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A Generalization of the Quasiconvex Optimization Problem

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In this paper the quasiconvex minimization problem is included in a problem defined by sets (instead of functions). Lagrangian conditions for both problems are then studied and related. Lagrangian conditions for the standard convex minimization problem are usually defined in terms of subdifferentials. Lagrangian conditions are defined here in terms of those functionals on the set of (quasi) convex functions which satisfy certain axioms.
Constraint qualifications valid at all points in the feasible set are also considered, in connection with questions of redundancy.

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1. Introduction

Given a minimization problem with objective function f and constraints $g_i(x) \leq 0$, $i \in I$, the lagrangian-type conditions at a feasible point s have typically the form

$$\exists \xi \in \boldsymbol{d}f(s) \;\; \exists J \in \wp_f(I(s)) \;\; \exists \xi_j \in \boldsymbol{d}g_j(s) \;\; \exists \lambda_j \geq 0, \;\; j \in J \;\text{ such that }\; \xi + \sum_{j \in J} \lambda_j \xi_j = 0$$

where $\wp_f(I(s))$ is the set of finite subsets of $I(s) := \{i \in I : g_i(s) = 0\}$, and $\boldsymbol{d}$ is the subdifferential or any of its generalizations or variants. Now $\boldsymbol{d}f(s)$ ($\boldsymbol{d}g_j(s)$) is a subset of the dual space, such as to make the lagrangian condition sufficient for optimality, at least in the convex case.
In this paper the quasiconvex minimization problem is included in a problem defined by sets (instead of functions). Lagrangian conditions for the problem with functions and the problem with sets are then studied and related. It follows that constraint qualifications (regularity conditions) for the quasiconvex minimization problem can be derived from those for the problem with sets.
Lagrangian conditions for the standard convex optimization problem are usually defined in terms of subdifferentials. Lagrangian conditions are defined here in terms of those functionals on the set of (quasi) convex functions which satisfy certain axioms.

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Constraint qualifications valid at all points in the feasible set are also considered, in connection with questions of redundancy.

2. The problem with sets

Let $F[\Im]$ be a locally convex (Hausdorff) real space with the topology $\Im$, and $X \subseteq F$. We consider a non-empty family $(S_i)_{i\in I}$ of convex sets in X. We write $S := \bigcap_{i\in I} S_i$. Let Ψ be the set of all the functions $T : X \to \Im$ such that $T(x)$ is convex, and $x \in \overline{T(x)}$ when $T(x) \neq \emptyset$. If $T \in \Psi$, we consider the problem:

$$T(x) \cap S = \emptyset \quad \text{subject to} \quad x \in S_i, \quad \text{(for every) } i \in I. \tag{$P(T)$}$$

Given $x \in X$, we write $\Psi(x) := \{T \in \Psi : x \text{ solves } P(T)\}$. For $M \subseteq F$, we denote by $K(M)$ the cone generated by M. The polar of M is $M^\rho := \{p \in F' : \langle p, x\rangle \leq 0, \forall x \in M\}$, and the epigraph of the support function of M (called "discriminant" in Gutiérrez [1]) is $\boldsymbol{D}(M) := \{(p,\alpha) \in F' \times \mathbb{R} : \langle p, x\rangle \leq \alpha, \forall x \in M\}$. M^ρ is then a (weakly) closed convex cone in the (topological) dual F', and $\boldsymbol{D}(M)$ a (weakly) closed convex cone in $F' \times \mathbb{R}$. For any set C, $\wp(C)$ is the set of its subsets, and $\wp_f(C)$ the set of its finite subsets.

Definition 2.1. $(S_i)_{i\in I}$ is said to satisfy property $\tilde{N}$ iff

$$\boldsymbol{D}\left(\bigcap_{i\in I} S_i\right) = \operatorname{co}\left[\bigcup_{i\in I} \boldsymbol{D}(S_i)\right].$$

Definition 2.2. Let $x \in X$; $(S_i)_{i\in I}$ is said to satisfy property $\tilde{N}(x)$ iff

$$\left(\bigcap_{i\in I}(S_i - x)\right)^\rho = \operatorname{co}\left[\bigcup_{i\in I}(S_i - x)^\rho\right].$$

Property $\tilde{N}$ is a global property, and property $\tilde{N}(x)$ is the corresponding local property. Property $\tilde{N}$ and property $\tilde{N}(x)$ could have been defined by the inclusions $\boldsymbol{D}(S) \subseteq \operatorname{co}\left[\bigcup_{i\in I} \boldsymbol{D}(S_i)\right]$ and $(S-x)^\rho \subseteq \operatorname{co}\left[\bigcup_{i\in I}(S_i - x)^\rho\right]$, respectively, since the reverse inclusions always hold.

If the sets S_i are closed cones with vertex at x, then $(S-x)^\rho = \overline{\operatorname{co}\left[\bigcup_{i\in I}(S_i - x)^\rho\right]}$ (cf. Köthe [2]). On the other hand, if the sets S_i are closed and $S \neq \emptyset$, then $\boldsymbol{D}(S) = \overline{\operatorname{co}\left[\bigcup_{i\in I} \boldsymbol{D}(S_i)\right]}$ (cf. Gutiérrez [1]).

Let $s \in S$. If the sets S_i are closed, a sufficient condition for property $\tilde{N}(s)$ is that $\left[\bigcap_{i\in I} \operatorname{int}(S_i)\right] \neq \emptyset$ (cf. Holmes [3]). In the finite-dimensional case it is enough that $\left[\bigcap_{i\in I} \operatorname{rint}(S_i)\right] \neq \emptyset$, and it is not necessary to require that the sets S_i are closed (cf. Rockafellar [4]). In a parallel way, if in the finite-dimensional case $\left[\bigcap_{i\in I} \operatorname{rint}(S_i)\right] \neq \emptyset$, then property $\tilde{N}$ holds (cf. Gutiérrez [1]).

If V_1 and V_2 are pointed convex cones and (V_1, V_2) satisfies property N of Jameson (i.e., given any neighbourhood U of 0, there exists some neighbourhood V of 0 such that

$(V_1+V)\cap(V_2+V)\subseteq(V_1\cap V_2)+U)$, then $(V_i)_{i=1,2}$ satisfies property $\tilde{N}(0)$ (cf. the proof of Theorem 1 in Jameson [5]).

Proposition 2.3. *If property $\tilde{N}$ is satisfied, then property $\tilde{N}(s)$ holds for every $s\in S$. When S is compact, property $\tilde{N}$ holds iff property $\tilde{N}(s)$ holds for every $s\in S$.*

Proof. Suppose that property $\tilde{N}$ holds, and let $s\in S$. If $p\in(S-s)^\rho$, then $(p,p(s))\in\boldsymbol{D}(S)$, and thus there exists $J\in\wp_f(I)$ such that $(p,p(s))=\sum_{j\in J}(p_j,\alpha_j)$, with $(p_j,\alpha_j)\in\boldsymbol{D}(S_j)$. Also $p_j(s)=\alpha_j$ for every $j\in J$ (obviously $\langle p_j,s\rangle\leq\alpha_j$; if $k\in J$ exists such that $\langle p_k,s\rangle<\alpha_k$, then $\langle p,s\rangle=\sum_{j\in J}\langle p_j,s\rangle<\sum_{j\in J}\alpha_j=\langle p,s\rangle$, which is a contradiction), and then $p_j\in(S_j-s)^\rho$. Therefore $p\in\operatorname{co}\left[\bigcup_{i\in I}(S_i-s)^\rho\right]$, and property $\tilde{N}(s)$ follows. Suppose now that S is compact, and $\tilde{N}(s)$ holds for every $s\in S$. If $(p,\alpha)\in\boldsymbol{D}(S)$, let $s_0\in S$ such that $\langle p,s_0\rangle=\sup\{\langle p,s\rangle : s\in S\}$. Thus $\langle p,s_0\rangle\leq\alpha$; from $\tilde{N}(s_0)$, it is immediate that $(p,\langle p,s_0\rangle)\in\operatorname{co}\left[\bigcup_{i\in I}\boldsymbol{D}(S_i)\right]$, and $(p,\alpha)\in\operatorname{co}\left[\bigcup_{i\in I}\boldsymbol{D}(S_i)\right]$. Hence property $\tilde{N}$ is satisfied. $\square$

If $T(x)=\emptyset$, then x obviously solves $P(T)$, regardless of $(S_i)_{i\in I}$ (unconstrained solution). Let $s\in S$. Consider the following condition:

$$\left.\begin{array}{l}\text{If } T(s)\neq\emptyset\text{, then:}\\ \exists p\in[(T(s)-s)^\rho\sim\{0\}]\ \ \exists J\in\wp_f(I)\ \ \exists p_j\in(S_j-s)^\rho,\ \ j\in J\\ \qquad\qquad\text{such that }\ p+\sum_{j\in J}p_j=0.\end{array}\right\}\qquad L(T,s)$$

$L(T,s)$ is a sufficient condition for s to solve problem $P(T)$ (if $s_0\in(T(s)\cap S)$ exists, then $\langle p+\sum_{j\in J}p_j,s_0-s\rangle<0$ (since $\langle p,s_0\rangle<\langle p,s\rangle$, as $T(s)$ is open), and this contradicts $L(T,s)$). On the other hand, if $\tilde{N}(s)$ holds, then $L(T,s)$ is a necessary condition for s to solve problem $P(T)$ (if s solves $P(T)$, then (applying the separation theorem and considering that $s\in(\overline{T(s)}\cap S)$) there exists $p\in F'$, $p\neq0$, such that $p\in(T(s)-s)^\rho$ and $-p\in(S-s)^\rho$, and thus $L(T,s)$ follows from $\tilde{N}(s)$). If $\dim(F)<\infty$, the last assertion is still true if $T(x)$ is not required to belong to $\Im$.

The weakest constraint qualifications are important in the study of necessary optimality conditions. Let $s\in S$. We say that s is a regular point if $L(T,s)$ holds for every $T\in\Psi(s)$. A constraint qualification for s is a condition imposed on $(S_i)_{i\in I}$ guaranteeing that s is a regular point. A weakest constraint qualification for s (WCQ-s) is a constraint qualification for s which holds iff s is a regular point.

Proposition 2.4. *Let $s\in S$. Then property $\tilde{N}(s)$ is a WCQ-s.*

Proof. We have seen above that property $\tilde{N}(s)$ is a constraint qualification for s. Suppose now that s is a regular point. Let $q\in(S-s)^\rho$, $q\neq0$. We define $T^\star:X\to\Im$ by $T^\star(x):=x+\{z\in F:\langle q,z\rangle>0\}$. Then $T^\star\in\Psi(s)$, and $L(T^\star,s)$ is satisfied. As $(T^\star(s)-s)^\rho=\{-\lambda q:\lambda\geq0\}$, we have $q\in\operatorname{co}\left[\bigcup_{i\in I}(S_i-s)^\rho\right]$, and property $\tilde{N}(s)$ follows. $\square$

In practice, it is most relevant to know if the family $(S_i)_{i\in I}$ satisfies a constraint qualification for every point of S, as condition $L(T,s)$ is used to delimit those points of S that can solve $P(T)$. A general constraint qualification is a condition imposed on $(S_i)_{i\in I}$ which guarantees that every point of S is regular, and a weakest general constraint qualification (WGCQ) is a general constraint qualification which holds iff every point of S is regular.

Corollary 2.5. *Property $\tilde{N}$ is a general constraint qualification. If S is compact, then property $\tilde{N}$ is a WGCQ.*

We say that S_k is redundant if $S = \bigcap_{i\in I\sim\{k\}} S_i$. If the sets S_i, $i \in I$, are closed and there exists $I' \subseteq I$ such that $\boldsymbol{D}(S) \subseteq \mathrm{co}\left[\bigcup_{i\in I'} \boldsymbol{D}(S_i)\right]$ (which is a general constraint qualification), then the constraints S_k, $k \in (I \sim I')$, are redundant (as $\boldsymbol{D}(S) = \boldsymbol{D}(\bigcap_{i\in I'} S_i)$, and cf. §20.7 in Köthe [2]). If S is non-empty and compact (alternatively to S compact: S closed and with non-empty interior, or S closed and $\dim(F) < \infty$), then the hypothesis that $\boldsymbol{D}(S) \subseteq \mathrm{co}\left[\bigcup_{i\in I'} \boldsymbol{D}(S_i)\right]$ can obviously be replaced by the following: $(S-s)^\rho \subseteq \mathrm{co}\left[\bigcup_{i\in I'} (S_i - s)^\rho\right]$ for every $s \in S$.

3. The problem with functions

In this section we suppose that X is convex and open. We consider a non-empty family $(g_i)_{i\in I}$ of quasiconvex functions $g_i : X \to \mathbb{R}$. A function $f : X \to \mathbb{R}$ is semistrictly quasiconvex iff

$$f(x) < f(z) \quad \text{implies that} \quad f(\lambda x + (1-\lambda)z) < f(z)$$

for $x, z \in X$ and $0 < \lambda < 1$. Convexity and strict quasiconvexity imply semistrict quasiconvexity (v. Avriel et al. [6]). Let Ψ' be the set of all the continuous semistrictly quasiconvex functions $f : X \to \mathbb{R}$. We shall refer in the sequel to "the convex case" when the functions g_i, $i \in I$, are convex and we take Ψ' as the set of all the continuous convex functions $f : X \to \mathbb{R}$. If $f \in \Psi'$, we consider the problem:

$$\operatorname{Min} f(x) \quad \text{subject to} \quad g_i(x) \le 0, \;\; i \in I. \qquad P'(f)$$

Given $x \in X$, we write $\Psi'(x) := \{f \in \Psi' : x \text{ solves } P'(f)\}$. For any function $h : X \to \mathbb{R}$, let $\pounds h(x) := \{z \in X : h(z) \le h(x)\}$, $\pounds^< h(x) := \{z \in X : h(z) < h(x)\}$ and $\pounds_0 h := \{z \in X : h(z) \le 0\}$. We take $S_i \equiv \pounds_0 g_i$, $i \in I$, and $T(x) \equiv \pounds^< f(x)$, $x \in X$. Then $P'(f)$ is a particular case of $P(T)$ ($f \in \Psi'$ entails that $x \in \overline{T(x)}$ when $T(x) \ne \emptyset$). Thus we can apply to $P'(f)$ the results obtained in the previous section.
The subdifferential ∂ is often used in lagrangian-type conditions in convex optimization, but other alternatives are possible. Let Δ (Δ_c) be the set of all the functionals $\boldsymbol{d}$ which to every quasiconvex (convex) function $h : X \to \mathbb{R}$ make correspond a function $\boldsymbol{d}h : X \to \wp(F')$ such that:
(i) $\boldsymbol{d}h(x) \subseteq (\pounds h(x) - x)^\rho, \quad \forall x \in X$
(ii) $\pounds^< h(x) = \emptyset \Leftrightarrow 0 \in \boldsymbol{d}h(x), \quad \forall x \in X$
(iii) $\pounds^< h(x) \ne \emptyset \Rightarrow K(\boldsymbol{d}h(x)) = (\pounds h(x) - x)^\rho \sim \{0\}, \quad \forall x \in X.$
By well known properties of the subdifferential, $\partial \in \Delta_c$.

The infradifferential δ is introduced in Gutiérrez [7].

Definition 3.1. The infradifferential $\delta h(x)$ of h at $x \in X$ is the set of those $\xi \in F'$ such that, for any $z \in X$ with $h(z) \leq h(x)$,

$$h(z) \geq h(x) + \langle \xi, z - x \rangle.$$

In [7] it is shown, in the convex case, that $\delta \in \Delta_c$ and that, if h does not reach its minimum in x, $K(\delta h(x)) = K(\partial h(x))$. The concept of infradifferential is closely related to that of lower subdifferential, independently introduced in Plastria [8] (v. also Penot and Volle [9]).
Let $\boldsymbol{d} \in \Delta$ (or $\boldsymbol{d} \in \Delta_c$ in the convex case) and $s \in S$. We write $I(s) := \{i \in I : g_i(s) = 0\}$. Consider the following condition:

$$\left.\begin{array}{c} \exists \xi \in \boldsymbol{d}f(s) \;\; \exists J \in \wp_f(I(s)) \;\; \exists \xi_j \in \boldsymbol{d}g_j(s) \;\; \exists \lambda_j \geq 0, \quad j \in J \\ \text{such that } \; \xi + \displaystyle\sum_{j \in J} \lambda_j \xi_j = 0. \end{array}\right\} \qquad L'(\boldsymbol{d}, f, s)$$

Obviously $L'(\boldsymbol{d}, f, s)$ implies $L(T, s)$, and thus $L'(\boldsymbol{d}, f, s)$ is a sufficient condition for s to solve problem $P'(f)$. Nevertheless, $L(T, s)$ not necessarily implies $L'(\boldsymbol{d}, f, s)$.
Using $L'(\boldsymbol{d}, f, s)$ instead of $L(T, s)$, the concepts of regular point, constraint qualification for s, weakest constraint qualification for s (WCQ-s), general constraint qualification, and weakest general constraint qualification (WGCQ), are defined similarly as in Section 1 (and the remark about the relevance of general constraint qualifications and WGCQ also applies here). It is obvious that any constraint qualification for s implies property $\tilde{N}(s)$.
Consider now the condition:

$$\pounds^{<} g_i(s) = \emptyset \;\Rightarrow\; K(\boldsymbol{d}g_i(s)) = (\pounds g_i(s) - s)^{\rho}, \quad \forall i \in I(s). \qquad W_0(\boldsymbol{d}, s)$$

If $W_0(\boldsymbol{d}, s)$ holds, then $L(T, s)$ implies $L'(\boldsymbol{d}, f, s)$. Therefore, property $\tilde{N}(s)$ and $W_0(\boldsymbol{d}, s)$ together form a constraint qualification for s. However, it is not a WCQ-s. Consider by contrast the following condition, introduced (with subdifferentials) in Wolkowicz [10]:

$$(S - s)^{\rho} \sim \{0\} = \operatorname{co}\left[\bigcup_{i \in I(s)} K(\boldsymbol{d}g_i(s))\right] \qquad W(\boldsymbol{d}, s)$$

(or the equivalent, $(S - s)^{\rho} \sim \{0\} \subseteq \operatorname{co}\left[\bigcup_{i \in I(s)} K(\boldsymbol{d}g_i(s))\right]$, for the reverse inclusion always holds). The next proposition was established (in the convex case and for subdifferentials) in Wolkowicz [10].

Proposition 3.2. *Let $\boldsymbol{d} \in \Delta$ (or $\boldsymbol{d} \in \Delta_c$ in the convex case) and $s \in S$. Then $W(\boldsymbol{d}, s)$ is a WCQ-s.*

Proof. It runs parallel to that of Proposition 2.4 (now taking into account (i), (ii) and (iii)). $\square$

In general, constraint qualifications fulfil two aims:

(a) Decompose any element of $(S-s)^\rho$ into a finite sum of elements of the polars $(S_i-s)^\rho$.
(b) Express every said element of $(S_i-s)^\rho$ as the product of an element of $\boldsymbol{d}g_i(s)$ and a non-negative scalar.

In the first constraint qualification for s that we have considered, property $\tilde{N}(s)$ corresponds to (a), and $W_0(\boldsymbol{d}, s)$ to (b). Nevertheless, this constraint qualification can be improved: if there exists certain decomposition of an element of $(S-s)^\rho$ in a finite sum of elements of the polars $(S_i-s)^\rho$, and some of these do not belong to $K(\boldsymbol{d}g_i(s))$, then there may exist still another decomposition in which all the corresponding elements of the polars $(S_i-s)^\rho$ belong to $K(\boldsymbol{d}g_i(s))$. With regard to the decomposition of an element of $(S-s)^\rho$, the only useful part of $(S_i-s)^\rho$ is $K(\boldsymbol{d}g_i(s))$.
If $\boldsymbol{d} \in \Delta$ (or $\boldsymbol{d} \in \Delta_c$ in the convex case) has special properties, then (b) may be obviated or facilitated. For example, the infradifferential δh of a convex function $h : X \to \mathbb{R}$ satisfies: $\pounds^{<}h(x) = \emptyset \Rightarrow K(\delta h(x)) = (\pounds h(x) - x)^\rho$ for every $x \in X$ (cf. Gutiérrez [7]); thus (in the convex case) $W_0(\delta, s)$ always holds, and property $\tilde{N}(s)$ is a WCQ-s.
Suppose that S is non-empty and compact (alternatively to S compact: S closed and with non-empty interior, or S closed and $\dim(F) < \infty$). Let $I_0 := \{i \in I/\boldsymbol{d}g_i(s) \subseteq \{0\}, \forall s \in S\}$, and be $I_0 \neq I$. If a general constraint qualification holds, then the constraints $g_k(x) \leq 0$, $k \in I_0$, are redundant (v. Proposition 3.2, and Section 2 *in fine*). Consider that $\boldsymbol{d}$ is the subdifferential ∂, in the convex case, and that the functions g_i, $i \in I$, are continuous (thus their subdifferential is not empty, and then: $i \in I_0$ iff g_i is constant in S and takes its minimum in S) and Gâteaux-differentiable; it is immediate that all the points of S are regular (i.e., a general constraint qualification holds) iff the constraints $g_k(x) \leq 0$, $k \in I_0$, are redundant and the system formed by the rest of the constraints satisfies property $\tilde{N}(s)$ for every $s \in S$.
Let $s \in S$. If s is not regular in the system $g_i(x) \leq 0$, $i \in I$, then we can try to find an equivalent quasiconvex system where s is regular. An s-regularization (regularization) of the system $g_i(x) \leq 0$, $i \in I$, is a system $\hat{g}_i(x) \leq 0$, $i \in I_r$, where s is regular (where every point of S is regular), with $\hat{g}_i : X \to \mathbb{R}$ quasiconvex and $\bigcap_{i \in I_r}\{x \in X/\hat{g}_i(x) \leq 0\} = S$. If $I_r = I$ and $\{x \in X/\hat{g}_i(x) \leq 0\} = S_i$ for every $i \in I$, then the s-regularization (regularization) is called simple. A necessary condition for $g_i(x) \leq 0$, $i \in I$, to admit a simple s-regularization is that property $\tilde{N}(s)$ holds; the converse is not true in general. However, there is a partial converse.

Proposition 3.3. *In the convex case, let* $\boldsymbol{d}$ *be the subdifferential* ∂*. If* F *is normable and the sets* S_i*,* $i \in I$*, are closed, then* $g_i(x) \leq 0$*,* $i \in I$*, admits a simple convex* s*-regularization (regularization) iff it satisfies property* $\tilde{N}(s)$ *(satisfies property* $\tilde{N}(s)$ *for every* $s \in S$*).*

Proof. Suppose that $g_i(x) \leq 0$, $i \in I$, satisfies property $\tilde{N}(s)$. We define $\hat{g}_i : X \to \mathbb{R}$ by $\hat{g}_i(x) := \mu(S_i, x)$, where μ is the distance given by the norm; $\hat{g}_i$ is convex and $S_i = \{x \in X/\hat{g}_i(x) \leq 0\}$. If $p \in (S_i - s)^\rho \sim \{0\}$, then $\hat{g}_i(x) \geq \|p\|^{-1}\langle p, x-s\rangle$ for every $x \in X$ (this is trivial if $\langle p, x\rangle \leq \langle p, s\rangle$; if $\langle p, x\rangle > \langle p, s\rangle$, then $\hat{g}_i(x) \geq \mu(\{z \in F/\langle p, z\rangle \leq \langle p, s\rangle\}, x) = \mu(\{z \in F/\langle p, z\rangle = \langle p, s\rangle\}, x) = \mu(\mathrm{Ker}(p), x-s)$, and consider now that $|\langle p, z\rangle| = \|p\|\mu(\mathrm{Ker}(p), z)$ for $z \in F$ (cf., e.g., Jameson [11])), and thus $p \in K(\partial\hat{g}_i(s))$. Hence $W_0(\partial, s)$ holds in the system $\hat{g}_i(x) \leq 0$, $i \in I$, and s is regular in it). □

In this statement, the hypothesis that F is normable can be replaced by that $\mathrm{int}(S_i) \neq \emptyset$ for every $i \in I$ (we define now $\hat{g}_i(x) := \pi(x - s_i) - 1$, where $s_i \in \mathrm{int}(S_i)$ and π is the Minkowsky functional of $(S_i - s_i)$). Therefore, in the convex case and for the subdifferential ∂, if the sets S_i, $i \in I$, are closed and $\left[\bigcap_{i \in I} \mathrm{int}(S_i)\right] \neq \emptyset$, then $g_i(x) \leq 0$, $i \in I$, admits a simple convex regularization.

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