

# THE DOUBLY CONNECTED MINIMAL SURFACES BETWEEN TWO CIRCLES IN PARALLEL PLANES

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**Abstract.** In this article, we revisit some results published by several mathematicians regarding existence and non-existence of doubly connected minimal surfaces between two circles in parallel planes. In particular, we propose new analytical interpretations and results for catenoidal solutions formed between two coaxial circles in parallel planes of arbitrary radii. We conclude by discussing the nature of the pair of the catenoidal solutions that arise from our proposed algorithm.

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## Contents

<b>1</b>	<b>Introduction and Historical Background</b>	<b>31</b>
<b>2</b>	<b>Arbitrary Circles as Boundary of Minimal Surfaces</b>	<b>33</b>
<b>3</b>	<b>Discussion on Stability of Catenoidal Solutions</b>	<b>41</b>
<b>4</b>	<b>Conclusion</b>	<b>44</b>
	<b>References</b>	<b>44</b>

## 1. Introduction and Historical Background

In this article, we study minimal surfaces representing soap films bounded by two circles in parallel planes. They are due to surface tension leading to surface area minimization, where the gravity can be neglected. We consider the case of two coaxial circles  $C_1$  and  $C_2$  of radius  $r_1$  and  $r_2$  respectively, which lie in two parallel planes situated at a distance  $h$  from one another. If  $h$  is sufficiently small, then the soap film obtained between the two circles will be a catenoid. From the early studies of Plateau [8], it was known that there exists a critical distance  $h_0$  (between