GRAPH OPERATIONS AND NEIGHBOR-INTEGRITY

Alpay Kirlangiç, Bornova-Izmir

(Received July 25, 2001, revised March 9, 2004)

Abstract. Let G be a graph. A vertex subversion strategy of G, say S, is a set of vertices in G whose closed neighborhood is removed from G. The survival-subgraph is denoted by G/S. The Neighbor-Integrity of G, $\mathrm{NI}(G)$, is defined to be $\mathrm{NI}(G) = \min\limits_{S \subseteq V(G)} \{|S| + c(G/S)\}$, where S is any vertex subversion strategy of G, and $c(G/S)$ is the maximum order of the components of G/S. In this paper we give some results connecting the neighbor-integrity and binary graph operations.

Keywords: vulnerability, integrity, neighbor-integrity

MSC 2000: 05C40, 05C85

1. Introduction

If we think of a graph as a model of a network, the vulnerability measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. In order to measure the vulnerability we have some parameters like connectivity, toughness, binding number, integrity and tenacity [4], [5], [6], [10]. But these parameters do not consider the effect which removal of a vertex has on the neighbors of that vertex. In a Spy Network, vertices correspond to stations or operatives, and edges represent lines of communication. If a station or an operative is captured, the adjacent stations will be betrayed and are therefore useless in the whole network. Therefore, instead of removing only vertices from a graph, we remove vertices and all of their adjacent vertices. The concept of Neighbor-Integrity was introduced as a measure of graph vulnerability in this sense by Margaret B. Cozzens and Shu-Shih Y. Wu [5].

Let G be a simple graph without loops and multiple edges and let u be any vertex in G. The set $N(u) = \{v \in V(G);\ v \neq u,\ v \text{ and } u \text{ are adjacent}\}$ is the open neighborhood of u, and $N[u] = \{u\} \cup N(u)$ denotes the closed neighborhood of u. A vertex u in G is said to be subverted if the closed neighborhood of u, $N[u]$, is removed from G. A set of vertices $S = \{u_1, u_2, \ldots, u_m\}$ is called a vertex subversion strategy of G if each of the vertices in S has been subverted from G. If S has been subverted from the graph G, then the survival subgraph is disconnected, a clique, or the empty graph (see [5]). The survival subgraph is denoted by G/S. The Neighbor-Integrity of a graph G is defined to be

$$\mathrm{NI}(G) = \min_{S \subseteq V(G)} \{|S| + c(G/S)\},$$

where S is any vertex subversion strategy of G, and $c(G/S)$ is the maximum order of the components of G/S [5].

Cozzens and Wu [5], [7], [8], [9] obtained several results on the neighbor-integrity. In Section 2 the known results on the neighbor-integrity are given. In Section 3 we give the neighbor-integrity of graphs obtained by binary graph operations.

2. Basic results

In this section we will review some of the known results.

Theorem 2.1 [5], [8]**.** *The neighbor-integrity of*

(a) *the complete graph K_n is* 1.

(b) *the path P_n is*

$$\mathrm{NI}(P_n) = \begin{cases} \lceil 2\sqrt{n+3}\rceil - 4, & \text{if } n \geqslant 2; \\ 1, & \text{if } n = 1. \end{cases}$$

If S achieves the neighbor-integrity of the graph P_n, then $|S| = \lceil \sqrt{n+3}\rceil - 1$.

(c) *the cycle C_n is*

$$\mathrm{NI}(C_n) = \begin{cases} \lceil 2\sqrt{n}\rceil - 3, & \text{if } n \geqslant 5; \\ 2, & \text{if } n = 4; \\ 1, & \text{if } n = 3. \end{cases}$$

If S achieves the neighbor-integrity of the graph C_n, then $|S| = \lceil \sqrt{n}\rceil - 1$.

Theorem 2.2 [5]**.** (a) *The size of a maximum matching in G is an upper bound for* $\mathrm{NI}(G)$.

(b) *The independence number of G is an upper bound for* $\mathrm{NI}(G)$.

(c) $\mathrm{NI}(G) = 1$ *if and only if G contains a spanning subgraph that is a star or G is a set of isolated vertices.*

Theorem 2.3 [5]. *Let C_n^k be the k-th power of a cycle, where $n \geqslant 3$ and $1 \leqslant k \leqslant \lfloor n/2 \rfloor$. Then*

$$\mathrm{NI}(C_n^k) = \begin{cases} \lceil 2\sqrt{n}\rceil - (2k+1), & \text{if } 1 \leqslant k < \frac{\sqrt{n}-1}{2}; \\ \lceil n/(2k+1)\rceil, & \text{otherwise.} \end{cases}$$

Theorem 2.4 [11]. (a) *For any graph G, $\mathrm{NI}(G \times P_n) \leqslant n\,\mathrm{NI}(G)$.*
(b) *For any graphs G and H, $\mathrm{NI}(G \times H) \geqslant \max\{\mathrm{NI}(G), \mathrm{NI}(H)\}$.*

3. Graph operations and neighbor-integrity

In this section we consider the binary graph operations. These operations are join, composition, product and corona of two graphs. The graphs G_1 and G_2 have disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$, respectively.

(a) Join

Definition 3.1. The *union* $G = G_1 \cup G_2$ of graphs G_1 and G_2 is the graph with $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1) \cup E(G_2)$. The *join* $G = G_1 + G_2$ of graphs G_1 and G_2 is the graph union $G_1 \cup G_2$ together with all the edges joining $V(G_1)$ and $V(G_2)$.

Definition 3.2. A subset S of $V(G)$ such that every edge of G has at least one end in S is called a *covering of* G. The number of vertices in a minimum covering of G is the *covering number* of G and is denoted by $\beta(G)$.

Theorem 3.1. *Let G_1 and G_2 be two graphs. Then*

$$\mathrm{NI}(G_1 + G_2) = \begin{cases} 1, & \text{if } \beta(G_1) = 1 \text{ or } \beta(G_2) = 1; \\ 2, & \text{otherwise.} \end{cases}$$

Proof. This proof is also valid up to symmetry for G_2. If $\beta(G_1) = 1$, then we can find a vertex u such that $u \in V(G_1)$ and $N[u] = V(G_1 + G_2)$. Hence $(G_1 + G_2)/\{u\}$ is empty and $c((G_1 + G_2)/\{u\}) = 0$. Therefore $\mathrm{NI}(G_1 + G_2) = 1$, if $\beta(G_1) = 1$. On the other hand, it is always true that $\mathrm{NI}(G_1 + G_2) \leqslant 2$ and it cannot be 1 if $\beta(G_1) \geqslant 2$.

This completes the proof. □

(b) Composition

Definition 3.3. The *composition* $G_1[G_2]$ of two graphs G_1 and G_2 has its vertex set $V(G_1) \times V(G_2)$, with (u_1, u_2) adjacent to (v_1, v_2) if either u_1 is adjacent to v_1 in G_1 or $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2.

Definition 3.4. A *vertex dominating set* for a graph G is a set S of vertices such that every vertex of G belongs to S or is adjacent to a vertex of S. The minimum cardinality of a vertex dominating set in a graph G is called the *vertex dominating number* of G and is denoted by $\gamma(G)$.

Theorem 3.2. *Let H be a graph of order $n \geqslant 2$ and let G be a graph. Then* $\mathrm{NI}(G[H]) = \min\{\mathrm{NI}(G) + c(G/S)(n-2),\ \gamma(G) + \min\limits_{v\in V(H)}\{c(H/v)\},\ \gamma(G)(1 + \min\limits_{v\in V(H)}\{\mathrm{NI}(H/v)\})\}$ *where $S \subseteq V(G)$ and $|S| + c(G/S) = \mathrm{NI}(G)$.*

Proof. Let X be a subset of $V(G[H])$ such that $|X| + c(G[H]/X) = \mathrm{NI}(G[H])$. The graph $G[H]$ contains n copies of G and let S be a set of removed vertices from any copy of G. Then we have two cases:

Case 1: If S is not a dominating set, then X must contain the vertices of every copy of G in $G[H]$. Hence $|X| = |S| < \gamma(G)$ and $c(G[H]/X) = c(G/S)(n-1)$. When S realizes the neighbor-integrity of G, we have

$$\mathrm{NI}(G[H]) = \min_{S\subseteq V(G)}\{|S| + c(G/S)(n-1)\} = \mathrm{NI}(G) + c(G/S)(n-2). \tag{1}$$

Case 2: If S is a dominating set, then $|X| \geqslant \gamma(G)$.

$$\text{If } |X| = \gamma(G), \text{ then } c(G[H]/X) = \min_{v\in V(H)}\{c(H/v)\}. \tag{2}$$

$$\text{If } |X| > \gamma(G), \text{ then } c(G[H]/X) = \min_{v\in V(H)}\{\mathrm{NI}(H/v)\}\gamma(G). \tag{3}$$

The theorem follows from (1), (2) and (3). □

Corollary 3.1. (a) $\mathrm{NI}(P_m[P_n]) = \min\{\lceil 2\sqrt{m+3}\rceil(n-1) + \lceil\sqrt{m+3}\rceil(2-n) - 3n+2,\ \lceil\frac{m}{3}\rceil + \lceil\frac{n-3}{2}\rceil,\ \lceil\frac{m}{3}\rceil(\lceil 2\sqrt{n}\rceil - 3)\}$,

(b) $\mathrm{NI}(P_m(C_n)] = \min\{\lceil 2\sqrt{m+3}\rceil(n-1) + \lceil\sqrt{m+3}\rceil(2-n) - 3n+2,\ \lceil\frac{m}{3}\rceil + n - 3,\ \lceil\frac{m}{3}\rceil(\lceil 2\sqrt{n}\rceil - 3)\}$,

(c) $\mathrm{NI}(C_m[P_n]) = \min\{(\lceil 2\sqrt{m}\rceil - 3)(n-1) + \lceil\sqrt{m}\rceil(2-n),\ \lceil\frac{m}{3}\rceil + \lceil\frac{n-3}{2}\rceil,\ \lceil\frac{m}{3}\rceil(\lceil 2\sqrt{n}\rceil - 3)\}$,

(d) $\mathrm{NI}(C_m[C_n]) = \min\{(\lceil 2\sqrt{m}\rceil - 3)(n-1) + \lceil\sqrt{m}\rceil(2-n),\ \lceil\frac{m}{3}\rceil + n - 3,\ \lceil\frac{m}{3}\rceil(\lceil 2\sqrt{n}\rceil - 3)\}$.

Proof. (a) follows from Theorem 3.2 and 2.1(b). The proof of the other parts is similar. □

(c) Product

Definition 3.5. The (Cartesian) product $G_1 \times G_2$ of graphs G_1 and G_2 also has $V(G_1) \times V(G_2)$ as its vertex set, but here (u_1, u_2) is adjacent to (v_1, v_2) if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1.

In the next theorem we give $\mathrm{NI}(K_2 \times P_n)$ for $5 \leqslant n < 34$ and a lower bound for $\mathrm{NI}(K_2 \times P_n)$ where $n \geqslant 34$. In Theorem 3.4, we compute the exact result of $\mathrm{NI}(K_2 \times P_n)$ for $n \geqslant 34$.

Theorem 3.3. *Let n be a positive integer. If $5 \leqslant n < 34$, then $\mathrm{NI}(K_2 \times P_n) = \lceil n/3 \rceil + 1$. Moreover, if $n \geqslant 34$, then $\mathrm{NI}(K_2 \times P_n) \geqslant \lceil 2\sqrt{2n+4} \rceil - 5$.*

Proof. Let $S \subseteq V(K_2 \times P_n)$ and let b be the maximum order of the components of $(K_2 \times P_n)/S$. Then we have two cases, depending on b:

Case 1: Let $b = 1$. In order to obtain the components of order 1, we have to remove $\lceil n/3 \rceil$ vertices. Hence $\mathrm{NI}(K_2 \times P_n) = \lceil n/3 \rceil + 1$.

Case 2: Let $b \geqslant 2$. If we remove $|S| = a$ vertices, then the number of components is at most $a + 1$. So

$$c((K_2 \times P_n)/S) \geqslant \frac{2n - 4a}{a+1}$$

and

$$\mathrm{NI}(K_2 \times P_n) \geqslant \min_a \left\{ a + \frac{2n - 4a}{a+1} \right\}.$$

The function $f(a) = a + \frac{2n-4a}{a+1}$ assumes its minimum value at $a = -1 + \sqrt{2n+4}$ and $f(-1 + \sqrt{2n+4}) = 2\sqrt{2n+4} - 5$. Since the neighbor-integrity is integer valued, we round this result up to get a lower bound and so $\mathrm{NI}(K_2 \times P_n) \geqslant \lceil 2\sqrt{2n+4} \rceil - 5$ if $b \geqslant 2$.

Consequently, we have

$$\mathrm{NI}(K_2 \times P_n) \geqslant \min\{\lceil n/3 \rceil + 1,\ \lceil 2\sqrt{2n+4} \rceil - 5\} \text{ for every } b.$$

One can easily show that

$$\lceil n/3 \rceil + 1 \leqslant \lceil 2\sqrt{2n+4} \rceil - 5 \text{ for } 5 \leqslant n < 34.$$

Therefore

$$\mathrm{NI}(K_2 \times P_n) = \lceil n/3 \rceil + 1 \text{ for } \ 5 \leqslant n < 34$$

and

$$\mathrm{NI}(K_2 \times P_n) \geqslant \lceil 2\sqrt{2n+4} \rceil - 5 \text{ for } n \geqslant 34.$$

Hence the proof is completed. □

Before we prove Theorem 3.4, we need the following lemma.

Lemma 3.1. *Let $a = -1 + \lfloor\sqrt{2n+4}\rfloor$, $b = -4 + \lceil\sqrt{2n+4}\rceil$ and $n \geqslant 0$. The following inequalities hold.*
(a) $2n - ab - 4a + 1 \leqslant 3b + 7$,
(b) $2n - ab - 3a - 1 \leqslant 3b + 6$.

Proof. (a) We shall show that $2n + 6 \leqslant \lceil\sqrt{2n+4}\rceil(2 + \lfloor\sqrt{2n+4}\rfloor)$. For every $n \geqslant 0$,

$$\begin{aligned}2n + 6 \leqslant \sqrt{2n+4}(1 + \sqrt{2n+4}) &\leqslant \sqrt{2n+4}(2 + \lfloor\sqrt{2n+4}\rfloor)\\ &\leqslant \lceil\sqrt{2n+4}\rceil(2 + \lfloor\sqrt{2n+4}\rfloor).\end{aligned}$$

The proof of part (b) can be reduced to a sequence of inequalities similar to those in (a). □

Theorem 3.4. *Let $a = -1 + \lfloor\sqrt{2n+4}\rfloor$, $b = -4 + \lceil\sqrt{2n+4}\rceil$ and $n \geqslant 34$. Then*

$$\mathrm{NI}(K_2 \times P_n) = \begin{cases} a + b, & \text{if } n \leqslant \frac{1}{2}(ab + 4a + b - 2);\\ a + b + 1, & \text{if } n > \frac{1}{2}(ab + 4a + b - 2) \text{ and } n \leqslant \frac{1}{2}(ab + 4a + 2b + 2);\\ a + b + 2, & \text{otherwise.}\end{cases}$$

Proof. Let $S \subseteq V(K_2 \times P_n)$ and let b be the maximum order of the components of $(K_2 \times P_n)/S$. If we remove $|S| = a$ vertices from any copy of P_n, then we have $a + 1$ components for $n \geqslant 34$. So we consider two cases, depending on b:

Case 1: Let b be an even number. If we remove $|S| = a$ vertices in such a way that the first component has $b - 1$ vertices and each of the $a - 1$ components have b vertices from any copy of P_n, then we have $a + 1$ components as shown in Figure 1. Let $\{x_1, x_2, \ldots, x_a\}$ be a set of removed vertices from any copy of P_n. Notice that Figure 1 shows a specific situation and we can select the vertices x_i from different copies of P_n.

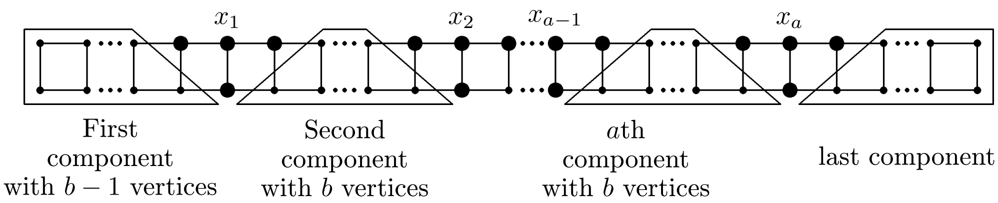

Figure 1

Our aim is to investigate whether some vertices should be deleted or not from the last component.

In this case, the last component has $2n-ab-4a+1$ vertices and $2n-ab-4a+1$ must be an odd number. By Lemma 3.1(a), we know that $2n-ab-4a+1 \leqslant 3b+7$. That is, we must remove at most two vertices from the last component. Hence we have the following three possibilities for the last component:

(a) The last component has at most $b-1$ vertices,

(b) The last component has at least b and at most $2b+3$ vertices,

(c) The last component has at least $2b+4$ and at most $3b+7$ vertices.

According to these possibilities, the neighbor-integrity of $K_2 \times P_n$ is equal to

$$(4) \qquad \begin{cases} a+b, \text{ if } n \leqslant \frac{1}{2}(ab+4a+b-2); \\ a+b+1, \text{ if } n > \frac{1}{2}(ab+4a+b-2) \text{ and } n \leqslant \frac{1}{2}(ab+4a+2b+2); \\ a+b+2, \text{ if } n > \frac{1}{2}(ab+4a+2b+2) \text{ and } n \leqslant \frac{1}{2}(ab+4a+3b+6). \end{cases}$$

C a s e 2: Let b be an odd number. If we remove $|S| = a$ vertices in such a way that the first component has b vertices and each of the $a-1$ components have $b-1$ vertices from any copy of P_n, then we have $a+1$ components as shown in Figure 2. Let $\{x_1, x_2, \ldots, x_a\}$ be a set of removed vertices from any copy of P_n. Notice that Figure 2 shows a specific situation and we can select the vertices x_i from different copies of P_n.

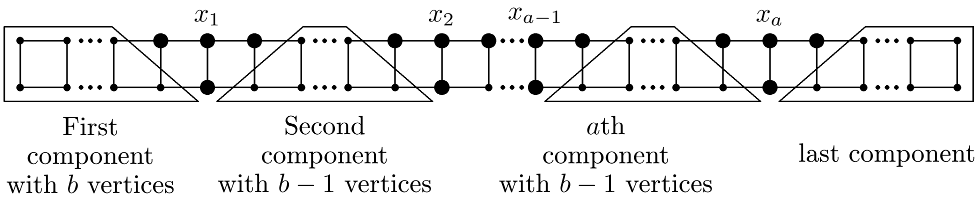

Figure 2

Our aim is to investigate whether some vertices should be deleted or not from the last component.

In this case, the last component has $2n-ab-3a-1$ vertices. By Lemma 3.1(b), we know that $2n-ab-3a-1 \leqslant 3b+6$. That is, we must remove at most two vertices from the last component. Hence we have three possibilities for the last component and so the neighbor-integrity of $K_2 \times P_n$ is equal to

$$(5) \qquad \begin{cases} a+b, \text{ if } n \leqslant \frac{1}{2}(ab+3a+b+1); \\ a+b+1, \text{ if } n > \frac{1}{2}(ab+3a+b+1) \text{ and } n \leqslant \frac{1}{2}(ab+3a+2b+4); \\ a+b+2, \text{ if } n > \frac{1}{2}(ab+3a+2b+4) \text{ and } n \leqslant \frac{1}{2}(ab+3a+3b+7). \end{cases}$$

By elementary arithmetical operations it follows from (4) and (5) that we have

$$\mathrm{NI}(K_2 \times P_n) = \begin{cases} a+b, \text{ if } n \leqslant \frac{1}{2}(ab+4a+b-2); \\ a+b+1, \text{ if } n > \frac{1}{2}(ab+4a+b-2) \text{ and } n \leqslant \frac{1}{2}(ab+4a+2b+2); \\ a+b+2, \text{ otherwise.} \end{cases}$$

□

Theorem 3.5. *Let n be a positive integer. If $5 \leqslant n < 39$, then*

$$\mathrm{NI}(K_2 \times C_n) = \begin{cases} (n/3)+2, & \textit{if } n = 3k \textit{ and } k \textit{ is odd}; \\ \lceil n/3 \rceil + 1, & \textit{otherwise.} \end{cases}$$

Moreover, if $n \geqslant 39$, then $\mathrm{NI}(K_2 \times C_n) \geqslant \lceil 2\sqrt{2n} \rceil - 4$.

P r o o f. Let $S \subseteq V(K_2 \times C_n)$ and let b be the maximum order of the components of $(K_2 \times C_n)/S$. Then we have two cases, depending on b:

C a s e 1: Let $b = 1$. If $n = 3k$ and k is odd, then we have to remove $(n/3)+1$ vertices and so $\mathrm{NI}(K_2 \times C_n) = (n/3)+2$. Otherwise we have to remove $\lceil n/3 \rceil$ vertices and so $\mathrm{NI}(K_2 \times C_n) = \lceil n/3 \rceil + 1$.

C a s e 2: Let $b \geqslant 2$. If we remove $|S| = a$ vertices, then the number of components is at most a. So

$$\mathrm{NI}(K_2 \times C_n) \geqslant \min_a \left\{ a + \frac{2n-4a}{a} \right\}$$

and $\mathrm{NI}(K_2 \times P_n) \geqslant \lceil 2\sqrt{2n+4} \rceil - 5$ if $b \geqslant 2$.

The rest of the proof is very similar to that of Theorem 3.3. □

Theorem 3.6. *Let $a = \lfloor \sqrt{2n} \rfloor$, $b = \lceil \sqrt{2n} \rceil - 4$ and $n \geqslant 39$. Then*

$$\mathrm{NI}(K_2 \times C_n) = \begin{cases} a+b-1, & \textit{if } n \leqslant \frac{1}{2}(ab+3a); \\ a+b, & \textit{if } n > \frac{1}{2}(ab+3a) \textit{ and } n \leqslant \frac{1}{2}(ab+4a); \\ a+b+1, & \textit{if } n > \frac{1}{2}(ab+4a) \textit{ and } n \leqslant \frac{1}{2}(ab+4a+b+4); \\ a+b+2, & \textit{otherwise.} \end{cases}$$

P r o o f. Let $S \subseteq V(K_2 \times C_n)$ and let b be the maximum order of the components of $(K_2 \times C_n)/S$. If we remove $|S| = a$ vertices from any copy of C_n, then the number of components is a. Now remove $|S| = a$ vertices in such a way that each of the $a-1$ components will have b vertices.

Our aim is to investigate whether some vertices should be deleted or not from the last component. Then we have two cases, depending on b:

Case 1: If b is an even number, then the last component has $2n - ab - 4a + b$ vertices.

Case 2: If b is an odd number, then the last component has $2n - ab - 3a + b - 1$ vertices.

The rest of the proof is very similar to that of Theorem 3.4. □

(d) Corona

Definition 3.6. The *corona* of two graphs G_1 (on n vertices) and G_2 is defined as the graph G obtained by taking one copy of G_1 of order n and n copies of G_2, and then joining the i'th vertex of G_1 to every vertex in the i'th copy of G_2. The corona of two graphs G_1 ad G_2 is denoted by $G_1 \circ G_2$.

Theorem 3.7. *Let G_1 and G_2 be graphs with orders m and n, respectively. Then*
(a) *If $m \leqslant n$, then* $\mathrm{NI}(G_1 \circ G_2) = m$.
(b) *If $m > n$, then* $\mathrm{NI}(G_1 \circ G_2) \geqslant n + 1$.

Proof. Let $S \subseteq V(G_1 \circ G_2)$. If $m \leqslant n$, then $S = V(G_1)$ and $c((G_1 \circ G_2)/S) = 0$. So $\mathrm{NI}(G_1 \circ G_2) = m$. Otherwise $S \subset V(G_1)$ and hence $c((G_1 \circ G_2)/S) \geqslant n$. Then $\mathrm{NI}(G_1 \circ G_2) \geqslant n + 1$.

The proof is completed. □

Definition 3.7. The *wheel* with m spokes, $W_{1,m}$, is a graph that contains an m-cycle and one additional vertex that is adjacent to all vertices of the cycle.

Theorem 3.8. *Let G be a graph of order n and $W_{1,m}$ a wheel graph. Then*

$$\mathrm{NI}(W_{1,m} \circ G) = \begin{cases} m + 1, & \text{if } m + 1 \leqslant n; \\ n + 1, & \text{otherwise.} \end{cases}$$

Proof. Let $S \subseteq V(W_{1,m})$ and u be a vertex which is adjacent to all the vertices of the m-cycle. The first part of the proof follows from Theorem 3.7. Otherwise, $S = \{u\}$ and $\mathrm{NI}(W_{1,m} \circ G) = n + 1$.

The proof is completed. □

References

[1] *Atıcı, M.; Kırlangıç, A.*: Counterexamples to the theorems of integrity of prisms and ladders. J. Comb. Math. Comb. Comput. *34* (2000), 119–127.
[2] *Bagga, K. S.; Beineke, L. W.; Goddard, W. D.; Lipman, M. J.; Pippert, R. E.*: A survey of integrity. Discrete Appl. Math. *37/38* (1992), 13–28.
[3] *Bagga, K. S.; Beineke, L. W.; Lipman, M. J.; Pippert, R. E.*: Edge-integrity: a survey. Discrete Math. *124* (1994), 3–12.
[4] *Barefoot, C. A.; Entringer, R.; Swart, H.*: Vulnerability in graphs—a comparative survey. J. Comb. Math. Comb. Comput. *1* (1987), 13–22.
[5] *Cozzens, M. B.*: Stability measures and data fusion networks. Graph Theory of New York *26* (1994), 8–14.
[6] *Cozzens, M. B.; Moazzami, D.; Stueckle, S.*: The tenacity of a graph. Graph theory, combinatorics, algorithms and applications (Alavi, Y. et al., eds.). Wiley, New York, 1995, pp. 1111–1122.
[7] *Cozzens, M. B.; Wu, S. Y.*: Edge-neighbor-integrity of trees. Australas J. Comb. *10* (1994), 163–174.
[8] *Cozzens, M. B.; Wu, S. Y.*: Vertex-neighbor-integrity of trees. Ars. Comb. *43* (1996), 169–180.
[9] *Cozzens, M. B.; Wu, S. Y.*: Bounds of edge-neighbor-integrity of graphs. Australas J. Comb. *15* (1997), 71–80.
[10] *Goddard, W.; Swart, H. C.*: On the toughness of a graph. Quaest. Math. *13* (1990), 217–232.
[11] *Gambrell, Marci J.*: Vertex-neighbor-integrity of magnifiers, expanders and hypercubes. Discrete Math. *216* (2000), 257–266.
[12] *Harary, F.*: Graph Theory. Addison-Wesley Publishing Company, 1969.
[13] *Kırlangıç, A.*: The edge-integrity of some graphs. J. Comb. Math. Comb. Comput. *37* (2001), 139–148.
[14] *Kırlangıç, A.; Ozan, A.*: The neighbour-integrity of total graphs. Int. J. Comput. Math. *76* (2000), 25–33.

Author's address: *Alpay Kırlangıç*, Department of Mathematics, Science Faculty, Ege University-35100, Bornova-Izmir, Turkey, e-mail: `kirlan@bornova.ege.edu.tr`.