1-n}/b, q^{1-n}/c \end{matrix}; q, \frac{q^{1-n}z}{bc}\right] \longrightarrow \frac{(z/q^n;q)_\infty}{(z;q)_\infty} {}_5\phi_4\left[\begin{matrix} q^{-n/2}, -q^{-n/2}, q^{1/2-n/2}, -q^{1/2-n/2}, q^{1-n}/bc \\ q^{1-n}/b, q^{1-n}/c, z/q^n, q/z \end{matrix}; q, q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3210

Description: Transformation formula ([1], (3.2.6)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, q^{-n} \\ e, d \end{matrix}; q, \frac{deq^n}{ab}\right] \longrightarrow \left(\frac{deq^{n-1}}{a}\right)^n \frac{(aq^{1-n}/d, aq^{1-n}/e;q)_n}{(d,e;q)_n} {}_3\phi_2\left[\begin{matrix} a, abq^{1-n}/de, q^{-n} \\ aq^{1-n}/e, aq^{1-n}/d \end{matrix}; q, \frac{q}{b}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3211

Description: Transformation formula ([1], Ex. 3.1) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a^2, b^2, z \\ ab\sqrt{q}, -ab\sqrt{q} \end{matrix}; q, q\right] \longrightarrow {}_3\phi_2\left[\begin{matrix} a^2, b^2, z^2 \\ a^2b^2q, 0 \end{matrix}; q^2, q^2\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3212

Description: Transformation formula ([1], Ex. 3.1, reversed) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a^2, b^2, z^2 \\ a^2b^2q, 0 \end{matrix}; q^2, q^2\right] \longrightarrow {}_3\phi_2\left[\begin{matrix} a^2, b^2, z \\ ab\sqrt{q}, -ab\sqrt{q} \end{matrix}; q, q\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3213

Description: Transformation formula ([1], Ex. 3.2(i)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, -b \\ b^2, az \end{matrix}; q, -z\right] \longrightarrow \frac{(z;q)_\infty}{(az;q)_\infty} {}_2\phi_1\left[\begin{matrix} a, aq \\ b^2q \end{matrix}; q^2, z^2\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3214

Description: Transformation formula ([1], Ex. 3.2(ii)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, -b \\ b^2, -az \end{matrix}; q, z\right] \longrightarrow \frac{(az^2; q^2)_\infty}{(z; q)_\infty(-az; q)_\infty} {}_2\phi_2\left[\begin{matrix} a, b^2/a \\ b^2q, az^2 \end{matrix}; q^2, aqz^2\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3215

Description: Transformation formula ([1], Ex. 3.3) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, q/a, z \\ c, -q \end{matrix}; q, q\right] \longrightarrow \frac{(-1, -qz/c; q)_\infty}{(-q/c, -z; q)_\infty} {}_3\phi_2\left[\begin{matrix} c/a, ac/q, z^2 \\ c^2, 0 \end{matrix}; q^2, q^2\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3216

Description: Transformation formula ([1], Ex. 3.3, reversed) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} c, a^2/cq, z^2 \\ a^2, 0 \end{matrix}; q^2, q^2\right] \longrightarrow \frac{(-q/a, -z; q)_\infty}{(-1, -qz/a; q)_\infty} {}_3\phi_2\left[\begin{matrix} a/c, cq/a, z \\ a, -q \end{matrix}; q, q\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3217

Description: Transformation formula ([1], (3.2.11)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, \frac{de}{abc}\right] \longrightarrow \frac{(de/ab, de/ac; q)_\infty}{(de/a, de/abc; q)_\infty} {}_7\phi_7\left[\begin{matrix} de/aq, \sqrt{deq}/\sqrt{a}, -\sqrt{deq}/\sqrt{a}, e/a, d/a, b, c \\ \sqrt{de}/\sqrt{aq}, -\sqrt{de}/\sqrt{aq}, d, e, de/ab, de/ac, 0 \end{matrix}; q, \frac{de}{bc}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3261

Description: Transformation formula ([1], (3.3.3); Appendix (III.33)) in form of a rule.

$$ \begin{aligned} {}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, \frac{de}{abc}\right] \longrightarrow & \frac{(e/b, e/c, cq/a, q/d; q)_\infty}{(c, cq/d, q/a, e/bc; q)_\infty} {}_3\phi_2\left[\begin{matrix} c, d/a, cq/e \\ cq/a, bcq/e \end{matrix}; q, \frac{bq}{d}\right] \\ & - \frac{(q/d, eq/d, b, c, d/a, de/bcq, bcq^2/de; q)_\infty}{(d/q, e, bq/d, cq/d, q/a, e/bc, bcq/e; q)_\infty} {}_3\phi_2\left[\begin{matrix} aq/d, bq/d, cq/d \\ q^2/d, eq/d \end{matrix}; q, \frac{de}{abc}\right] \end{aligned} $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3262

Description: Transformation formula ([1], (3.3.1); Appendix (III.34)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};q,\frac{de}{abc}\right] \longrightarrow \frac{(e/b,e/c;q)_\infty}{(e,e/bc;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} d/a,b,c \\ d,bcq/e \end{matrix};q,q\right] + \frac{(d/a,b,c,de/bc;q)_\infty}{(d,e,bc/e,de/abc;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} e/b,e/c,de/abc \\ de/bc,eq/bc \end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3263

Description: Transformation formula ([1], (3.3.3), reversed; Appendix (III.33)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};q,\frac{de}{abc}\right] \longrightarrow \frac{(dq/bc,e/c,a,b,abq/de,de/ab;q)_\infty}{(aq/e,q/c,d,ab/d,de/abc,e;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} q/b,de/abc,d/b \\ dq/ab,dq/bc \end{matrix};q,\frac{bq}{e}\right] + \frac{(aq/c,d/b,d/a,q/e;q)_\infty}{(aq/e,q/c,d,d/ab;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} aq/d,e/c,a \\ abq/d,aq/c \end{matrix};q,\frac{bq}{e}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3264

Description: Transformation formula ([1], (3.3.1), reversed; Appendix (III.34)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};q,q\right] \longrightarrow \frac{(bcq/e,q/e;q)_\infty}{(cq/e,bq/e;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} d/a,b,c \\ d,bcq/e \end{matrix};q,\frac{aq}{e}\right] - \frac{(q/e,a,b,c,dq/e;q)_\infty}{(cq/e,bq/e,d,e/q,aq/e;q)_\infty}\,{}_3\phi_2\left[\begin{matrix} cq/e,bq/e,aq/e \\ dq/e,q^2/e \end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3265

Description: Transformation formula ([1], (3.4.1); Appendix (III.35)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a,b,c \\ aq/b,aq/c \end{matrix};q,\frac{aqx}{bc}\right] \longrightarrow \frac{(ax;q)_\infty}{(x;q)_\infty}\,{}_5\phi_4\left[\begin{matrix} \sqrt{a},-\sqrt{a},\sqrt{a}\sqrt{q},-\sqrt{a}\sqrt{q},aq/bc \\ aq/b,aq/c,ax,q/x \end{matrix};q,q\right] + \frac{(a,aq/bc,aqx/b,aqx/c;q)_\infty}{(aq/b,aq/c,aqx/bc,1/x;q)_\infty}\,{}_5\phi_4\left[\begin{matrix} \sqrt{a}x,-\sqrt{a}x,\sqrt{a}\sqrt{q}x,-\sqrt{a}\sqrt{q}x,aqx/bc \\ aqx/b,aqx/c,qx,ax^2 \end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3266

Description: Transformation formula ([1], (3.5.2)) in form of a rule.

$$ {}_3\phi_2\left[\begin{matrix} a^2,x^2,y^2 \\ a^2b^2,b^2x^2y^2 \end{matrix};q^2,b^2q\right] \longrightarrow \frac{(b^2;q^2)_\infty}{(a^2b^2;q^2)_\infty}\frac{(a,-ab^2;q)_\infty}{(-1,b^2;q)_\infty}\,{}_5\phi_4\left[\begin{matrix} -a,-bx,bx,-by,by \\ -q,-ab^2,-bxy,bxy \end{matrix};q,q\right] + \frac{(b^2;q^2)_\infty}{(a^2b^2;q^2)_\infty}\frac{(-a,ab^2;q)_\infty}{(-1,b^2;q)_\infty}\,{}_5\phi_4\left[\begin{matrix} a,bx,-bx,by,-by \\ -q,ab^2,bxy,-bxy \end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3267

Description: Transformation formula ([1], Ex. 3.6) in form of a rule.

$$
{}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, q\right] \longrightarrow \frac{(q/e, abq/e, acq/e, d/a; q)_\infty}{(d, aq/e, bq/e, cq/e; q)_\infty} {}_3\phi_2\left[\begin{matrix} a, aq/e, abcq/de \\ abq/e, acq/e \end{matrix}; q, \frac{d}{a}\right] - \frac{(q/e, a, b, c, dq/e; q)_\infty}{(e/q, aq/e, bq/e, cq/e, d; q)_\infty} {}_3\phi_2\left[\begin{matrix} aq/e, bq/e, cq/e \\ q^2/e, dq/e \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3268

Description: Transformation formula ([1], Ex. 3.6, reversed) in form of a rule.

$$
{}_3\phi_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; q, \frac{de}{abc}\right] \longrightarrow \frac{(de/bc, b, d/a, e/a; q)_\infty}{(b/a, d, e, de/abc; q)_\infty} {}_3\phi_2\left[\begin{matrix} a, d/b, e/b \\ de/bc, aq/b \end{matrix}; q, q\right] + \frac{(a, d/b, e/b, de/ac; q)_\infty}{(d, e, de/abc, a/b; q)_\infty} {}_3\phi_2\left[\begin{matrix} b, d/a, e/a \\ bq/a, de/ac \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3269

Description: Transformation formula ([1], Ex. 3.8, reversed) in form of a rule.

$$
{}_3\phi_2\left[\begin{matrix} a, b, 0 \\ c, x \end{matrix}; q, q\right] \longrightarrow \frac{(q/c, abq/c, x/a; q)_\infty}{(aq/c, bq/c, x; q)_\infty} {}_2\phi_1\left[\begin{matrix} a, aq/c \\ abq/c \end{matrix}; q, \frac{x}{a}\right] - \frac{(a, b, qx/c, q/c; q)_\infty}{(c/q, aq/c, bq/c, x; q)_\infty} {}_3\phi_2\left[\begin{matrix} aq/c, bq/c, 0 \\ q^2/c, qx/c \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4201

Description: Transformation formula ([1], Ex. 3.4, reversed) in form of a rule.

$$
{}_4\phi_2\left[\begin{matrix} a, b, -b, q^{-n} \\ aq^{1-n}/d, b^2 \end{matrix}; q, -\frac{q}{d}\right] \longrightarrow a^{-n} \frac{(d; q)_n}{(d/a; q)_n} {}_4\phi_3\left[\begin{matrix} a, aq, q^{1-n}, q^{-n} \\ d, dq, b^2 q \end{matrix}; q^2, q^2\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4301

Description: Transformation formula ([1], (2.10.4); Appendix (III.15)) in form of a rule.

$$
{}_4\phi_3\left[\begin{matrix} a, b, c, q^{-n} \\ e, f, abcq^{1-n}/ef \end{matrix}; q, q\right] \longrightarrow a^n \frac{(e/a, f/a; q)_n}{(e, f; q)_n} {}_4\phi_3\left[\begin{matrix} q^{-n}, a, acq^{1-n}/ef, abq^{1-n}/ef \\ abcq^{1-n}/ef, aq^{1-n}/e, aq^{1-n}/f \end{matrix}; q, q\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4302

Description: Transformation formula ([1], (3.2.9); Appendix (III.16)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, q^{-n} \\ e, f, abcq^{1-n}/ef \end{matrix}; q, q\right] \longrightarrow \frac{(a, ef/ab, ef/ac; q)_n}{(e, f, ef/abc; q)_n} {}_4\phi_3\left[\begin{matrix} q^{-n}, e/a, f/a, ef/abc \\ ef/ab, ef/ac, q^{1-n}/a \end{matrix}; q, q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4303

Description: Transformation formula ([1], (2.5.1), reversed; Appendix (III.19)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, q^{-n} \\ e, f, abcq^{1-n}/ef \end{matrix}; q, q\right] \longrightarrow \frac{(ef/ab, ef/ac; q)_n}{(ef/a, ef/abc; q)_n} {}_8W_7(\frac{ef}{aq}; \frac{f}{a}, \frac{e}{a}, b, c, q^{-n}; q, \frac{efq^n}{bc}) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4304

Description: Transformation formula ([1], (2.10.7); Appendix (III.20)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, q^{-n} \\ d, e, abcq^{1-n}/de \end{matrix}; q, q\right] \longrightarrow \frac{(deq^n/c, deq^n/b, deq^n/a, deq^n/abc; q)_\infty}{(deq^n/bc, deq^n/ac, deq^n/ab, deq^n; q)_\infty} {}_8W_7(deq^{-1+n}; a, b, c, dq^n, eq^n; q, \frac{de}{abc}) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4305

Description: Transformation formula ([1], (3.10.13); Appendix (III.21)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a^2, b^2, c, d \\ ab\sqrt{q}, -ab\sqrt{q}, -cd \end{matrix}; q, q\right] \longrightarrow {}_4\phi_3\left[\begin{matrix} a^2, b^2, c^2, d^2 \\ a^2b^2q, -cd, -cdq \end{matrix}; q^2, q^2\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4306

Description: Transformation formula ([1], (3.10.13), reversed; Appendix (III.21), reversed) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a^2, b^2, c^2, d^2 \\ a^2b^2q, -cd, -cdq \end{matrix}; q^2, q^2\right] \longrightarrow {}_4\phi_3\left[\begin{matrix} a^2, b^2, c, d \\ ab\sqrt{q}, -ab\sqrt{q}, -cd \end{matrix}; q, q\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4307

Description: Transformation formula ([1], (8.8.3); Appendix (III.22)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, abz, ab/z \\ ab\sqrt{q}, -ab\sqrt{q}, -ab \end{matrix}; q, q\right]^2 \longrightarrow {}_5\phi_4\left[\begin{matrix} a^2, b^2, ab, abz, ab/z \\ ab\sqrt{q}, -ab\sqrt{q}, -ab, a^2b^2 \end{matrix}; q, q\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4308

Description: Transformation formula ([1], Ex. 2.2) in form of a rule.

$$ {}_4W_3(a; b; q, t) \longrightarrow \frac{(aq, bt; q)_\infty}{(t, aq/b; q)_\infty} {}_2\phi_1\left[\begin{matrix} 1/b, t \\ bqt \end{matrix}; q, aq\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4309

Description: Transformation formula ([1], Ex. 2.13(i)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, d \\ aq/b, aq/c, aq/d \end{matrix}; q, \frac{a^2q^3}{b^2c^2d^2}\right] \longrightarrow \frac{(aq^2/bcd, a^3q^3/b^2c^2d^2; q)_\infty}{(a^2q^2/bcd, a^2q^3/b^2c^2d^2; q)_\infty} {}_{10}W_9\Big(\frac{a^2q}{bcd}; \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}; q, \frac{aq^2}{bcd}\Big) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4310

Description: Transformation formula ([1], Ex. 2.13(ii)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, d \\ aq/b, aq/c, aq/d \end{matrix}; q, -\frac{aq^2}{bcd}\right] \longrightarrow \frac{(aq, -q, a^{3/2}q^2/bcd, -a^{3/2}q^2/bcd; q)_\infty}{(a^2q^2/bcd, -aq^2/bcd, \sqrt{a}q, -\sqrt{a}q; q)_\infty} {}_8W_7\Big(\frac{a^2q}{bcd}; \sqrt{a}, -\sqrt{a}, \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}; q, -q\Big) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4311

Description: Transformation formula ([1], (3.4.8)) in form of a rule.

$$ {}_4W_3\Big(a; b; q, \frac{x}{b^2q}\Big) \longrightarrow \frac{(ax^2/b^2, x/bq; q)_\infty}{(aqx/b, x^2/b^2q; q)_\infty} {}_8W_7\Big(\frac{ax}{b}; \sqrt{aq}, -\sqrt{aq}, \sqrt{a}q, -\sqrt{a}q, x; q, \frac{x}{b^2q}\Big) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4312

Description: Transformation formula ([1], Ex. 3.4) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, aq, q^{1-n}, q^{-n} \\ d, dq, b^2q \end{matrix}; q^2, q^2\right] \longrightarrow a^n \frac{(d/a;q)_n}{(d;q)_n} {}_4\phi_2\left[\begin{matrix} a, b, -b, q^{-n} \\ aq^{1-n}/d, b^2 \end{matrix}; q, -\frac{q}{d}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4361

Description: Transformation formula ([1], Ex. 3.16) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, -\sqrt{a}q, b, c \\ -\sqrt{a}, aq/b, aq/c \end{matrix}; q, x\right] \longrightarrow \left(1 - \frac{bcx}{\sqrt{aq}}\right) \frac{(bcx;q)_\infty}{(bcx/aq;q)_\infty} {}_5\phi_4\left[\begin{matrix} \sqrt{a}, -\sqrt{a}q, \sqrt{aq}, -\sqrt{aq}, aq/bc \\ aq/b, aq/c, aq^2/bcx, bcx \end{matrix}; q, q\right] $$
$$ + (1-\sqrt{a}) \frac{(aq, aq/bc, cx, bx; q)_\infty}{(aq/b, aq/c, x, aq/bcx; q)_\infty} {}_5\phi_4\left[\begin{matrix} bcx/\sqrt{a}q, -bcx/\sqrt{a}, bcx/\sqrt{aq}, -bcx/\sqrt{aq}, x \\ cx, bx, bcx/a, b^2c^2x^2/aq \end{matrix}; q, q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4362

Description: Transformation formula ([1], (2.10.10), reversed; Appendix (III.36)) in form of a rule.

$$ {}_4\phi_3\left[\begin{matrix} a, b, c, d \\ e, f, abcdq/ef \end{matrix}; q, q\right] \longrightarrow \frac{(ef/ab, ef/ac, ef/ad, ef/abcd; q)_\infty}{(ef/a, ef/abc, ef/abd, ef/acd; q)_\infty} {}_8W_7\left(\frac{ef}{aq}; \frac{f}{a}, \frac{e}{a}, b, c, d; q, \frac{ef}{bcd}\right) $$
$$ - \frac{(ef/abcd, a, b, c, d, e^2f/abcd, ef^2/abcd; q)_\infty}{(ef/abc, ef/abd, ef/acd, e, f, ef/bcd, abcd/ef; q)_\infty} {}_4\phi_3\left[\begin{matrix} ef/abc, ef/abd, ef/acd, ef/bcd \\ e^2f/abcd, ef^2/abcd, efq/abcd \end{matrix}; q, q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5401

Description: Transformation formula ([1], (2.8.3); Appendix (III.25)) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} a, b, c, d, q^{-n} \\ aq/b, aq/c, aq/d, b^2c^2d^2q^{-2-n}/a^2 \end{matrix}; q, q\right] \longrightarrow \frac{(aq^2/bcd, a^3q^3/b^2c^2d^2; q)_n}{(a^2q^2/bcd, a^2q^3/b^2c^2d^2; q)_n} $$
$$ {}_{12}W_{11}\left(\frac{a^2q}{bcd}; \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}, \sqrt{a}, -\sqrt{a}, \sqrt{a}\sqrt{q}, -\sqrt{a}\sqrt{q}, \frac{a^3q^{3+n}}{b^2c^2d^2}, q^{-n}; q, q\right) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5402

Description: Transformation formula ([1], (2.8.4); Appendix (III.26)) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} q^{-n}, b, c, d, e \\ q^{1-n}/b, q^{1-n}/c, q^{1-n}/d, b^2c^2d^2eq^{-2+2n} \end{matrix}; q, q\right] \longrightarrow \frac{(q^{2-2n}/bcde, q^{3-3n}/b^2c^2d^2; q)_n}{(q^{2-2n}/bcd, q^{3-3n}/b^2c^2d^2e; q)_n} $$
$$ {}_{12}W_{11}\left(\frac{q^{1-2n}}{bcd}; \frac{q^{1-n}}{cd}, \frac{q^{1-n}}{bd}, \frac{q^{1-n}}{bc}, q^{\frac{-n}{2}}, -q^{\frac{-n}{2}}, q^{\frac{1}{2}-\frac{n}{2}}, -q^{\frac{1}{2}-\frac{n}{2}}, e, \frac{q^{3-3n}}{b^2c^2d^2e}; q, q\right) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5403

Description: Transformation formula ([1], (3.10.4), reversed) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} x^2, y^2, a, aq, q^{-2n} \\ a^2, ab, abq, q^{2-2n}x^2y^2/a^2b^2 \end{matrix}; q^2, q^2\right] \longrightarrow \frac{(a^2b^2/x^2, a^2b^2/y^2; q^2)_n}{(a^2b^2, a^2b^2/x^2y^2; q^2)_n} {}_{10}W_9(-\frac{ab}{q}; b, x, -x, y, -y, -q^{-n}, q^{-n}; q, \frac{a^3b^2q^{2n}}{x^2y^2}) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5404

Description: Transformation formula ([1], Ex. 2.26, Appendix (III.14), reversed) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} q^{1-n}/b\,c, -q^{1/2-n/2}, q^{1/2-n/2}, -q^{-n/2}, q^{-n/2} \\ q^{1-n}/b, q^{1-n}/c, z/q^n, q/z \end{matrix}; q, q\right] \longrightarrow \frac{(z; q)_\infty}{(z/q^n; q)_\infty} {}_3\phi_2\left[\begin{matrix} q^{-n}, b, c \\ q^{1-n}/b, q^{1-n}/c \end{matrix}; q, \frac{q^{1-n}\,z}{b\,c}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5405

Description: Transformation formula ([1], (8.8.3), Appendix (III.22), reversed) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} a^2, b^2, a\,b, z, a^2\,b^2/z \\ a\,b\,\sqrt{q}, -\left(a\,b\,\sqrt{q}\right), -\left(a\,b\right), a^2\,b^2 \end{matrix}; q, q\right] \longrightarrow {}_4\phi_3\left[\begin{matrix} a, b, z, a^2\,b^2/z \\ a\,b\,\sqrt{q}, -\left(a\,b\,\sqrt{q}\right), -\left(a\,b\right) \end{matrix}; q, q\right]^2 $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5461

Description: Transformation formula ([1], (3.4.4)) in form of a rule.

$$ {}_5W_4(a; b, c; q, \frac{\sqrt{aq}x}{bc}) \longrightarrow \left(1 - x^2\right) \frac{(\sqrt{a}q^{3/2}x; q)_\infty}{(x/\sqrt{aq}; q)_\infty} {}_5\phi_4\left[\begin{matrix} \sqrt{aq}, -\sqrt{aq}, \sqrt{a}q, -\sqrt{a}q, aq/bc \\ aq/b, aq/c, \sqrt{a}q^{3/2}x, \sqrt{a}q^{3/2}/x \end{matrix}; q, q\right] + \frac{(aq, aq/bc, \sqrt{aq}x/b, \sqrt{aq}x/c; q)_\infty}{(aq/b, aq/c, \sqrt{aq}x/bc, \sqrt{aq}/x; q)_\infty} {}_5\phi_4\left[\begin{matrix} x, -x, \sqrt{q}x, -\sqrt{q}x, \sqrt{aq}x/bc \\ \sqrt{aq}x/b, \sqrt{aq}x/c, \sqrt{q}x/\sqrt{a}, qx^2 \end{matrix}; q, q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5462

Description: Transformation formula ([1], (3.4.4), reversed, first form) in form of a rule.

$$ {}_5\phi_4\left[\begin{matrix} \sqrt{aq}, -\sqrt{aq}, \sqrt{a}q, -\sqrt{a}q, bc/aq \\ b, c, \sqrt{a}q^{3/2}x, \sqrt{a}q^{3/2}/x \end{matrix}; q, q\right] \longrightarrow \frac{1}{1-x^2} \frac{(x/\sqrt{aq}; q)_\infty}{(\sqrt{a}q^{3/2}x; q)_\infty} {}_5W_4(a; \frac{aq}{b}, \frac{aq}{c}; q, \frac{bcx}{(aq)^{\frac{3}{2}}}) - \frac{1}{1-x^2} \frac{(x/\sqrt{aq}, aq, bc/aq, bx/\sqrt{aq}, cx/\sqrt{aq}; q)_\infty}{(\sqrt{a}q^{3/2}x, b, c, bcx/(aq)^{3/2}, \sqrt{aq}/x; q)_\infty} {}_5\phi_4\left[\begin{matrix} x, -x, \sqrt{q}x, -\sqrt{q}x, bcx/(aq)^{3/2} \\ bx/\sqrt{aq}, cx/\sqrt{aq}, \sqrt{q}x/\sqrt{a}, qx^2 \end{matrix}; q, q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5463

Description: Transformation formula ([1], (3.4.4), reversed, second form) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} x, -x, \sqrt{q}x, -\sqrt{q}x, bc/\sqrt{aq}x \\ b, c, \sqrt{q}x/\sqrt{a}, qx^2 \end{matrix}; q, q\right]
\longrightarrow \frac{(b\sqrt{aq}/x, c\sqrt{aq}/x, bc/\sqrt{aq}x, \sqrt{aq}/x; q)_\infty}{(aq, bc/x^2, b, c; q)_\infty} {}_5W_4(a; \frac{\sqrt{aq}x}{b}, \frac{\sqrt{aq}x}{c}; q, \frac{bc}{\sqrt{aq}x})
- (1-x^2)\,\frac{(\sqrt{a}q^{3/2}x, b\sqrt{aq}/x, c\sqrt{aq}/x, bc/\sqrt{aq}x, \sqrt{aq}/x; q)_\infty}{(x/\sqrt{aq}, aq, bc/x^2, b, c; q)_\infty} {}_5\phi_4\left[\begin{matrix} \sqrt{aq}, -\sqrt{aq}, \sqrt{a}q, -\sqrt{a}q, bc/x^2 \\ b\sqrt{aq}/x, c\sqrt{aq}/x, \sqrt{a}q^{3/2}x, \sqrt{a}q^{3/2}/x \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5464

Description: Transformation formula ([1], (3.5.2), reversed) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} a, x, -x, y, -y \\ -q, ab^2, xy/b, -xy/b \end{matrix}; q, q\right]
\longrightarrow \frac{(a^2b^2; q^2)_\infty}{(b^2; q^2)_\infty} \frac{(-1, b^2; q)_\infty}{(-a, ab^2; q)_\infty} {}_3\phi_2\left[\begin{matrix} a^2, x^2/b^2, y^2/b^2 \\ a^2b^2, x^2y^2/b^2 \end{matrix}; q^2, b^2q\right] - \frac{(a, -ab^2; q)_\infty}{(-a, ab^2; q)_\infty} {}_5\phi_4\left[\begin{matrix} -a, -x, x, -y, y \\ -q, -ab^2, -xy/b, xy/b \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5465

Description: Transformation formula ([1], (3.5.7), reversed) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} \sqrt{a}, -\sqrt{a}, c, d, e \\ \sqrt{ab}, -\sqrt{ab}, a, cdeq/ab \end{matrix}; q, q\right] \longrightarrow -\frac{(abq, a^3b^3/c^2d^2e^2; q^2)_\infty}{(aq, a^3b^2/c^2d^2e^2; q^2)_\infty} \frac{(c, d, e, a^2b/cde, ab/cde; q)_\infty}{(cde/ab, ab, ab/cd, ab/ce, ab/de; q)_\infty}
{}_5\phi_4\left[\begin{matrix} a^{3/2}b/cde, -a^{3/2}b/cde, ab/cd, ab/ce, ab/de \\ (ab)^{3/2}/cde, -(ab)^{3/2}/cde, a^2b/cde, abq/cde \end{matrix}; q, q\right]
+ \frac{(ab/c, ab/d, ab/e, ab/cde; q)_\infty}{(ab, ab/cd, ab/ce, ab/de; q)_\infty} {}_{10}W_9(\frac{ab}{q}; b, c, cq, d, dq, e, eq; q^2, \frac{a^3b^2}{c^2d^2e^2})
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5466

Description: Transformation formula ([1], Ex. 3.16, reversed, first form) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} a, -aq, a\sqrt{q}, -a\sqrt{q}, bc/a^2q \\ b, c, x, a^2q^2/x \end{matrix}; q, q\right] \longrightarrow \frac{1}{(1-\frac{aq}{x})} \frac{(q/x; q)_\infty}{(a^2q^2/x; q)_\infty} {}_4\phi_3\left[\begin{matrix} a^2, -aq, a^2q/b, a^2q/c \\ -a, b, c \end{matrix}; q, \frac{bc}{a^2x}\right]
- \frac{(1-a)}{(1-\frac{aq}{x})} \frac{(q/x, a^2q, bc/a^2q, bq/x, cq/x; q)_\infty}{(a^2q^2/x, b, c, bc/a^2x, x/q; q)_\infty} {}_5\phi_4\left[\begin{matrix} aq/x, -aq^2/x, aq^{3/2}/x, -aq^{3/2}/x, bc/a^2x \\ bq/x, cq/x, q^2/x, a^2q^3/x^2 \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5467

Description: Transformation formula ([1], Ex. 3.16, reversed, second form) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} a, -aq, a\sqrt{q}, -a\sqrt{q}, bcx/a^2q^2 \\ b, c, x, a^2q \end{matrix}; q, q\right]
$$

$$
\longrightarrow \frac{1}{(1-\frac{aq}{x})} \frac{(bq/x, cq/x, bcx/a^2q^2, q/x; q)_\infty}{(a^2q^3/x^2, bc/a^2q, b, c; q)_\infty} {}_4\phi_3\left[\begin{matrix} a^2q^2/x^2, -aq^2/x, a^2q^2/bx, a^2q^2/cx \\ -aq/x, bq/x, cq/x \end{matrix}; q, \frac{bcx}{a^2q^2}\right]
$$

$$
- \frac{(1-a)}{(1-\frac{aq}{x})} \frac{(a^2q^2/x, bq/x, cq/x, bcx/a^2q^2, q/x; q)_\infty}{(x/q, a^2q^3/x^2, bc/a^2q, b, c; q)_\infty} {}_5\phi_4\left[\begin{matrix} aq/x, -aq^2/x, aq^{3/2}/x, -aq^{3/2}/x, bc/a^2q \\ bq/x, cq/x, q^2/x, a^2q^2/x \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5468

Description: Transformation formula ([1], Ex. 2.25) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} a, b, c, d, e \\ aq/b, aq/c, aq/d, b^2c^2d^2e/a^2q^2 \end{matrix}; q, q\right]
$$

$$
\longrightarrow -\frac{(a, b, c, d, e, a^2q^3/b^2c^2d^2e, a^3q^4/b^3c^2d^2e, a^3q^4/b^2c^3d^2e, a^3q^4/b^2c^2d^3e; q)_\infty}{(aq/b, aq/c, aq/d, b^2c^2d^2e/a^2q^3, a^3q^3/b^2c^2d^2e, a^2q^3/bc^2d^2e, a^2q^3/b^2cd^2e, a^2q^3/b^2c^2de, a^2q^3/b^2c^2d^2; q)_\infty}
$$

$$
{}_5\phi_4\left[\begin{matrix} a^2q^3/b^2c^2d^2, a^3q^3/b^2c^2d^2e, a^2q^3/bc^2d^2e, a^2q^3/b^2cd^2e, a^2q^3/b^2c^2de \\ a^2q^4/b^2c^2d^2e, a^3q^4/b^3c^2d^2e, a^3q^4/b^2c^3d^2e, a^3q^4/b^2c^2d^3e \end{matrix}; q, q\right]
$$

$$
+ \frac{(aq^2/bcd, a^2q^2/bcde, a^3q^3/b^2c^2d^2, a^2q^3/b^2c^2d^2e; q)_\infty}{(a^2q^2/bcd, a^3q^3/b^2c^2d^2e, a^2q^3/b^2c^2d^2, aq^2/bcde; q)_\infty}
$$

$$
{}_{12}W_{11}(\frac{a^2q}{bcd}; \sqrt{a}, -\sqrt{a}, \sqrt{a}\sqrt{q}, -\sqrt{a}\sqrt{q}, \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}, e, \frac{a^3q^3}{b^2c^2d^2e}; q, q)
$$

$$
+ \frac{(a, e, aq/cd, aq/bd, aq/bc, a^2q^3/b^2c^2d^2e, a^2q^3/b^2cde, a^2q^3/bc^2de, a^2q^3/bcd^2e; q)_\infty}{(aq/b, aq/c, aq/d, a^3q^3/b^2c^2d^2e, a^2q^3/bc^2d^2e, a^2q^3/b^2cd^2e, a^2q^3/b^2c^2de, a^2q^3/b^2c^2d^2, bcde/aq^2; q)_\infty}
$$

$$
\frac{(a^5q^7/b^4c^4d^4e^2; q)_\infty}{(a^4q^6/b^3c^3d^3e^2; q)_\infty} {}_{12}W_{11}(\frac{a^4q^5}{b^3c^3d^3e^2}; \frac{a^{\frac{3}{2}}q^2}{bcde}, -\frac{a^{\frac{3}{2}}q^2}{bcde}, \frac{a^{\frac{3}{2}}q^{\frac{5}{2}}}{bcde}, -\frac{a^{\frac{3}{2}}q^{\frac{5}{2}}}{bcde}, \frac{aq^2}{bcd}, \frac{a^3q^3}{b^2c^2d^2e}, \frac{a^2q^3}{bc^2d^2e}, \frac{a^2q^3}{b^2cd^2e}, \frac{a^2q^3}{b^2c^2de}; q, q)
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5469

Description: Transformation formula ([1], (3.4.1), reversed; Appendix (III.35)) in form of a rule.

$$
{}_5\phi_4\left[\begin{matrix} \sqrt{a}, -\sqrt{a}, \sqrt{a}\sqrt{q}, -\sqrt{a}\sqrt{q}, bc/aq \\ b, c, ax, q/x \end{matrix}; q, q\right] \longrightarrow \frac{(x; q)_\infty}{(ax; q)_\infty} {}_3\phi_2\left[\begin{matrix} a, aq/b, aq/c \\ b, c \end{matrix}; q, \frac{bcx}{aq}\right]
$$

$$
- \frac{(x, a, bc/aq, bx, cx; q)_\infty}{(ax, b, c, bcx/aq, 1/x; q)_\infty} {}_5\phi_4\left[\begin{matrix} \sqrt{a}x, -\sqrt{a}x, \sqrt{a}\sqrt{q}x, -\sqrt{a}\sqrt{q}x, bcx/aq \\ bx, cx, qx, ax^2 \end{matrix}; q, q\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6501

Description: Transformation formula ([1], Ex. 2.14(ii)) in form of a rule.

$$
{}_6\phi_5\left[\begin{matrix} a, \sqrt{a}q, b, c, d, q^{-n} \\ \sqrt{a}, aq/b, aq/c, aq/d, b^2c^2d^2q^{-1-n}/a^2 \end{matrix}; q, q\right]
\longrightarrow \frac{(aq/bcd, a^3q^2/b^2c^2d^2, -a^{3/2}q^2/bcd; q)_n}{(a^2q^2/bcd, a^2q^2/b^2c^2d^2, -a^{3/2}q/bcd; q)_n} {}_{12}W_{11}(\frac{a^2q}{bcd}; \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}, \sqrt{a}q, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, \frac{a^3q^{2+n}}{b^2c^2d^2}, q^{-n}; q, q)
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7601

Description: Transformation formula ([1], (2.8.5); Appendix (III.27)) in form of a rule.

$$
{}_7\phi_6\left[\begin{matrix} a, \sqrt{a}q, -\sqrt{a}q, b, c, d, q^{-n} \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d, b^2c^2d^2/a^2q^n \end{matrix}; q, q\right] \longrightarrow \frac{(1-a^3q^{1+2n}/b^2c^2d^2)}{(1-a^3q/b^2c^2d^2)} \frac{(a/bcd, a^3q/b^2c^2d^2; q)_n}{(a^2q^2/bcd, a^2q/b^2c^2d^2; q)_n}
{}_{12}W_{11}(\frac{a^2q}{bcd}; \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}, \sqrt{a}\sqrt{q}, -\sqrt{a}\sqrt{q}, \sqrt{a}q, -\sqrt{a}q, \frac{a^3q^{1+n}}{b^2c^2d^2}, q^{-n}; q, q)
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7701

Description: Transformation formula ([1], (3.2.11), reversed) in form of a rule.

$$
{}_7\phi_7\left[\begin{matrix} a, \sqrt{a}q, -\sqrt{a}q, b, c, d, e \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/d, aq/e, 0 \end{matrix}; q, \frac{a^2q^2}{bcde}\right] \longrightarrow \frac{(aq, aq/de; q)_\infty}{(aq/d, aq/e; q)_\infty} {}_3\phi_2\left[\begin{matrix} aq/bc, d, e \\ aq/b, aq/c \end{matrix}; q, \frac{aq}{de}\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8701

Description: Transformation formula ([1], (2.10.10) terminated; Appendix (III.17)) in form of a rule.

$$
{}_8W_7(a; b, c, d, e, f; q, \frac{a^2q^2}{bcdef}) \longrightarrow \frac{(aq, aq/de, aq/df, aq/ef; q)_\infty}{(aq/d, aq/e, aq/f, aq/def; q)_\infty} {}_4\phi_3\left[\begin{matrix} aq/bc, d, e, f \\ aq/b, aq/c, def/a \end{matrix}; q, q\right],
$$

provided the ${}_8\phi_7$ series converges and the ${}_4\phi_3$ series terminates.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8702

Description: Transformation formula ([1], (2.5.1); Appendix (III.18)) in form of a rule.

$$
{}_8W_7(a; b, c, d, e, q^{-n}; q, \frac{a^2q^{2+n}}{bcde}) \longrightarrow \frac{(aq, aq/de; q)_n}{(aq/d, aq/e; q)_n} {}_4\phi_3\left[\begin{matrix} aq/bc, d, e, q^{-n} \\ aq/b, aq/c, de/aq^n \end{matrix}; q, q\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8703

Description: Transformation formula ([1], (2.10.7), reversed; Appendix (III.20), reversed) in form of a rule.

$$ {}_8W_7(a; b, c, d, e, \frac{aq^{1+n}}{e}; q, \frac{aq^{1-n}}{bcd}) \longrightarrow \frac{(aq/cd, aq/bd, aq/bc, aq; q)_\infty}{(aq/d, aq/c, aq/b, aq/bcd; q)_\infty} {}_4\phi_3 \left[\begin{matrix} b, c, d, q^{-n} \\ aq/e, bcd/a, e/q^n \end{matrix} ; q, q \right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8704

Description: Transformation formula ([1], (2.10.1); Appendix (III.23)) in form of a rule.

$$ {}_8W_7(a; b, c, d, e, f; q, \frac{a^2q^2}{bcdef}) \longrightarrow \frac{(aq, aq/ef, a^2q^2/bcde, a^2q^2/bcdf; q)_\infty}{(aq/e, aq/f, a^2q^2/bcd, a^2q^2/bcdef; q)_\infty} {}_8W_7(\frac{a^2q}{bcd}; \frac{aq}{cd}, \frac{aq}{bd}, \frac{aq}{bc}, e, f; q, \frac{aq}{ef}) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8705

Description: Transformation formula ([1], (2.10.1), iterated; Appendix (III.24)) in form of a rule.

$$ {}_8W_7(a; b, c, d, e, f; q, \frac{a^2q^2}{bcdef}) \longrightarrow \frac{(aq, b, a^2q^2/bdef, a^2q^2/bcef, a^2q^2/bcdf, a^2q^2/bcde; q)_\infty}{(aq/c, aq/d, aq/e, aq/f, a^3q^3/b^2cdef, a^2q^2/bcdef; q)_\infty} {}_8W_7(\frac{a^3q^2}{b^2cdef}; \frac{aq}{bc}, \frac{aq}{bd}, \frac{aq}{be}, \frac{aq}{bf}, \frac{a^2q^2}{bcdef}; q, b) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8706

Description: Transformation formula ([1], Ex. 2.13(ii), reversed) in form of a rule.

$$ {}_8W_7(a; b, -b, c, d, \frac{a^2q}{b^2cd}; q, -q) \longrightarrow \frac{(aq, -aq/b^2, bq, -bq; q)_\infty}{(b^2q, -q, aq/b, -aq/b; q)_\infty} {}_4\phi_3 \left[\begin{matrix} b^2, b^2c/a, b^2d/a, aq/cd \\ aq/c, aq/d, b^2cd/a \end{matrix} ; q, -\frac{aq}{b^2} \right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8707

Description: Transformation formula ([1], (3.4.7), reversed) in form of a rule.

$$ {}_8W_7(b; \sqrt{a}, -\sqrt{a}, \sqrt{aq}, -\sqrt{aq}, x; q, \frac{b^2q}{a^2x}) \longrightarrow \frac{(bq, b^2q/a^2; q)_\infty}{(bq/a, b^2q/a; q)_\infty} {}_2\phi_1 \left[\begin{matrix} a, ax/b \\ bq/x \end{matrix} ; q, \frac{b^2q}{a^2x} \right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8708

Description: Transformation formula ([1], (3.4.8), reversed) in form of a rule.

$$ {}_8W_7(b;\sqrt{aq},-\sqrt{aq},\sqrt{a}q,-\sqrt{a}q,x;q,\frac{b^2}{a^2qx}) \longrightarrow \frac{(bq,b^2/a^2q;q)_\infty}{(b^2/a,b/aq;q)_\infty} {}_4W_3(a;\frac{ax}{b};q,\frac{b^2}{a^2qx}) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8709

Description: Transformation formula ([1], (3.5.4), reversed) in form of a rule.

$$ {}_8W_7(b;x,-x,\frac{\sqrt{bq}}{\sqrt{a}},-\frac{\sqrt{bq}}{\sqrt{a}},a;q,\frac{bq}{x^2}) \longrightarrow \frac{(bq/a,b^2q^2/a^2x^2;q^2)_\infty}{(abq,b^2q^2/x^2;q^2)_\infty}\frac{(abq/x^2,bq;q)_\infty}{(bq/x^2,bq/a;q)_\infty} {}_2\phi_1\left[\begin{matrix}a^2,ax^2/b\\ abq^2/x^2\end{matrix};q^2,\frac{b^2q^2}{a^2x^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8710

Description: Transformation formula ([1], (3.5.10)) in form of a rule.

$$ \begin{aligned} &{}_8W_7(a;b,c,cq,d,dq;q^2,\frac{a^2q^2}{bc^2d^2}) \\ &\longrightarrow \frac{(aq,aq/bc,aq/cd,-aq/cd,aq/\sqrt{b}d,-aq/\sqrt{b}d;q)_\infty}{(aq/b,aq/c,aq/d,-aq/d,aq/\sqrt{b}cd,-aq/\sqrt{b}cd;q)_\infty} {}_8W_7(-\frac{a}{d};c,\sqrt{b},-\sqrt{b},\frac{\sqrt{aq}}{d},-\frac{\sqrt{aq}}{d};q,\frac{aq}{bc}) \end{aligned} $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8711

Description: Transformation formula ([1], (3.5.10), reversed) in form of a rule.

$$ \begin{aligned} &{}_8W_7(a;b,-b,d,-d,c;q,\frac{a^2q^2}{b^2cd^2}) \\ &\longrightarrow \frac{(a^2q^2/b^2d^2,a^2q^2/cd^2,-aq,aq,-aq/bc,aq/bc;q)_\infty}{(a^2q^2/d^2,a^2q^2/b^2cd^2,-aq/c,aq/c,-aq/b,aq/b;q)_\infty} {}_8W_7(\frac{a^2q}{d^2};b^2,c,cq,-\frac{aq}{d^2},-\frac{aq^2}{d^2};q^2,\frac{a^2q^2}{b^2c^2}) \end{aligned} $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8761

Description: Transformation formula ([1], (2.10.10); Appendix (III.36)) in form of a rule.

$$ \begin{aligned} &{}_8W_7(a;b,c,d,e,f;q,\frac{a^2q^2}{bcdef}) \longrightarrow \frac{(aq,aq/de,aq/df,aq/ef;q)_\infty}{(aq/d,aq/e,aq/f,aq/def;q)_\infty} {}_4\phi_3\left[\begin{matrix}aq/bc,d,e,f\\ aq/b,aq/c,def/a\end{matrix};q,q\right] \\ &\quad + \frac{(aq,aq/bc,d,e,f,a^2q^2/bdef,a^2q^2/cdef;q)_\infty}{(aq/b,aq/c,aq/d,aq/e,aq/f,a^2q^2/bcdef,def/aq;q)_\infty} {}_4\phi_3\left[\begin{matrix}aq/de,aq/df,aq/ef,a^2q^2/bcdef\\ a^2q^2/bdef,a^2q^2/cdef,aq^2/def\end{matrix};q,q\right] \end{aligned} $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8762

Description: Transformation formula ([1], (2.11.1); Appendix (III.37)) in form of a rule.

$$
\begin{aligned}
&{}_8W_7(a;b,c,d,e,f;q,\frac{a^2q^2}{bcdef})\\
&\longrightarrow -\frac{(aq,b/a,bq/c,bq/d,bq/e,bq/f,d,e,f,aq/bc,bdef/a^2,a^2q/bdef;q)_\infty}{(a/b,aq/c,aq/d,aq/e,aq/f,bd/a,be/a,bf/a,def/a,aq/def,q/c,b^2q/a;q)_\infty}\\
&\qquad {}_8W_7(\frac{b^2}{a};b,\frac{bc}{a},\frac{bd}{a},\frac{be}{a},\frac{bf}{a};q,\frac{a^2q^2}{bcdef})\\
&\quad +\frac{(aq,aq/de,aq/df,aq/ef,eq/c,fq/c,b/a,bef/a;q)_\infty}{(aq/d,aq/e,aq/f,aq/def,q/c,efq/c,be/a,bf/a;q)_\infty}\\
&\qquad {}_8W_7(\frac{ef}{c};\frac{aq}{bc},\frac{aq}{cd},\frac{ef}{a},e,f;q,\frac{bd}{a})
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8763

Description: Transformation formula ([1], (2.11.1), reversed; Appendix (III.37)) in form of a rule.

$$
\begin{aligned}
&{}_8W_7(a;b,c,d,e,f;q,\frac{a^2q^2}{bcdef})\\
&\longrightarrow \frac{(cef/a,fq/d,eq/d,c/a,aq/ef,aq,aq/bf,aq/be;q)_\infty}{(efq/d,cf/a,ce/a,q/d,aq/f,aq/e,aq/bef,aq/b;q)_\infty}{}_8W_7(\frac{ef}{d};\frac{aq}{bd},\frac{ef}{a},\frac{aq}{cd},e,f;q,\frac{bc}{a})\\
&\quad +\frac{(aq,a^2q^2/bdef,cq/b,aq^2/bde,aq^2/bdf,aq/cd,e,f,b,a^2q^2/bcef,bcef/a^2q;q)_\infty}{(cf/a,ce/a,q/d,aq/f,aq/e,aq/b,bef/aq,aq/d,a^2q^2/bcdef,aq/c,a^2q^3/b^2def;q)_\infty}\\
&\qquad {}_8W_7(\frac{a^2q^2}{b^2def};\frac{aq}{bd},\frac{q}{b},\frac{a^2q^2}{bcdef},\frac{aq}{bf},\frac{aq}{be};q,\frac{bc}{a})
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8764

Description: Transformation formula ([1], Ex. 2.15) in form of a rule.

$$
\begin{aligned}
&{}_8W_7(a^2;ab,ac,ad,ae,af;q,\frac{q^2}{abcdef})\\
&\longrightarrow -\frac{a}{b}\frac{(a^2q,ac,b/a,c/a,bq/d,bq/e,bq/f,q/bd,q/be,q/bf;q)_\infty}{(aq/d,aq/e,aq/f,q/ad,q/ae,q/af,b^2q,bc,a/b,c/b;q)_\infty}{}_8W_7(b^2;ab,bc,bd,be,bf;q,\frac{q^2}{abcdef})\\
&\quad -\frac{a}{c}\frac{(a^2q,ab,b/a,c/a,cq/d,cq/e,cq/f,q/cd,q/ce,q/cf;q)_\infty}{(aq/d,aq/e,aq/f,q/ad,q/ae,q/af,c^2q,bc,a/c,b/c;q)_\infty}{}_8W_7(c^2;ac,bc,cd,ce,cf;q,\frac{q^2}{abcdef})
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10901

Description: Transformation formula ([1], (2.9.1); Appendix (III.28)) in form of a rule.

$$ {}_{10}W_9(a;b,c,d,e,f,\frac{a^3q^{2+n}}{bcdef},q^{-n};q,q) \longrightarrow \frac{(aq,aq/ef,a^2q^2/bcde,a^2q^2/bcdf;q)_n}{(aq/e,aq/f,a^2q^2/bcdef,a^2q^2/bcd;q)_n}\,{}_{10}W_9(\frac{a^2q}{bcd};\frac{aq}{cd},\frac{aq}{bd},\frac{aq}{bc},e,f,\frac{a^3q^{2+n}}{bcdef},q^{-n};q,q) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10902

Description: Transformation formula ([1], Ex. 2.13(i), reversed) in form of a rule.

$$ {}_{10}W_9(a;b,-b,b\sqrt{q},-b\sqrt{q},c,d,\frac{a^2q}{b^2cd};q,\frac{aq}{b^2}) \longrightarrow \frac{(aq,a^2q/b^4;q)_\infty}{(aq/b^2,a^2q/b^2;q)_\infty}\,{}_4\phi_3\left[\begin{matrix} b^2,b^2c/a,b^2d/a,aq/cd \\ aq/c,aq/d,b^2cd/a \end{matrix};q,\frac{a^2q}{b^4}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10903

Description: Transformation formula ([1], Ex. 2.19) in form of a rule.

$$ {}_{10}W_9(a;b,c,d,e,f,g,q^{-n};q,q) \longrightarrow e^n\frac{(aq,aq/ce,aq/de,aq/ef,aq/eg,b;q)_n}{(aq/c,aq/d,aq/e,aq/f,aq/g,b/e;q)_n}\,{}_{10}W_9(\frac{e}{bq^n};e,\frac{aq}{bc},\frac{aq}{bd},\frac{aq}{bf},\frac{aq}{bg},\frac{e}{aq^n},q^{-n};q,q) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10904

Description: Transformation formula ([1], (3.10.4)) in form of a rule.

$$ {}_{10}W_9(a;b,x,-x,y,-y,-q^{-n},q^{-n};q,-\frac{a^3q^{3+2n}}{bx^2y^2}) \longrightarrow \frac{(a^2q^2,a^2q^2/x^2y^2;q^2)_n}{(a^2q^2/x^2,a^2q^2/y^2;q^2)_n}\,{}_5\phi_4\left[\begin{matrix} q^{-2n},x^2,y^2,-aq/b,-aq^2/b \\ x^2y^2/a^2q^{2n},a^2q^2/b^2,-aq,-aq^2 \end{matrix};q^2,q^2\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10905

Description: Transformation formula ([1], Ex. 3.21(iii)) in form of a rule.

$$ {}_{10}\phi_9\left[\begin{matrix} a,\sqrt{a}q,-\sqrt{a}q,b,c,a/bc,C/Aq^n,1/BCq^n,B/Aq^n,q^{-n} \\ \sqrt{a},-\sqrt{a},aq/b,aq/c,bcq,1/Cq^n,BC/Aq^n,1/Bq^n,1/Aq^n \end{matrix};q,q\right] \longrightarrow \frac{(aq,bq,cq,aq/bc,Aq/B,Aq/C,BCq;q)_n}{(Aq,Bq,Cq,Aq/BC,aq/b,aq/c,bcq;q)_n}\,{}_{10}\phi_9\left[\begin{matrix} A,\sqrt{A}q,-\sqrt{A}q,B,C,A/BC,c/aq^n,1/bcq^n,b/aq^n,q^{-n} \\ \sqrt{A},-\sqrt{A},Aq/B,Aq/C,BCq,1/cq^n,bc/aq^n,1/bq^n,1/aq^n \end{matrix};q,q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10906

Description: Transformation formula ([3], (7.7), reversed) in form of a rule.

$$
{}_{10}W_9(a^2q^n; c, d, e, aq^{\frac{1}{2}+n}, -aq^{\frac{1}{2}+n}, \frac{a^4q^{1+n}}{cde}, q^{-n}; q, -q^{1+n})
\longrightarrow \frac{(a^2q/c, a^2q/d, a^2q/e, a^2q/cde; q)_n}{(a^2q, a^2q/cd, a^2q/ce, a^2q/de; q)_n} {}_{12}W_{11}(a^2; q^{-2n}, c, cq, d, dq, e, eq, \frac{a^4q^{1+n}}{cde}, \frac{a^4q^{2+n}}{cde}; q^2, q^2)
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10907

Description: Transformation formula ([3], (7.8), reversed) in form of a rule.

$$
{}_{10}W_9(\frac{a^2}{e}; \frac{a\sqrt{q}}{e}, -\frac{a\sqrt{q}}{e}, c, d, e, \frac{a^4q^{1+n}}{cde}, q^{-n}; q, -\frac{q}{e})
\longrightarrow \frac{(a^2q/c, a^2q/d, a^2q/e, a^2q/cde; q)_n}{(a^2q, a^2q/cd, a^2q/ce, a^2q/de; q)_n} {}_{12}W_{11}(a^2; e^2, c, cq, d, dq, \frac{a^4q^{1+n}}{cde}, \frac{a^4q^{2+n}}{cde}, q^{1-n}, q^{-n}; q^2, q^2)
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10961

Description: Transformation formula ([1], (2.12.9); Appendix (III.39)) in form of a rule.

$$
\begin{aligned}
&{}_{10}W_9(a; b, c, d, e, f, g, \frac{a^3q^2}{bcdefg}; q, q) \\
&\longrightarrow \frac{(aq, b/a, f, g, a^3q^2/bcdefg, bq/f, bq/g, b^2cdefg/a^3q, aq/de, aq/ce, aq/cd; q)_\infty}{(b^2cde/a^2, a^2q/bcde, aq/c, aq/d, aq/e, aq/f, aq/g, bcdefg/a^2q, bc/a, bd/a, be/a; q)_\infty} \\
&\quad \frac{(bde/a, bce/a, bcd/a; q)_\infty}{(bf/a, bg/a, a^2q^2/cdefg; q)_\infty} {}_{10}W_9(\frac{b^2cde}{a^2q}; b, \frac{bc}{a}, \frac{bd}{a}, \frac{be}{a}, \frac{bcdef}{a^2q}, \frac{bcdeg}{a^2q}, \frac{aq}{fg}; q, q) \\
&\quad + \frac{(aq, b/a, a^2q^2/cdef, a^2q^2/cdeg, bfg/a, bcdef/a^2q, bcdeg/a^2q, aq/fg; q)_\infty}{(a^2q^2/cde, bcde/a^2q, aq/f, aq/g, bcdefg/a^2q, bf/a, bg/a, a^2q^2/cdefg; q)_\infty} \\
&\quad {}_{10}W_9(\frac{a^2q}{cde}; b, \frac{aq}{de}, \frac{aq}{ce}, \frac{aq}{cd}, f, g, \frac{a^3q^2}{bcdefg}; q, q) \\
&\quad - \frac{(aq, b/a, c, d, e, f, g, a^3q^2/bcdefg, bq/c, bq/d, bq/e, bq/f; q)_\infty}{(b^2q/a, a/b, aq/c, aq/d, aq/e, aq/f, aq/g, bcdefg/a^2q, bc/a, bd/a, be/a, bf/a; q)_\infty} \\
&\quad \frac{(bq/g, b^2cdefg/a^3q; q)_\infty}{(bg/a, a^2q^2/cdefg; q)_\infty} {}_{10}W_9(\frac{b^2}{a}; b, \frac{bc}{a}, \frac{bd}{a}, \frac{be}{a}, \frac{bf}{a}, \frac{bg}{a}, \frac{a^2q^2}{cdefg}; q, q)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10962

Description: Transformation formula ([1], (3.5.7)) in form of a rule.

$$ {}_{10}W_9(a;b,c,cq,d,dq,e,eq;q^2,\frac{a^3q^3}{bc^2d^2e^2}) \longrightarrow \frac{(aq,aq/cd,aq/ce,aq/de;q)_\infty}{(aq/c,aq/d,aq/e,aq/cde;q)_\infty} {}_5\phi_4\left[\begin{matrix}\sqrt{aq}/\sqrt{b},-\sqrt{aq}/\sqrt{b},c,d,e\\ \sqrt{aq},-\sqrt{aq},aq/b,cde/a\end{matrix};q,q\right] $$
$$ + \frac{(aq^2,a^3q^3/c^2d^2e^2;q^2)_\infty}{(aq^2/b,a^3q^3/bc^2d^2e^2;q^2)_\infty}\frac{(c,d,e,a^2q^2/bcde;q)_\infty}{(aq/c,aq/d,aq/e,cde/aq;q)_\infty} $$
$$ {}_5\phi_4\left[\begin{matrix}(aq)^{3/2}/\sqrt{b}cde,-(aq)^{3/2}/\sqrt{b}cde,aq/cd,aq/ce,aq/de\\ (aq)^{3/2}/cde,-(aq)^{3/2}/cde,a^2q^2/bcde,aq^2/cde\end{matrix};q,q\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T10963

Description: Transformation formula ([1], Ex. 2.30) in form of a rule.

$$ {}_{10}W_9(a;b,c,d,e,f,g,\frac{a^3q^2}{bcdefg};q,q) $$
$$ \longrightarrow -\frac{(bq/f,bq/g,b^2cdefg/a^3q,aq,c,d,e,f,g,a^3q^2/bcdefg,b/a,bq/c,bq/d,bq/e;q)_\infty}{(aq/f,aq/g,bcdefg/a^2q,b^2q/a,bc/a,bd/a,be/a,bf/a,bg/a,a^2q^2/cdefg,a/b,aq/c,aq/d,aq/e;q)_\infty} $$
$$ {}_{10}W_9(\frac{b^2}{a};b,\frac{bc}{a},\frac{bd}{a},\frac{be}{a},\frac{bf}{a},\frac{bg}{a},\frac{a^2q^2}{cdefg};q,q) $$
$$ + \frac{(beg/a,bfg/a,aq,b/a,a^3q^2/bcdefg,b^2cdefg/a^3q,aq/cg,aq/dg,aq/eg,aq/fg,bcg/a,bdg/a;q)_\infty}{(be/a,bf/a,b^2cdefg^2/a^3q,bg/a,a^3q^2/bcdefg^2,aq/g,aq/c,aq/d,aq/e,aq/f,bc/a,bd/a;q)_\infty} $$
$$ {}_{10}W_9(\frac{b^2cdefg^2}{a^3q^2};b,\frac{bdefg}{a^2q},\frac{bcefg}{a^2q},\frac{bcdfg}{a^2q},\frac{bcdeg}{a^2q},\frac{bg}{a},g;q,q) $$
$$ + \frac{(a^2q^2/cdfg,a^2q^2/cdeg,aq,b/a,g,bq/g,bdefg/a^2q,bcefg/a^2q,bcdfg/a^2q,bcdeg/a^2q;q)_\infty}{(be/a,bf/a,a^3q^3/cdefg^2,a^2q^2/cdefg,bcdefg^2/a^3q^2,bcdefg/a^2q,aq/c,aq/d,aq/e,aq/f;q)_\infty} $$
$$ \frac{(a^2q^2/defg,a^2q^2/cefg;q)_\infty}{(bc/a,bd/a;q)_\infty}{}_{10}W_9(\frac{a^3q^2}{cdefg^2};b,\frac{aq}{cg},\frac{aq}{dg},\frac{aq}{eg},\frac{aq}{fg},\frac{a^2q^2}{cdefg},\frac{a^3q^2}{bcdefg};q,q) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121101

Description: Transformation formula ([1], (2.8.3), reversed; Appendix (III.25), reversed) in form of a rule.

$$ {}_{12}W_{11}(a;b,c,d,\frac{a\sqrt{q}}{\sqrt{b}\sqrt{c}\sqrt{d}},-\frac{a\sqrt{q}}{\sqrt{b}\sqrt{c}\sqrt{d}},\frac{aq}{\sqrt{b}\sqrt{c}\sqrt{d}},-\frac{aq}{\sqrt{b}\sqrt{c}\sqrt{d}}, $$
$$ bcdq^n,q^{-n};q,q) \longrightarrow \frac{(aq,b^2c^2d^2/a^2q;q)_n}{(bcd/a,bcd;q)_n}{}_5\phi_4\left[\begin{matrix}a^2q/bcd,aq/cd,aq/bd,aq/bc,q^{-n}\\ aq/b,aq/c,aq/d,a^2q^{2-n}/b^2c^2d^2\end{matrix};q,q\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121102

Description: Transformation formula ([1], (2.8.4), reversed; Appendix (III.26), reversed) in form of a rule.

$$
{}_{12}W_{11}(\sqrt{b}\sqrt{c}\sqrt{d}q^{-\frac{1}{2}-\frac{n}{2}};b,c,d,q^{\frac{-n}{2}},-q^{\frac{-n}{2}},q^{\frac{1}{2}-\frac{n}{2}},-q^{\frac{1}{2}-\frac{n}{2}},e,\frac{bcd}{e};q,q)
\longrightarrow \frac{(\sqrt{b}\sqrt{c}\sqrt{d}q^{1/2-n/2},bcd/e;q)_n}{(\sqrt{b}\sqrt{c}\sqrt{d}q^{1/2-n/2}/e,bcd;q)_n}
{}_5\phi_4\left[\begin{matrix}q^{-n},\sqrt{b}q^{1/2-n/2}/\sqrt{c}\sqrt{d},\sqrt{c}q^{1/2-n/2}/\sqrt{b}\sqrt{d},\sqrt{d}q^{1/2-n/2}/\sqrt{b}\sqrt{c},e\\ \sqrt{c}\sqrt{d}q^{1/2-n/2}/\sqrt{b},\sqrt{b}\sqrt{d}q^{1/2-n/2}/\sqrt{c},\sqrt{b}\sqrt{c}q^{1/2-n/2}/\sqrt{d},eq^{1-n}/bcd\end{matrix};q,q\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121103

Description: Transformation formula ([1], (2.8.5), reversed; Appendix (III.27), reversed) in form of a rule.

$$
{}_{12}W_{11}(a;b,c,d,\frac{aq}{\sqrt{b}\sqrt{c}\sqrt{d}},-\frac{aq}{\sqrt{b}\sqrt{c}\sqrt{d}},\frac{aq^{\frac{3}{2}}}{\sqrt{b}\sqrt{c}\sqrt{d}},-\frac{aq^{\frac{3}{2}}}{\sqrt{b}\sqrt{c}\sqrt{d}},
bcdq^{-2+n},q^{-n};q,q) \longrightarrow \frac{(1-bcd/q^2)}{(1-bcdq^{-2+2n})}\frac{(aq,b^2c^2d^2/a^2q^3;q)_n}{(bcd/aq^2,bcd/q^2;q)_n}
{}_7\phi_6\left[\begin{matrix}a^2q/bcd,aq^{3/2}/\sqrt{b}\sqrt{c}\sqrt{d},-aq^{3/2}/\sqrt{b}\sqrt{c}\sqrt{d},aq/cd,aq/bd,aq/bc,q^{-n}\\ a\sqrt{q}/\sqrt{b}\sqrt{c}\sqrt{d},-a\sqrt{q}/\sqrt{b}\sqrt{c}\sqrt{d},aq/b,aq/c,aq/d,a^2q^{4-n}/b^2c^2d^2\end{matrix};q,q\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121104

Description: Transformation formula ([1], Ex. 2.14(ii), reversed) in form of a rule.

$$
{}_{12}W_{11}(a;c,d,\frac{a^2q}{bcd},\sqrt{b}q,-\sqrt{b},\sqrt{bq},-\sqrt{bq},\frac{a^2q^n}{b},q^{-n};q,q)
\longrightarrow \frac{(aq,a^2/b^2,-a/\sqrt{b};q)_n}{(a/b,a^2/b,-aq/\sqrt{b};q)_n}{}_6\phi_5\left[\begin{matrix}b,\sqrt{b}q,bc/a,bd/a,aq/cd,q^{-n}\\ \sqrt{b},aq/c,aq/d,bcd/a,b^2q^{1-n}/a^2\end{matrix};q,q\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121105

Description: Transformation formula ([3], (7.7)) in form of a rule.

$$
{}_{12}W_{11}(a^2;c,cq,d,dq,e,eq,\frac{a^4q^{1+n}}{cde},\frac{a^4q^{2+n}}{cde},q^{-2n};q^2,q^2)
\longrightarrow \frac{(a^2q,a^2q/cd,a^2q/ce,a^2q/de;q)_n}{(a^2q/c,a^2q/d,a^2q/e,a^2q/cde;q)_n}{}_{10}W_9(a^2q^n;c,d,e,aq^{\frac{1}{2}+n},-aq^{\frac{1}{2}+n},\frac{a^4q^{1+n}}{cde},q^{-n};q,-q^{1+n})
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121106

Description: Transformation formula ([3], (7.8)) in form of a rule.

$$
{}_{12}W_{11}(a^2; e^2, c, cq, d, dq, \frac{a^4 q^{1+n}}{cde}, \frac{a^4 q^{2+n}}{cde}, q^{1-n}, q^{-n}; q^2, q^2)
\longrightarrow \frac{(a^2 q, a^2 q/cd, a^2 q/ce, a^2 q/de; q)_n}{(a^2 q/c, a^2 q/d, a^2 q/e, a^2 q/cde; q)_n} {}_{10}W_9(\frac{a^2}{e}; \frac{a\sqrt{q}}{e}, -\frac{a\sqrt{q}}{e}, c, d, e, \frac{a^4 q^{1+n}}{cde}, q^{-n}; q, -\frac{q}{e})
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T121161

Description: Transformation formula ([1], Ex. 2.25, reversed) in form of a rule.

$$
\begin{aligned}
&{}_{12}W_{11}(a; \sqrt{b}, -\sqrt{b}, \sqrt{b}\sqrt{q}, -\sqrt{b}\sqrt{q}, c, d, \frac{a^2 q}{bcd}, e, \frac{a^2 q}{be}; q, q) \\
&\longrightarrow \frac{(aq, a^2q/be, a^2q/b^2, aq/be; q)_\infty}{(aq/b, aq/e, a^2q/b, a^2q/b^2e; q)_\infty} {}_5\phi_4\left[\begin{matrix} b, bc/a, bd/a, aq/cd, e \\ aq/c, aq/d, bcd/a, b^2e/a^2 \end{matrix}; q, q\right] \\
&\quad + \frac{(aq, aq/be, b, bc/a, bd/a, aq/cd, e, a^3q^2/b^2ce, a^3q^2/b^2de, acdq/be; q)_\infty}{(aq/b, aq/e, a^2q/b, aq/c, aq/d, bcd/a, b^2e/a^2q, acq/be, adq/be, a^3q^2/b^2cde; q)_\infty} \\
&\qquad {}_5\phi_4\left[\begin{matrix} a^2q/b^2, a^2q/be, acq/be, adq/be, a^3q^2/b^2cde \\ a^2q^2/b^2e, a^3q^2/b^2ce, a^3q^2/b^2de, acdq/be \end{matrix}; q, q\right] \\
&\quad - \frac{(aq, aq/be, b, e, c, d, a^2q/bcd, a^2q^2/bce, a^2q^2/bde, cdq/e, a^4q^3/b^3e^2; q)_\infty}{(aq/b, aq/e, a^2q/b, aq/c, aq/d, bcd/a, acq/be, adq/be, a^3q^2/b^2cde, be/aq, a^3q^3/b^2e^2; q)_\infty} \\
&\qquad {}_{12}W_{11}(\frac{a^3q^2}{b^2e^2}; \frac{aq}{\sqrt{be}}, -\frac{aq}{\sqrt{be}}, \frac{aq^{\frac{3}{2}}}{\sqrt{be}}, -\frac{aq^{\frac{3}{2}}}{\sqrt{be}}, \frac{aq}{b}, \frac{a^2q}{be}, \frac{acq}{be}, \frac{adq}{be}, \frac{a^3q^2}{b^2cde}; q, q)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

TeX

Description: Switch that changes the output of TeXForm to be usable with Plain-TEX and LATEX. By default the output of TeXForm is usable with AMS-TEX.

Usage: `TeX.`

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AMSTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= TeXForm[ph[{a,b},{c},q,z]]

Out[2]//TeXForm=
{} _{2} \phi _{1} \!  \left [ \matrix \let \over / a, b\\ \let \over /
 c\endmatrix ;q, {\displaystyle z} \right ]
```

```
In[3]:= TeX

In[4]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[5]:= TeXForm[ph[{a,b},{c},q,z]]

Out[5]//TeXForm=
{} _{2} \phi _{1} \!  \left [ \matrix { \let \over / a, b\cr \let \over / c}
 ;q, {\displaystyle z} \right ]
```

See also: AmSTeX, AmSLaTeX, LaTeX, TeXMat, TeXphW.

TeXMat

Description: Function that writes (to be precise: appends) an expression `Expr` in `InputForm` to a file `[name].m` and the `TeXForm` of `Expr` to the file `[name].tex`. The expressions are numbered automatically. The number can be reset by `SchreibeZahl`. The string `comment` is optional. It allows to place the comment `comment` above the expression and the number in each of the two files.

Usage: : `TeXMat[Expr,name,comment]`.

Example(s):

```
In[1]:= TeXMat[pq[a,2*n,q^3],filename]

In[2]:= TeXMat[ph[{a,b},{c},q,z],filename,"A basic hypergeometric _2\phi_1
        series"]

In[3]:= !type filename.m
A[1]:=
pq[a, 2*n, q^3]
"A basic hypergeometric _2\\phi_1  series"
A[2]:=
ph[{a, b}, {c}, q, z]

In[3]:= !type filename.tex
A[1]:=
({\let \over / a}; {q^3}) _{2\,n}
"A basic hypergeometric _2\\phi_1  series"
A[2]:=
{} _{2} \phi _{1} \!  \left [ \matrix \let \over / a, b\\ \let \over /
  c\endmatrix ;q, {\displaystyle z} \right ]
```

See also: AmSTeX, AmSLaTeX, LaTeX, TeX, TeXphW, SchreibeZahl.

TeXphW

Description: Switch that toggles between writing very well-poised basic hypergeometric series in terms of `W` and in terms of `ph`, respectively, when written in TeXForm. By default very well-poised hypergeometric series are written in terms of `W`.

Usage: `TeXphW`.

Example(s):

```
In[1]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.

In[2]:= ph[{a,q*Sqrt[a],-q*Sqrt[a],b,c},{Sqrt[a],-Sqrt[a],a*q/b,a*q/c},q,z]

           [ a, Sqrt[a] q, -(Sqrt[a] q), b, c       ]
           [                                        ]
Out[2]=  φ [                        a q  a q   ; q, z ]
        5 4[  Sqrt[a], -Sqrt[a], ---, ---           ]
           [                      b    c            ]

In[3]:= TeXForm[%]

Out[3]//TeXForm=
{} _{5} W _{4} ({\displaystyle a; b, c}; q, {\displaystyle z})

In[4]:= TeXphW

In[5]:= hypqAttributes

Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses ph[] for very well-poised basic hypergeometric series.

In[6]:= TeXForm[%2]

Out[6]//TeXForm=
{} _{5} \phi _{4} \!  \left [ \matrix \let \over / a, {\sqrt{a}} q, -\left(
 {\sqrt{a}} q \right) , b, c\\ \let \over / {\sqrt{a}}, -{\sqrt{a}}, {{a
 q}\over b}, {{a q}\over c}\endmatrix ;q, {\displaystyle z} \right ]

In[7]:= TeXphW

In[8]:= hypqAttributes
```

```
Automatic evaluation of pq and ph is inactive.
Automatic cancelling in ph is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses W[] for very well-poised basic hypergeometric series.
```

See also: `ph, W, hypqAttributes.`

Tgl2101

Description: Transformation formula ([1], (1.4.1); Appendix (III.1)) in form of an equation. It is the same transformation as that in `T2101`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2102

Description: Transformation formula ([1], (1.4.5); Appendix (III.2)) in form of an equation. It is the same transformation as that in `T2102`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2103

Description: Transformation formula ([1], (1.4.6); Appendix (III.3)) in form of an equation. It is the same transformation as that in `T2103`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2104

Description: Transformation formula ([1], (1.5.4); Appendix (III.4)) in form of an equation. It is the same transformation as that in `T2104`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2105

Description: Transformation formula ([1], (3.2.4), nonterminating; Appendix (III.5)) in form of an equation. It is the same transformation as that in `T2105`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2106

Description: Transformation formula ([1], (1.5.6); Appendix (III.6)) in form of an equation. It is the same transformation as that in `T2106`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2107

Description: Transformation formula ([1], Ex. 1.15(iii); Appendix (III.7)) in form of an equation. It is the same transformation as that in `T2107`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2108

Description: Transformation formula ([1], Ex. 1.15(ii); Appendix (III.8)) in form of an equation. It is the same transformation as that in `T2108`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2109

Description: Transformation formula ([1], Ex. 2.2, reversed) in form of an equation. It is the same transformation as that in `T2109`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2110

Description: Transformation formula ([1], (3.4.7)) in form of an equation. It is the same transformation as that in `T2110`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2111

Description: Transformation formula ([1], (3.5.4)) in form of an equation. It is the same transformation as that in `T2111`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2112

Description: Transformation formula ([1], Ex. 3.2(i), reversed) in form of an equation. It is the same transformation as that in `T2112`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2161

Description: Transformation formula ([1], (3.3.5); Appendix (III.31)) in form of an equation. It is the same transformation as that in `T2161`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2162

Description: Transformation formula ([1], Appendix (III.32)) in form of an equation. It is the same transformation as that in `T2162`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2163

Description: Transformation formula ([1], Ex. 3.8) in form of an equation. It is the same transformation as that in `T2163`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2201

Description: Transformation formula ([1], (1.5.4), reversed; Appendix (III.4), reversed) in form of an equation. It is the same transformation as that in `T2201`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2202

Description: Transformation formula ([1], Ex. 3.2(ii), reversed) in form of an equation. It is the same transformation as that in `T2202`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3101

Description: Transformation formula ([1], Ex. 1.15(ii), reversed; Appendix (III.8), reversed) in form of an equation. It is the same transformation as that in `T3101`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3201

Description: Transformation formula ([1], (3.2.4), reversed; Appendix (III.5), reversed) in form of an equation. It is the same transformation as that in `T3201`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3202

Description: Transformation formula ([1], (1.5.6), reversed; Appendix (III.6), reversed) in form of an equation. It is the same transformation as that in `T3202`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3203

Description: Transformation formula ([1], Ex. 1.15(iii), reversed; Appendix (III.7), reversed) in form of an equation. It is the same transformation as that in `T3203`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3204

Description: Transformation formula ([1], (3.2.7); Appendix (III.9)) in form of an equation. It is the same transformation as that in `T3204`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3205

Description: Transformation formula ([1], (3.2.10); Appendix (III.10)) in form of an equation. It is the same transformation as that in `T3205`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3206

Description: Transformation formula ([1], (3.2.3); Appendix (III.11)) in form of an equation. It is the same transformation as that in `T3206`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3207

Description: Transformation formula ([1], (3.2.2); Appendix (III.12)) in form of an equation. It is the same transformation as that in `T3207`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3208

Description: Transformation formula ([1], (3.2.5); Appendix (III.13)) in form of an equation. It is the same transformation as that in `T3208`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3209

Description: Transformation formula ([1], Ex. 2.26; Appendix (III.14)) in form of an equation. It is the same transformation as that in `T3209`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3210

Description: Transformation formula ([1], (3.2.6)) in form of an equation. It is the same transformation as that in `T3210`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3211

Description: Transformation formula ([1], Ex. 3.1) in form of an equation. It is the same transformation as that in `T3211`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3212

Description: Transformation formula ([1], Ex. 3.1, reversed) in form of an equation. It is the same transformation as that in `T3212`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3213

Description: Transformation formula ([1], Ex. 3.2(i)) in form of an equation. It is the same transformation as that in `T3213`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3214

Description: Transformation formula ([1], Ex. 3.2(ii)) in form of an equation. It is the same transformation as that in `T3214`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3215

Description: Transformation formula ([1], Ex. 3.3) in form of an equation. It is the same transformation as that in `T3215`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3216

Description: Transformation formula ([1], Ex. 3.3, reversed) in form of an equation. It is the same transformation as that in `T3216`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3217

Description: Transformation formula ([1], (3.2.11)) in form of an equation. It is the same transformation as that in `T3217`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3261

Description: Transformation formula ([1], (3.3.3); Appendix (III.33)) in form of an equation. It is the same transformation as that in `T3261`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3262

Description: Transformation formula ([1], (3.3.1); Appendix (III.34)) in form of an equation. It is the same transformation as that in `T3262`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3265

Description: Transformation formula ([1], (3.4.1); Appendix (III.35)) in form of an equation. It is the same transformation as that in `T3265`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3266

Description: Transformation formula ([1], (3.5.2)) in form of an equation. It is the same transformation as that in `T3266`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3267

Description: Transformation formula ([1], Ex. 3.6) in form of an equation. It is the same transformation as that in `T3267`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3268

Description: Transformation formula ([1], Ex. 3.6, reversed) in form of an equation. It is the same transformation as that in `T3268`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3269

Description: Transformation formula ([1], Ex. 3.8, reversed) in form of an equation. It is the same transformation as that in `T3269`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4201

Description: Transformation formula ([1], Ex. 3.4, reversed) in form of an equation. It is the same transformation as that in `T4201`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4301

Description: Transformation formula ([1], (2.10.4); Appendix (III.15)) in form of an equation. It is the same transformation as that in `T4301`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4302

Description: Transformation formula ([1], (3.2.9); Appendix (III.16)) in form of an equation. It is the same transformation as that in `T4302`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4303

Description: Transformation formula ([1], (2.5.1), reversed; Appendix (III.19)) in form of an equation. It is the same transformation as that in `T4303`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4304

Description: Transformation formula ([1], (2.10.7); Appendix (III.20)) in form of an equation. It is the same transformation as that in `T4304`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4305

Description: Transformation formula ([1], (3.10.13); Appendix (III.21)) in form of an equation. It is the same transformation as that in `T4305`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4306

Description: Transformation formula ([1], (3.10.13), reversed; Appendix (III.21), reversed) in form of an equation. It is the same transformation as that in `T4306`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4307

Description: Transformation formula ([1], (8.8.3); Appendix (III.22)) in form of an equation. It is the same transformation as that in `T4307`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4308

Description: Transformation formula ([1], Ex. 2.2) in form of an equation. It is the same transformation as that in `T4308`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4309

Description: Transformation formula ([1], Ex. 2.13(i)) in form of an equation. It is the same transformation as that in `T4309`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4310

Description: Transformation formula ([1], Ex. 2.13(ii)) in form of an equation. It is the same transformation as that in `T4310`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4311

Description: Transformation formula ([1], (3.4.8)) in form of an equation. It is the same transformation as that in `T4311`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4312

Description: Transformation formula ([1], Ex. 3.4) in form of an equation. It is the same transformation as that in `T4312`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4361

Description: Transformation formula ([1], Ex. 3.16) in form of an equation. It is the same transformation as that in `T4361`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5401

Description: Transformation formula ([1], (2.8.3); Appendix (III.25)) in form of an equation. It is the same transformation as that in `T5401`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5402

Description: Transformation formula ([1], (2.8.4); Appendix (III.26)) in form of an equation. It is the same transformation as that in `T5402`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5403

Description: Transformation formula ([1], (3.10.4), reversed) in form of an equation. It is the same transformation as that in `T5403`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5404

Description: Transformation formula ([1], Ex. 2.26, Appendix (III.14), reversed) in form of an equation. It is the same transformation as that in `T5404`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5405

Description: Transformation formula ([1], (8.8.3), Appendix (III.22), reversed) in form of an equation. It is the same transformation as that in `T5405`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5461

Description: Transformation formula ([1], (3.4.4)) in form of an equation. It is the same transformation as that in `T5461`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5462

Description: Transformation formula ([1], (3.4.4), reversed, first form) in form of an equation. It is the same transformation as that in `T5462`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5463

Description: Transformation formula ([1], (3.4.4), reversed, second form) in form of an equation. It is the same transformation as that in `T5463`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5464

Description: Transformation formula ([1], (3.5.2), reversed) in form of an equation. It is the same transformation as that in `T5464`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5465

Description: Transformation formula ([1], (3.5.7), reversed) in form of an equation. It is the same transformation as that in `T5465`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5466

Description: Transformation formula ([1], Ex. 3.16, reversed, first form) in form of an equation. It is the same transformation as that in `T5466`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5467

Description: Transformation formula ([1], Ex. 3.16, reversed, second form) in form of an equation. It is the same transformation as that in `T5467`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5468

Description: Transformation formula ([1], Ex. 2.25) in form of an equation. It is the same transformation as that in `T5468`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl6501

Description: Transformation formula ([1], Ex. 2.14(ii)) in form of an equation. It is the same transformation as that in `T6501`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7601

Description: Transformation formula ([1], (2.8.5); Appendix (III.27)) in form of an equation. It is the same transformation as that in `T7601`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7701

Description: Transformation formula ([1], (3.2.11), reversed) in form of an equation. It is the same transformation as that in `T7701`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8701

Description: Transformation formula ([1], (2.10.10), terminated; Appendix (III.17)) in form of an equation. It is the same transformation as that in `T8701`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8702

Description: Transformation formula ([1], (2.5.1); Appendix (III.18)) in form of an equation. It is the same transformation as that in `T8702`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8703

Description: Transformation formula ([1], (2.10.7), reversed; Appendix (III.20), reversed) in form of an equation. It is the same transformation as that in `T8703`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8704

Description: Transformation formula ([1], (2.10.1); Appendix (III.23)) in form of an equation. It is the same transformation as that in `T8704`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8705

Description: Transformation formula ([1], (2.10.1), iterated; Appendix (III.24)) in form of an equation. It is the same transformation as that in `T8705`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8706

Description: Transformation formula ([1], Ex. 2.13(ii), reversed) in form of an equation. It is the same transformation as that in `T8706`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8707

Description: Transformation formula ([1], (3.4.7), reversed) in form of an equation. It is the same transformation as that in `T8707`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8708

Description: Transformation formula ([1], (3.4.8), reversed) in form of an equation. It is the same transformation as that in `T8708`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8709

Description: Transformation formula ([1], (3.5.4), reversed) in form of an equation. It is the same transformation as that in `T8709`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8710

Description: Transformation formula ([1], (3.5.10)) in form of an equation. It is the same transformation as that in `T8710`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8711

Description: Transformation formula ([1], (3.5.10), reversed) in form of an equation. It is the same transformation as that in `T8711`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8761

Description: Transformation formula ([1], (2.10.10); Appendix (III.36)) in form of an equation. It is the same transformation as that in `T8761`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8762

Description: Transformation formula ([1], (2.11.1); Appendix (III.37)) in form of an equation. It is the same transformation as that in `T8762`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8764

Description: Transformation formula ([1], Ex. 2.15) in form of an equation. It is the same transformation as that in `T8764`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10901

Description: Transformation formula ([1], (2.9.1); Appendix (III.28)) in form of an equation. It is the same transformation as that in `T10901`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10902

Description: Transformation formula ([1], Ex. 2.13(i), reversed) in form of an equation. It is the same transformation as that in `T10902`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10903

Description: Transformation formula ([1], Ex. 2.19) in form of an equation. It is the same transformation as that in `T10903`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10904

Description: Transformation formula ([1], (3.10.4)) in form of an equation. It is the same transformation as that in `T10904`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10905

Description: Transformation formula ([1], Ex. 3.21(iii)) in form of an equation. It is the same transformation as that in `T10905`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10906

Description: Transformation formula ([3], (7.7), reversed) in form of an equation. It is the same transformation as that in `T10906`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10907

Description: Transformation formula ([3], (7.8), reversed) in form of an equation. It is the same transformation as that in `T10907`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10961

Description: Transformation formula ([1], (2.12.9); Appendix (III.39)) in form of an equation. It is the same transformation as that in `T10961`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10962

Description: Transformation formula ([1], (3.5.7)) in form of an equation. It is the same transformation as that in `T10962`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl10963

Description: Transformation formula ([1], Ex. 2.30) in form of an equation. It is the same transformation as that in `T10963`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121101

Description: Transformation formula ([1], (2.8.3), reversed; Appendix (III.25), reversed) in form of an equation. It is the same transformation as that in `T121101`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121102

Description: Transformation formula ([1], (2.8.4), reversed; Appendix (III.26), reversed) in form of an equation. It is the same transformation as that in `T121102`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121103

Description: Transformation formula ([1], (2.8.5), reversed; Appendix (III.27), reversed) in form of an equation. It is the same transformation as that in `T121103`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121104

Description: Transformation formula ([1], Ex. 2.14(ii), reversed) in form of an equation. It is the same transformation as that in `T121104`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121105

Description: Transformation formula ([3], (7.7)) in form of an equation. It is the same transformation as that in `T121105`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121106

Description: Transformation formula ([3], (7.8)) in form of an equation. It is the same transformation as that in `T121106`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl121161

Description: Transformation formula ([1], Ex. 2.25, reversed) in form of an equation. It is the same transformation as that in `T121161`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tli2101

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.4.1); Appendix (III.1)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2101`.

Example(s):

```
In[1]:= ph[{a,b},{c},q,z]

                [ a, b          ]
Out[1]=  ϕ      [        ; q, z ]
        2 1     [   c           ]

In[2]:= %/.Tli2101

                 [ a, b        ]      (a, b z; q)         [ c             ]
                 [             ]                 ∞        [ -, z          ]
Out[2]= {1,  ϕ   [      ; q, z ], {{----------- ϕ        [ a    ; q, a   ]},
           2 1   [  c          ]      (c, z; q)    2 1    [               ]
                                               ∞          [ b z           ]

                                   (b, a z; q)         [ c             ]
                                              ∞        [ -, z          ]
⟩     {phPerm[2, 1, u], T2101}}, {{------------ ϕ      [ b    ; q, b   ]}, T2101}}
                                   (c, z; q)    2 1    [               ]
                                            ∞          [ a z           ]
```

The first entry in this list counts the number of iterations of `Tli`-rules (cf. `Tli2104`), the second entry displays the original expression to which the `Tli`-rules are applied. The subsequent entries of the list always display an expression together with the sequence of rules that have to be applied to obtain this expression from the original expression. For instance, the above list says that the number of iterations is 1, the original expression is ${}_2\phi_1\left[\begin{matrix}a,b\\c\end{matrix};q,z\right]$, and (if the parameters of the original series are permuted) by the application of `T2101` two different expressions can be obtained from the original series. The first of them is obtained by first permuting the upper parameters by `phPerm[2,1,u]` and then applying `T2101`.

```
In[3]:= %1/.phPerm[2,1,u]/.T2101

         (a, b z; q)          [ c             ]
                    ∞         [ -, z          ]
Out[3]= ------------ ϕ        [ a    ; q, a   ]
         (c, z; q)    2 1     [               ]
                  ∞           [ b z           ]
```

For an example of how to iterate `Tli`-rules see `Tli2104`.

See also: `Tli2104, TransListe, Gleichung.`

Tli2102

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.4.5); Appendix (III.2)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2102`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2103

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.4.6); Appendix (III.3)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2103`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2104

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.5.4); Appendix (III.4)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2104`.

Example(s):

Here we demonstrate the iterated application of `Tli`-rules. We continue the `Tli2101`-example. For information of how to read the resulting listing confer `Tli2101`.

```
In[4]:= %2/.Tli2104

                                           (a z; q)        c
                  a, b                             ∞        -, a
Out[4]= {2,  φ  [       ; q, z ], {{{---------  φ  [ b      ; q, b z ]}, T2101},
           2 1    c                         (z; q)    2 2
                                                 ∞         a z, c

                 (b z; q)       c
                         ∞      -, b
〉      T2104}, {{{---------  φ  [ a      ; q, a z ]}, {phPerm[2, 1, u], T2101}},
                  (z; q)    2 2
                       ∞        b z, c

                 (a z, b z; q)          a b z
                              ∞     z, -----
〉      T2104}, {{{-------------  φ  [          c   ; q, c ]},
                   (c, z; q)     2 2
                            ∞        b z, a z

〉       {phPerm[2, 1, u], T2101}}, {phPerm[2, 1, u], T2104}},
```

```
        (b z, a z; q)       a b z
                     oo   z, -----
>    {{{-------------- phi       c   ; q, c }, T2101},
         (c, z; q)     2 2
                  oo      a z, b z

>     {phPerm[2, 1, u], T2104}}}
```

```
In[5]:= %1/.phPerm[2,1,u]/.T2101/.phPerm[2,1,u]/.T2104

        (a z, b z; q)       a b z
                     oo   z, -----
Out[5]= -------------- phi       c   ; q, c
         (c, z; q)     2 2
                  oo      b z, a z
```

See also: `Tli2101, TransListe, Gleichung.`

Tli2105

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.4), nonterminating; Appendix (III.5)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2105`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2106

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.5.6); Appendix (III.6)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2106`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2107

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 1.15(iii); Appendix (III.7)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2107`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2108

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 1.15(ii); Appendix (III.8)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2108`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2109

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.2, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2109`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2110

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.7)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2110`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2111

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.4)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2111`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2112

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.2(i), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2112`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2161

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.3.5); Appendix (III.31)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2161`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2162

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Appendix (III.32)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2162`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2163

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.8) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2163`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2201

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.5.4), reversed; Appendix (III.4), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2201`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli2202

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.2(ii), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T2202`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3101

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 1.15(ii), reversed; Appendix (III.8), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3101`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3201

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.4), reversed; Appendix (III.5), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3201`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3202

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (1.5.6), reversed; Appendix (III.6), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3202`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3203

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 1.15(iii), reversed; Appendix (III.7), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3203`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3204

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.7); Appendix (III.9)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3204`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3205

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.10); Appendix (III.10)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3205`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3206

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.3); Appendix (III.11)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3206`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3207

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.2); Appendix (III.12)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3207`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3208

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.5); Appendix (III.13)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3208`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3209

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.26; Appendix (III.14)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3209`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3210

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.6)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3210`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3211

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.1) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3211`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3212

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.1, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3212`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3213

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.2(i)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3213`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3214

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.2(ii)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3214`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3215

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.3) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3215`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3216

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.3, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3216`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3217

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.11)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3217`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3261

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.3.3); Appendix (III.33)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3261`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3262

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.3.1); Appendix (III.34)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3262`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3265

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.1); Appendix (III.35)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3265`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3266

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.2)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3266`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3267

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.6) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3267`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3268

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.6, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3268`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli3269

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.8, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T3269`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4201

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.4, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4201`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4301

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.4); Appendix (III.15)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4301`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4302

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.9); Appendix (III.16)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4302`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4303

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.5.1), reversed; Appendix (III.19)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4303`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4304

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.7); Appendix (III.20)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4304`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4305

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.10.13); Appendix (III.21)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4305`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4306

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.10.13), reversed; Appendix (III.21), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4305`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4307

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (8.8.3); Appendix (III.22)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4306`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4308

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.2) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4308`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4309

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.13(i)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4309`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4310

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.13(ii)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4310`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4311

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.8)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4311`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4312

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.4) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4312`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli4361

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.16) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T4361`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5401

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.3); Appendix (III.25)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5401`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5402

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.4); Appendix (III.26)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5402`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5403

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.10.4), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5403`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5404

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.26, Appendix (III.14), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5404`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5405

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (8.8.3), Appendix (III.22), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5405`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5461

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.4)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5461`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5462

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.4), reversed, first form) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5462`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5463

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.4), reversed, second form) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5463`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5464

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.2), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5464`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5465

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.7), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5465`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5466

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.16, reversed, first form) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5466`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5467

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.16, reversed, second form) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5467`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli5468

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.25) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T5468`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli6501

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.14(ii)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T6501`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli7601

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.5); Appendix (III.27)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T7601`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli7701

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.2.11), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T7701`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8701

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.10), terminated; Appendix (III.17)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8701`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8702

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.5.1); Appendix (III.18)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8702`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8703

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.7), reversed; Appendix (III.20), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8703`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8704

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.1); Appendix (III.23)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8704`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8705

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.1), iterated; Appendix (III.24)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8705`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8706

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.13(ii), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8706`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8707

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.7), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8707`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8708

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.4.8), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8708`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8709

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.4), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8709`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8710

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.10)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8710`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8711

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.10), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8711`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8761

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.10.10); Appendix (III.36)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8761`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8762

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.11.1); Appendix (III.37)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8762`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli8764

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.15) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T8764`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10901

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.9.1); Appendix (III.28)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10901`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10902

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.13(i), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10902`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10903

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.19) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10903`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10904

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.10.4)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10904`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10905

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 3.21(iii)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10905`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10906

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([3], (7.7), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10906`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10907

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([3], (7.8), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10907`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10961

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.12.9); Appendix (III.39)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10961`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10962

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (3.5.7)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10962`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli10963

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.30) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T10963`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121101

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.3), reversed; Appendix (III.25), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121101`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121102

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.4), reversed; Appendix (III.26), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121102`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121103

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], (2.8.5), reversed; Appendix (III.27), reversed). of an equation together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121103`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121104

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.14(ii), reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121104`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121105

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([3], (7.7)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121105`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121106

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([3], (7.8)) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121106`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

Tli121161

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 2.25, reversed) together with information of how to get them. Also iterated use is possible. The transformation is the same as that in `T121161`.

See also: `Tli2101, Tli2104, TransListe, Gleichung.`

TListe

Description: Rule that gives for a basic hypergeometric series a list of applicable transformation formulas. Each entry of this list has the format {`ArgumentPermutations,T`⟨`number`⟩}, where `ArgumentPermutations` is a sequence of reorderings of the parameters of the basic hypergeometric series (given in terms of `phPerm` and `phTausche`) and `T`⟨`number`⟩ is the name of the transformation in form of a rule which can be applied subsequently. You should be aware that `TListe` automatically applies `phOrdne` before checking which transformation could be applied.

Important Note: If the value returned by `TListe` is the empty set this does *not* mean that no transformation can be applied. You always must remember that the list of transformations included in this package is a list of *basic* transformations. There are numerous special cases of these transformations which are not contained in this list as a separate transformation. The examples below should illustrate these remarks.

Usage: `Expr/.TListe.`

Example(s):

```
In[1]:= ph[{a,b},{c},q,z]
```

$$\text{Out[1]=}\ {}_{2}\phi_{1}\left[\begin{matrix} a,\ b \\ c \end{matrix}\ ;\ q,\ z\right]$$

```
In[2]:= %/.TListe
```

```
Be sure to apply "phOrdne" before using the following information!

Out[2]= {{T2101}, {T2102}, {T2103}, {T2104}, {T2161}, {T2162}}

In[3]:= ph[{d,b^2,q^-n,a^2},{-a*b*Sqrt[q],a*b*Sqrt[q],-q^-n*d},q,q]

                         2   -n   2
                   d, b , q   , a

Out[3]= phi                                       d   ; q, q
        4 3 -(a b Sqrt[q]), a b Sqrt[q], -(--)
                                             n
                                            q

In[4]:= %/.TListe
Is n a nonnegative integer?
[y|n]:    y
Is n a nonnegative integer?
[y|n]:    y
Is n a nonnegative integer?
[y|n]:    y
Is n a nonnegative integer?
[y|n]:    y
Is n a nonnegative integer?
[y|n]:    y

Be sure to apply "phOrdne" before using the following information!

Out[4]= {{T4301}, {T4302}, {T4303}, {T4304},

>     {phPerm[2,3,1,4,u], phPerm[3,1,2,1], T4305}}

In[5]:= %3/.phOrdne/.phPerm[2,3,1,4,u]/.phPerm[3,1,2,1]/.T4305
Is n a nonnegative integer?
[y|n]:    y

                 2   2   2   -2 n
                b , a , d , q
                                                2   2
Out[5]= phi   2  2      d         1 - n ; q , q
        4 3 a  b  q, -(--), -(d q      )
                         n
                        q
```

Now we consider two examples illustrating the note above. Though none of the implemented transformations can be applied, both series can be transformed, the first by a limiting case of Watson's transformation, the second by a number of specialized very well-poised ${}_8\phi_7$ transformations. These facts are also observed by using this package.

```
In[6]:= ph[{a,q*Sqrt[a],-q*Sqrt[a],b,c,d,e},{Sqrt[a],-Sqrt[a],
        a*q/b,a*q/c,a*q/d,a*q/e,0},q,a^2*q^2/(b*c*d*e)]

             a, Sqrt[a] q, -(Sqrt[a] q), b, c, d, e          2  2
                                                            a  q
Out[6]=  phi                   a q  a q  a q  a q   ; q, -------
        7 7  Sqrt[a], -Sqrt[a], ---, ---, ---, ---, 0     b c d e
                                 b    c    d    e

In[7]:= %/.TListe

Out[7]= {}

In[8]:= Tgl8702
Do you want to set values for the equation?  [y|n]:  n
Do you want to set a value for q in the equation?  [y|n]:  n

                                                        -n         2  2 + n
              a, Sqrt[a] q, -(Sqrt[a] q), b, c, d, e, q           a  q
Out[8]=  phi                                                  ; q, ---------
        8 7                    a q  a q  a q  a q     1 + n       b c d e
             Sqrt[a], -Sqrt[a], ---, ---, ---, ---, a q
                                 b    c    d    e

                 a q        -n                 a q
                 ---, d, e, q               (a q, ---; q)
                 b c                              d e    n
 >          ==  phi                  ; q, q  --------------
              4 3 a q  a q  d e               a q  a q
                  ---, ---, ----              (---, ---; q)
                   b    c     n                d    e     n
                             a q

In[9]:= Limes[%,n->Infinity]

             a, Sqrt[a] q, -(Sqrt[a] q), b, c, d, e          2  2
                                                            a  q
Out[9]=  phi                   a q  a q  a q  a q   ; q, -------   ==
        7 7  Sqrt[a], -Sqrt[a], ---, ---, ---, ---, 0     b c d e
                                 b    c    d    e
```

```
                          a q
                          ---, d, e
             a q          b c              a q
   (a q;q)  (---;q)    ϕ             ; q, ---
          ∞  d e    ∞ 3 2 a q  a q          d e
                          ---, ---
                           b    c
⟩    ----------------------------------------
                a q       a q
               (---;q)   (---;q)
                d     ∞    e    ∞
```

```
In[10]:= ph[{q*Sqrt[a],-q*Sqrt[a],q,c,d,e,f},{Sqrt[a],-Sqrt[a],a*q/c,
         a*q/d,a*q/e,a*q/f},q,a^2*q^2/(q*c*d*e*f)]

            Sqrt[a] q, -(Sqrt[a] q), q, c, d, e, f        2
                                                        a  q
Out[10]=  ϕ                            a q  a q  a q  a q ; q, -------
         7 6 Sqrt[a], -Sqrt[a], ---, ---, ---, ---       c d e f
                                 c    d    e    f

In[11]:= %/.TListe

Out[11]= {}

In[12]:= %%/.phEinf
Add the parameter:  a

            a, Sqrt[a] q, -(Sqrt[a] q), q, c, d, e, f        2
                                                           a  q
Out[12]=  ϕ                               a q  a q  a q  a q ; q, -------
         8 7 a, Sqrt[a], -Sqrt[a], ---, ---, ---, ---       c d e f
                                    c    d    e    f

In[13]:= %/.TListe

Be sure to apply "phOrdne" before using the following information!

Out[13]= {{T8701}, {T8704}, {T8705}, {T8761}, {T8762}}
```

See also: SListe, phPerm, phTausche, TransListe.

---

trans

Description: $(a;q)_n \to (q^{1-n}/a;q)_n(-a)^n q^{\binom{n}{2}}$.

Usage: `Expr/.trans`.

Example(s):

```
In[1]:= pq[a,n,q^2]

              2
Out[1]= (a; q )
               n

In[2]:= %/.trans

                           2   2 - 2 n
             n  n  -n + n   q          2
Out[2]= (-1)  a  q         (--------; q )
                               a        n
```

See also: `Ers, PosListe, ManipulationsListe.`

TransListe

Description: List of all transformation formulas.

Usage: `TransListe.`

See also: `TransListe$gl, Liste.`

TransListe$gl

Description: List of all transformation formulas.

Usage: `TransListe$gl.`

See also: `TransListe.`

W

Description: `W[a,List,q,z]` is the very well-poised basic hypergeometric series.

Usage: `W[a,List,q,z].`

Example(s):

```
In[1]:= W[a,{b,c},q,z]

             [ a, Sqrt[a] q, -(Sqrt[a] q), b, c           ]
             [                                            ]
Out[1]=  φ   [                            a q  a q   ; q, z ]
        5 4  [   Sqrt[a], -Sqrt[a], ---, ---              ]
             [                       b    c               ]
```

See also: `ph, TeXphW, PQ, phFormat.`

---

zerl

Description: $(a;q)_n \to (a;q)_m (aq^m;q)_{n-m}$,
$(a;q)_\infty \to (a;q)_m (aq^m;q)_\infty$.

The parameter `m` has to be entered on request.

Usage: `Expr/.zerl`.

Example(s):

```
In[1]:= pq[a,2*n]

Out[1]= (a; q)
              2 n

In[2]:= %/.zerl
bottom-split by:  m+n

                            m + n
Out[2]= (a; q)       (a q      ; q)
              m + n                -m + n
```

See also: `Ers`, `PosListe`, `ManipulationsListe`.

---

zus1

Description: $(a;q)_n (aq^n;q)_m \to (a;q)_{n+m}$,
$(a;q)_n (aq^n;q)_\infty \to (a;q)_\infty$.

Usage: `Expr/.zus1`.

Example(s):

```
In[1]:= pq[a,2*n]*pq[a*q^(2*n),m-n]

                          2 n
Out[1]= (a; q)     (a q    ; q)
              2 n              m - n

In[2]:= %/.zus1

Out[2]= (a; q)
              m + n

In[3]:= pq[a,2*n]*pqinf[a*q^(2*n)]

              2 n
Out[3]= (a q    ;q)  (a; q)
                   ∞        2 n

In[4]:= %/.zus1
```

```
Out[4]= (a;q)
               ∞
```

See also: `zus2, zus3, erw1, erw2, Ers, PosListe, ManipulationsListe.`

---

zus2

Description: $(a;q)_n/(a;q)_m \to (aq^m;q)_{n-m}$,
$(a;q)_\infty/(a;q)_m \to (aq^m;q)_\infty$.

Usage: `Expr/.zus2.`

Example(s):

```
In[1]:= pq[a,m]/pq[a,n]*pq[b,m+n]

        (a; q)  (b; q)
              m       m + n
Out[1]= -------------------
              (a; q)
                    n

In[2]:= %/.zus2

                                n
Out[2]= (b; q)      (a q  ; q)
              m + n          m - n

In[3]:= pq[a,m]/pqinf[a]*pq[b,m+n]

        (a; q)  (b; q)
              m       m + n
Out[3]= -------------------
              (a;q)
                   ∞

In[4]:= %/.zus2

        (b; q)
              m + n
Out[4]= -----------
             m
         (a q ;q)
                 ∞
```

See also: `zus1, zus3, erw1, erw2, Ers, PosListe, ManipulationsListe.`

---

zus3

Description: $(a;q)_n/(b;q)_m \to (a;q)_{n-m}$,

provided $aq^n = bq^m$, and

$$(a;q)_\infty/(aq^n;q)_\infty \to (a;q)_n.$$

Usage: `Expr/.zus3.`

Example(s):

```
In[1]:= pq[a*q^m,n]/pq[a*q^n,m]

              m
        (a q ; q)
                 n
Out[1]= ----------
              n
        (a q ; q)
                 m

In[2]:= %/.zus3

              m
Out[2]= (a q ; q)
                 -m + n
In[3]:= pqinf[a*q^n]/pqinf[a*q]

              n
        (a q ;q)
                ∞
Out[3]= ---------
        (a q;q)
               ∞

In[4]:= %/.zus3

               1
Out[4]= --------------
        (a q; q)
                -1 + n
```

See also: `zus1, zus2, erw1, erw2, Ers, PosListe, ManipulationsListe.`

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