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ON A GENERALIZATION OF A RESULT OF ZHANG AND YANG

Abhijit Banerjee and Sujoy Majumder

Abstract. In this paper we find out the specific form of a meromorphic function when a generalized linear expression of the function share a small function with its k-th derivative counterpart. Our result will improve and generalize a few existing results, especially that of Zhang and Yang [J. L. Zhang and L. Z. Yang, A power of a meromorphic function sharing a small function with its derivative, Ann. Acad. Sci. Fenn. Math., 34 (2009), 249-260].

1. Introduction, definitions and results

In this paper, by a meromorphic function we will always mean meromorphic function in the complex plane $\mathbb{C}$. We adopt the standard notations of the Nevanlinna theory of meromorphic functions as explained in [3]. It will be convenient to let E denote any set of positive real numbers of finite linear measure, not necessarily the same at each occurrence. For a non-constant meromorphic function h, we denote by $T(r,h)$ the Nevanlinna characteristic of h and by $S(r,h)$ any quantity satisfying $S(r,h) = o\{T(r,h)\}$, as $r \to \infty$ and $r \notin E$.

Let f and g be two non-constant meromorphic functions and let a be a complex number. We say that f and g share a CM, provided that $f - a$ and $g - a$ have the same zeros with the same multiplicities. Similarly, we say that f and g share a IM, provided that $f - a$ and $g - a$ have the same zeros ignoring multiplicities. In addition, we say that f and g share ∞ CM, if $1/f$ and $1/g$ share 0 CM, and we say that f and g share ∞ IM, if $1/f$ and $1/g$ share 0 IM.

A meromorphic function a is said to be a small function of f provided that $T(r,a) = S(r,f)$, that is $T(r,a) = o(T(r,f))$ as $r \to \infty$, $r \notin E$. If a is a small function we define that f and g share a CM if $f - a$ and $g - a$ share 0, CM.

During the last four decades the uniqueness theory of entire and meromorphic functions has become a prominent branch of the value distribution theory (see [12]). Rubel-Yang [7] first established the fact that when f and its derivative f' share two complex values CM then they are identical. In 1979, improving the result

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in [6] analogous result corresponding to IM sharing was obtained by Mues and Steinmetz [4].

In course of time many researchers such as Brück [1], Gundersen [2], Yang [8] et al. became more involved to find out the relation between an entire or meromorphic function with its higher derivatives or more general (linear) differential expressions, sharing one value. To find the most specific form of the function, Yang-Zhang [9] (see also [13]) first considered the uniqueness of a power of a meromorphic (entire) function $F = f^n$ and its derivative F' when they share certain value.

The paper is devoted to the specific type of form of the function first used by Yang-Zhang [9]. To this end we are invoking the following results which elaborates the gradual developments to this setting of meromorphic functions. Zhang [13] proved the following theorem, which improved all the results obtained in [9].

Theorem A. [13] *Let f be a non-constant meromorphic function, n, k be positive integers and $a(z)(\not\equiv 0, \infty)$ be a meromorphic small function of f. Suppose $f^n - a$ and $(f^n)^{(k)} - a$ share the value 0 CM and*

$$(n-k-1)(n-k-4) > 3k+6.$$

Then $f^n \equiv (f^n)^{(k)}$, and f assumes the form

$$f(z) = ce^{\frac{\lambda}{n}z},$$

where c is a nonzero constant and $\lambda^k = 1$.

In 2009, Zhang-Yang [14] further improved the above results in the following manner.

Theorem B. [14] *Let f be a non-constant meromorphic function, n, k be positive integers and $a(z)(\not\equiv 0, \infty)$ be a meromorphic small function of f. Suppose $f^n - a$ and $(f^n)^{(k)} - a$ share the value 0 CM and*

$$n > k + 1 + \sqrt{k+1}.$$

Then $f^n \equiv (f^n)^{(k)}$, and f assumes the form

$$f(z) = ce^{\frac{\lambda}{n}z},$$

where c is a nonzero constant and $\lambda^k = 1$.

A recent development in the uniqueness theory of meromorphic functions is the introduction of the notion of weighted sharing of values [5]. This measures a gradual increment from IM (ignoring multiplicities) sharing to CM (counting multiplicities) sharing.

Let k be a non-negative integer or infinity. For $a \in \mathbb{C} \cup \{\infty\}$ we denote by $E_k(a; f)$ the set of all a-points of f, where an a-point of multiplicity m is counted m times if $m \leq k$ and $k+1$ times if $m > k$. If for two meromorphic functions f

and g we have $E_k(a;f) = E_k(a;g)$, then we say that f and g share the value a with weight k.

The IM and CM sharing respectively correspond to weight 0 and ∞. If a is a small function we define that f and g share a IM or a CM or with weight l according as $f-a$ and $g-a$ share $(0,0)$ or $(0,\infty)$ or $(0,l)$ respectively.

Throughout this paper, we always use $P(f)$ to denote an arbitrary polynomial in f of degree n as follows,

$$P(f) = a_n f^n + a_{n-1} f^{n-1} + \cdots + a_0.$$

Now observing the above results the following questions are inevitable.

Question 1: Can the lower bound of n be further reduced in Theorem B?

Question 2: What happens if f^n is replaced by a general linear expression in f, namely of the form $P(f)$ in Theorem B?

Question 3: Can the "CM" sharing in Theorem B be reduced to finite weight sharing?

In this paper, taking the possible answer of the above questions into background we obtain our main result. To proceed further we require the following.

Let
$$P(f) = a_n (f-c_{l_1})^{l_1}(f-c_{l_2})^{l_2}\cdots(f-c_{l_s})^{l_s}, \tag{1.1}$$
where a_i $(i=0,1,\dots,n-1)$, $a_n \neq 0$ and c_{l_j} $(j=1,2,\dots,s)$ are distinct finite complex numbers and $l_1,l_2,\dots,l_s$, s,n and k are all positives integers with $\sum_{i=1}^{s} l_i = n$. Let $l = \max\{l_1,l_2,\dots,l_s\}$. We set an arbitrary non-zero polynomial $P_1(f_1)$ by

$$P_1(f_1) = a_n \prod_{\substack{i=1\\ l_i\neq l}}^{s} (f_1 + c_l - c_{l_i})^{l_i} = b_m f_1^m + b_{m-1} f_1^{m-1} + \cdots + b_0, \tag{1.2}$$

where $a_n = b_m$, $f_1 = f - c_l$ and $m = n - l$. Obviously

$$P(f) = f_1^l P_1(f_1). \tag{1.3}$$

Let $P_1(f_1) = b_m f_1^m + b_{m-1} f_1^{m-1} + \cdots + b_1 f_1 + b_0 = b_m \prod_{i=1}^{r}(f_1-\alpha_i)^{l_i}$, where $r = s-1$, $\alpha_i = c_{l_i} - c_l$, $i = 1,2,\dots,r$ be the distinct zeros of $P_1(f_1)$. For $i = 1,2,\dots,r$ we define

$$l_i^* = \begin{cases} k+1, & \text{if } l_i > k+1 \\ l_i, & \text{if } l_i \leq k+1. \end{cases}$$

Theorem 1. *Let f be a non-constant meromorphic function and $a(z)(\not\equiv 0,\infty)$ be a meromorphic small function of f. Suppose $P(f)-a$ and $[P(f)]^{(k)} - a$ share $(0, r+3)$ and*

$$n > \max\{k+m+1, k + \sum_{i=1}^{r} l_i^* + 2\}. \tag{1.4}$$

Then $P_1(f_1)$ *reduces to a non-zero monomial, namely* $P_1(f_1) = b_i f_1^i \not\equiv 0$ *for some* $i \in \{0, 1, \ldots, m\}$*; and* $f_1^{l+i} \equiv (f_1^{l+i})^{(k)}$*, where* f_1 *assumes the form* $f_1(z) = ce^{\frac{\lambda}{l+i}z}$*, i.e.,*

$$f(z) = ce^{\frac{\lambda}{l+i}z} + c_l,$$

where c *is a nonzero constant and* $\lambda^k = 1$*.*

Corollary 1. *Since* $n = m + l$*, (1.4) can also be replaced by*

$$l > \max\{k+1, k + \sum_{i=1}^{r} l_i^* + 2 - m\}.$$

So we see that if $P_1(f_1)$ *has at least one factor with multiplicity* $l_i > k+1$*,* $i = 1, 2, \ldots, r$ *then* $\sum_{i=1}^{r} l_i^* \leq m - 1$*, and so Theorem 1 would have been true for* $l > k + 1$*. Otherwise we always have* $l > k + 2$*.*

Corollary 2. *When* $m = 0$*, i.e.,* $P(f)$ *is of the form* $f^l = f^n$*, then* $P_1(f_1) =$ *constant and so* $r = 0$*. Thus conclusion of Theorem B can be obtained when* $n > k + 2$ *and* $f^n - a$ *and* $(f^n)^{(k)} - a$ *share* $(0, 3)$*.*

Remark 1. The following two examples show that the conclusion of Theorem 1 ceases to hold for $n = 1$.

Example 1. Let $f(z) = \frac{z+1}{1+e^{-z}}$. Then f and f' share $(1, \infty)$, but $f(z) \not\equiv f'(z)$.

Example 2. Let

$$f(z) = \frac{\frac{1}{2}z^2 + b}{1 + ce^{-z}},$$

where b and $c \neq 0$ are constants. Then $f(z) - z$ and $f'(z) - z$ share $(0, \infty)$, but $f(z) \not\equiv f'(z)$.

We are now going to explain the following definitions and notations which will be used in the paper.

Definition 1. [6] Let p be a positive integer and $a \in \mathbb{C} \cup \{\infty\}$. $N(r, a; f \mid\geq p)$ ($\overline{N}(r, a; f \mid\geq p)$) denotes the counting function (reduced counting function) of those a-points of f whose multiplicities are not less than p.

Definition 2. [5] Let f, g share a value $(a, 0)$. We denote by $\overline{N}_*(r, a; f, g)$ the reduced counting function of those a-points of f whose multiplicities differ from the multiplicities of the corresponding a-points of g. Clearly $\overline{N}_*(r, a; f, g) \equiv \overline{N}_*(r, a; g, f)$.

Definition 3. [11] For $a \in \mathbb{C} \cup \{\infty\}$ and a positive integer p we denote by $N_p(r, a; f)$ the sum $\overline{N}(r, a; f) + \overline{N}(r, a; f \mid\geq 2) + \cdots + \overline{N}(r, a; f \mid\geq p)$. Clearly $N_1(r, a; f) = \overline{N}(r, a; f)$.

2. Lemmas

In this section we present lemmas which will be needed in the sequel.

Lemma 1. [10] *Let f be a non-constant meromorphic function and let $a_n(z)(\not\equiv 0)$, $a_{n-1}(z), \ldots, a_0(z)$ be meromorphic functions such that $T(r, a_i(z)) = S(r, f)$ for $i = 0, 1, 2, \ldots, n$. Then*

$$T(r, a_n f^n + a_{n-1} f^{n-1} + \cdots + a_1 f + a_0) = nT(r, f) + S(r, f).$$

Lemma 2. [6] *If $N(r, 0; f^{(k)} \mid f \neq 0)$ denotes the counting function of those zeros of $f^{(k)}$ which are not zeros of f, where a zero of $f^{(k)}$ is counted according to its multiplicity then*

$$N(r, 0; f^{(k)} \mid f \neq 0) \leq k\overline{N}(r, \infty; f) + N(r, 0; f \mid< k) + k\overline{N}(r, 0; f \mid\geq k) + S(r, f).$$

Lemma 3. *Let $F = \frac{P(f)}{a(z)}$ and $G = \frac{[P(f)]^{(k)}}{a(z)}$, where $P(f)$ is defined as in* (1.1) *and they share* $(1, p)$. *Then*

$$\overline{N}_*(r, 1; F, G) \leq \frac{1}{p}\{\overline{N}(r, \infty; f) + \overline{N}(r, 0; f) + (s-1)T(r, f)\} + S(r, f).$$

Proof. Clearly, in view of Lemma 2 we have

$$\begin{aligned}
\overline{N}_*(r, 1; F, G) &\leq \overline{N}(r, 1; F \mid\geq p+1) + S(r, f) \\
&\leq \frac{1}{p}\,(N(r, 1; F) - \overline{N}(r, 1; F)) + S(r, f) \\
&\leq \frac{1}{p}\,N(r, 0; F^{'} \mid F \neq 0) + S(r, f) \\
&\leq \frac{1}{p}\{\overline{N}(r, \infty; f) + \overline{N}(r, 0; f_1) + \overline{N}(r, 0; P_1(f_1))\} + S(r, f) \\
&\leq \frac{1}{p}\{\overline{N}(r, \infty; f) + \overline{N}(r, 0; f) + (s-1)T(r, f)\} + S(r, f). \quad \blacksquare
\end{aligned}$$

3. Proof of the theorem

Proof of Theorem 1. Let F and G be defined as in Lemma 3. Then F and G share $(1, s+2)$ except for the zeros and poles of $a(z)$ and so

$$\overline{N}(r, 1; F) = \overline{N}(r, 1; G) + S(r, f).$$

Suppose

$$\Phi = \frac{1}{F}\left(\frac{G^{'}}{G-1} - \frac{F^{'}}{F-1}\right) = \frac{G}{F}\left(\frac{G^{'}}{G-1} - \frac{G^{'}}{G}\right) - \left(\frac{F^{'}}{F-1} - \frac{F^{'}}{F}\right). \tag{3.1}$$

We now consider the following two cases:

Case 1: $\Phi \equiv 0$.

On integration we get

$$F - 1 \equiv c(G - 1), \tag{3.2}$$

where c is a nonzero constant. This implies that $\overline{N}(r,\infty;f) = S(r,f)$. Let $c \neq 1$. Then from (3.2) we get

$$\frac{1}{F} \equiv \frac{1}{c-1}\left(c\frac{G}{F} - 1\right). \tag{3.3}$$

Now using (1.3), (3.3) and Lemma 1 we get

$$\begin{aligned}
nT(r,f) &= T(r,F) + O(1)\\
&\leq T(r,\frac{G}{F}) + S(r,f)\\
&= N(r,\infty;\frac{[P(f)]^{(k)}}{P(f)}) + m(r,\infty;\frac{[P(f)]^{(k)}}{P(f)}) + S(r,f)\\
&\leq N_k(r,0;P(f)) + k\overline{N}(r,\infty;f) + S(r,f)\\
&\leq N_k(r,0;f_1^l P_1(f_1)) + S(r,f)\\
&\leq k\,\overline{N}(r,0;f_1) + m\,T(r,f_1) + S(r,f)\\
&\leq (k+m)\,T(r,f) + S(r,f),
\end{aligned}$$

which is impossible since $n > k+m+1$.

Hence $c = 1$. From (3.2) we get $F \equiv G$, i.e.,

$$f_1^l P_1(f_1) \equiv [f_1^l P_1(f_1)]^{(k)}. \tag{3.4}$$

We now prove that $P_1(\omega_1) = b_i\omega_1^i \not\equiv 0$ for some $i \in \{0,1,\ldots,m\}$. On the contrary we assume that $P_1(\omega_1) = b_m\omega_1^m + b_{m-1}\omega_1^{m-1} + \cdots + b_1\omega_1 + b_0$, where at least two of $b_0, b_1, \ldots, b_{m-1}, b_m$ are nonzero. Without loss of generality, we assume that $b_s, b_t \neq 0$, where $s \neq t$, $s,t = 0,1,2,\ldots,m$.

From (3.4) it is clear that f_1 is an entire function. Also since $l > k+1$, it follows from (3.4) that 0 is a Picard Exceptional Value of f_1. So we have $f_1 = e^{\alpha}$, where α is a non-constant entire function. Then by induction we get

$$b_i[f_1^{l+i} - (f_1^{l+i})^{(k)}] = t_i(\alpha', \alpha'', \ldots, \alpha^{(k)})e^{(l+i)\alpha}, \tag{3.5}$$

where $t_i(\alpha', \alpha'', \ldots, \alpha^{(k)})$ $(i = 0,1,2,\ldots,m)$ are differential polynomials in α', α'', $\ldots$, $\alpha^{(k)}$.

From (3.4) and (3.5) we obtain

$$t_m(\alpha', \alpha'', \ldots, \alpha^{(k)})e^{m\alpha} + \cdots + t_1(\alpha', \alpha'', \ldots, \alpha^{(k)})e^{\alpha} + t_0(\alpha', \alpha'', \ldots, \alpha^{(k)}) \equiv 0. \tag{3.6}$$

Since $T(r,t_i) = S(r,f)$ $(i = 0,1,\ldots,m)$, by the Borel unicity theorem (Theorem 1.52 [12]), (3.6) gives $t_i \equiv 0$ $(i = 0,1,\ldots,m)$. As $b_s, b_t \neq 0$, from (3.5) we have

$$f_1^{l+s} \equiv (f_1^{l+s})^{(k)} \quad \text{and} \quad f_1^{l+t} \equiv (f_1^{l+t})^{(k)},$$

which is a contradiction. Actually in this case we get two different forms of $f_1(z)$ simultaneously. Hence $P_1(\omega_1) = b_i\omega_1^i \not\equiv 0$ for some $i \in \{0, 1, \ldots, m\}$. So from (3.4) we get $f_1^{l+i} \equiv [f_1^{l+i}]^{(k)}$, where $i \in \{0, 1, \ldots, m\}$. Clearly f_1 assumes the form $f_1(z) = ce^{\frac{\lambda}{l+i}z}$, i.e.,

$$f(z) = ce^{\frac{\lambda}{l+i}z} + c_l,$$

where c is a nonzero constant and $\lambda^k = 1$.

Case 2: $\Phi \not\equiv 0$.

Clearly $F \not\equiv G$. Now from the fundamental estimate of logarithmic derivative it follows that

$$m(r, \Phi) = S(r, f). \tag{3.7}$$

Also from (3.1) we get $m(r, \Phi) = S(r, f)$ and

$$m(r, F) \leq m(r, \frac{1}{\Phi}) + S(r, f). \tag{3.8}$$

If z_0 is a pole of f with multiplicity $p \geq 1$ such that $a(z_0) \neq 0, \infty$, then

$$\Phi(z) = O((z - z_0)^{p-1}). \tag{3.9}$$

Let z_1 be a zero of f_1 with multiplicity q such that $a(z_1) \neq 0, \infty$. Then z_1 will be a zero of F and F' with multiplicities lq and $lq - 1$ respectively. By the given condition we have $l > k + 1$. On the other hand z_1 will be a zero of G' with multiplicity $lq - (k + 1)$. As a result we have

$$\frac{F'(z)}{F(z) - 1} - \frac{G'(z)}{G(z) - 1} = O((z - z_1)^{lq-(k+1)}).$$

Consequently,

$$\frac{1}{\Phi(z)} = O((z - z_1)^{k+1}). \tag{3.10}$$

Then z_1 will be a pole of Φ with multiplicity $k + 1$.

Let z_{q_i} be a zero of $f_1 - \alpha_i$, with multiplicity q_i such that $a(z_{q_i}) \neq 0, \infty$, where $i = 1, 2, \ldots, r$. Then z_{q_i} will be a zero of F with multiplicity l_iq_i, where $i = 1, 2, \ldots, r$. Clearly z_{q_i} will be a zero of $F^{'}$ with multiplicity $l_iq_i - 1$, where $i = 1, 2, \ldots, r$.

On the other hand z_{q_i} will be a zero of G with multiplicity $l_iq_i - k(i = 1, 2, \ldots, r)$, provided $l_iq_i > k$. Clearly z_{q_i} will be a zero of G' with multiplicity $l_iq_i - (k+1)(i = 1, 2, \ldots, r)$, provided $l_iq_i > k+1$. So when $l_iq_i > k+1$, we have

$$\frac{F'z)}{F(z) - 1} - \frac{G'(z)}{G(z) - 1} = O((z - z_{q_i})^{l_iq_i-(k+1)}).$$

Therefore

$$\frac{1}{\Phi(z)} = \begin{cases} O((z - z_{q_i})^{k+1}), & \text{if } l_iq_i > k + 1 \\ O((z - z_{q_i})^{l_iq_i}), & \text{if } l_iq_i \leq k + 1. \end{cases} \tag{3.11}$$

Consequently, z_{q_i} will be a pole of Φ with multiplicity at least l_i^*, where $i = 1, 2, \ldots, r$.

Also, if z_2 $(a(z_2) \neq 0, \infty)$ is a common zero of $F-1$ and $G-1$ with different multiplicities, then z_2 will be a pole of Φ. Thus

$$N(r,\infty;\Phi) \leq (k+1)\overline{N}(r,0;f_1)+\sum_{i=1}^{r} l_i^* N(r,\alpha_i;f_1)+\overline{N}_*(r,1;F,G)+S(r,f). \quad (3.12)$$

Then from (3.1), (3.7), (3.9), (3.12), Lemmas 2 and 3 we get

$$\begin{aligned}
N(r,\infty;F) - \overline{N}(r,\infty;F) &\leq N(r,0;\Phi) + S(r,f)\\
&\leq T(r,\frac{1}{\Phi}) - m(r,\frac{1}{\Phi}) + S(r,f)\\
&\leq T(r,\Phi) - m(r,\frac{1}{\Phi}) + S(r,f)\\
&= N(r,\infty;\Phi) + m(r,\Phi) - m(r,\frac{1}{\Phi}) + S(r,f)\\
&\leq (k+1)\overline{N}(r,0;f_1) + \sum_{i=1}^{r} l_i^* N(r,\alpha_i;f_1) + \overline{N}_*(r,1;F,G) - m(r,\frac{1}{\Phi}) + S(r,f)\\
&\leq (k+1)\overline{N}(r,0;f_1) + \Big(\sum_{i=1}^{r} l_i^*\Big)T(r,f_1) + \frac{r+2}{r+3}T(r,f) - m(r,\frac{1}{\Phi}) + S(r,f).
\end{aligned} \quad (3.13)$$

Now using (3.8), (3.13) and Lemma 1 we get

$$\begin{aligned}
n\,T(r,f) &= (m+l)T(r,f) = T(r,F) + O(1)\\
&= N(r,\infty;F) + m(r,F) + O(1)\\
&\leq N(r,\infty;F) + m(r,\frac{1}{\Phi}) + S(r,f)\\
&\leq (k+1)\overline{N}(r,0;f_1) + \overline{N}(r,\infty;F) + \Big(\sum_{i=1}^{r} l_i^*\Big)T(r,f_1) + \frac{r+2}{r+3}T(r,f) + S(r,f)\\
&= (k+1)\overline{N}(r,0;f_1) + \overline{N}(r,\infty;f) + \Big(\sum_{i=1}^{r} l_i^*\Big)T(r,f_1) + \tfrac{r+2}{r+3}T(r,f) + S(r,f)\\
&\leq \Big(k + \sum_{i=1}^{r} l_i^* + \frac{r+2}{r+3} + 2\Big)T(r,f) + S(r,f),
\end{aligned}$$

which is impossible since $n > k + \sum_{i=1}^{r} l_i^* + 2$ and $0 < \frac{r+2}{r+3} < 1$.

This completes the proof of the theorem. ∎

4. An open question

Keeping other conditions intact can the sharing condition in Theorem 1 be relaxed to $(0,2)$ so that conclusion remains the same?

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A.B.: Department of Mathematics, University of Kalyani, West Bengal 741235, India

E-mail: `abanerjee_kal@yahoo.co.in, abanerjee_kal@rediffmail.com`

S.M.: Department of Mathematics, Katwa College, Burdwan, 713130, India

E-mail: `sujoy.katwa@gmail.com, smajumder05@yahoo.in`