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PARAMETRIC REALIZATION OF THE LORENTZ TRANSFORMATION GROUP IN PSEUDO-EUCLIDEAN SPACES

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Abstract. The Lorentz transformation group $\mathrm{SO}(m,n)$, $m,n \in \mathbb{N}$, is a group of Lorentz transformations of order (m,n), that is, a group of special linear transformations in a pseudo-Euclidean space $\mathbb{R}^{m,n}$ of signature (m,n) that leave the pseudo-Euclidean inner product invariant. A parametrization of $\mathrm{SO}(m,n)$ is presented, giving rise to the composition law of Lorentz transformations of order (m,n) in terms of parameter composition. The parameter composition, in turn, gives rise to a novel group-like structure that $\mathbb{R}^{m,n}$ possesses, called a bi-gyrogroup. Bi-gyrogroups form a natural generalization of gyrogroups where the latter form a natural generalization of groups. Like the abstract gyrogroup, the abstract bi-gyrogroup can play a universal computational role which extends far beyond the domain of pseudo-Euclidean spaces.

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