

CLASSICALLY INTEGRABLE NON-LINEAR SIGMA MODELS AND THEIR GEOMETRIC PROPERTIES

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Abstract. General classes of non-linear sigma models originating from a specified action are developed and studied. Models can be grouped and considered within a single mathematical structure this way. The geometrical properties of these models and the theories they describe are developed in detail. The zero curvature representation of the equations of motion are found. Those representations which have a spectral parameter are of importance here. Some new models with Lax pairs which depend on a spectral parameter are found. Some particular classes of solutions are worked out and discussed.

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1. Introduction

Classical integrability refers to the possibility of determining as many conserved quantities as there are degrees of freedom present in a dynamical system. The problem of exploring the nature of dynamical systems which are integrable has been a subject of great interest recently. It may be that these conserved quantities