Journal de Théorie des Nombres de Bordeaux **12** (2000), 1–12

An almost-sure estimate for the mean of generalized Q-multiplicative functions of modulus 1

par Jean-Loup MAUCLAIRE

Abstract. Let $Q = (Q_k)_{k \geq 0}$, $Q_0 = 1$, $Q_{k+1} = q_k Q_k$, $q_k \geq 2$, be a Cantor scale, $\mathbf{Z}_Q$ the compact projective limit group of the groups $\mathbf{Z}/Q_k\mathbf{Z}$, identified to $\prod_{0 \leq j \leq k-1} \mathbf{Z}/q_j\mathbf{Z}$, and let μ be its normalized Haar measure. To an element $x = \{a_0, a_1, a_2, ...\}$, $0 \leq a_k \leq q_{k+1} - 1$, of $\mathbf{Z}_Q$ we associate the sequence of integral valued random variables $x_k = \sum_{0 \leq j \leq k} a_j Q_j$. The main result of this article is that, given a complex Q-multiplicative function g of modulus 1, we have

$$\lim_{x_k \to x} \Big(\frac{1}{x_k} \sum_{n \leq x_k - 1} g(n) - \prod_{0 \leq j \leq k} \frac{1}{q_j} \sum_{0 \leq a < q_j} g(aQ_j) \Big) = 0 \quad \mu\text{-a.e.}$$

Jean-Loup Mauclaire
Théorie des Nombres (UMR 7586)
15 rue du Chevaleret
F–75013 Paris cedex
E-mail : mauclai@ccr.jussieu.fr

Manuscrit reçu le 14 septembre 1998.