Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 165032, 8 pages
doi:10.1155/2011/165032

Research Article

The Iterative Solution for Electromagnetic Field Coupling to Buried Wires

Xufeng Zhang, Jianshu Luo, and Zipeng Zhao

College of Science, National University of Defense Technology, Changsha 410073, China

Correspondence should be addressed to Xufeng Zhang, zxf12023@163.com

Received 9 January 2011; Revised 12 March 2011; Accepted 1 April 2011

Academic Editor: Ben T. Nohara

By integrating the electric field integral equation and the transmission line equation, an iterative solution for the electromagnetic field coupling to buried wires is obtained. At first we establish the integral equation which the difference between solutions of the integral equation and the telegraph equation satisfies. Then the solution of the telegraph equation is used to approximate the solution of this integral equation. Every following step of iteration is an improvement on the transmission line solution, and with several iterations, a well approximation to the solution of electric field integral equation can be obtained.

1. Introduction

The electromagnetic field coupling to buried wires plays an important role in many engineering applications, such as geophysical probes, power or communications cables, and grounding systems (see [1–7]). The integral equation approach or transmission line (TL) model (see [1–6]) can be used to resolve this problem. Actually the transmission line model is an approximation of the integral equation approach. This method is very efficient for infinite or at least very long buried wires, but if the line is of finite length, this method cannot give an accurate solution (see [1–3]). So if the buried wires of the finite length are of interest, the integral equation approach has to be used. However, the integral equation usually should be resolved by numerical methods, such as method of moment (MoM) (see [1, 8]) and boundary element method (BEM) (see [2–6]), which may cost much computation time for long lines.

This paper presents an iterative solution for the electromagnetic field coupling to buried wires. At first we establish the integral equation which the difference between solutions of the electric field integral equation and the telegraph equation satisfies. Then the solution of the telegraph equation is used to approximate the solution of this integral equation. By this way, the iterative solution is obtained. Taking the solution of the telegraph equation as initial value, a satisfying result can be obtained just after several iterations.

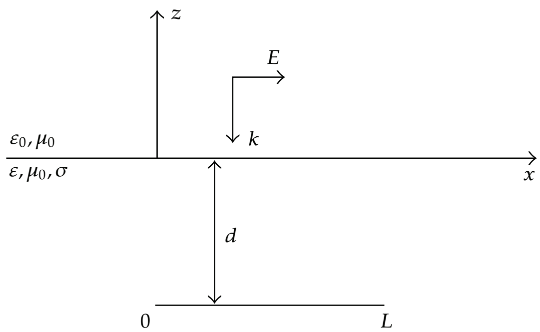


Figure 1: Thin wire buried in a lossy ground.

Finally, several numerical examples are presented, which indicates that the results obtained only after several steps iteration are in good agreement with the results obtained by MoM.

Actually, the iterative method is a semianalytic method, which requires only one integration along the line length for every space point in every step of iteration; so compared with numerical methods, such as MoM or BEM, this method is easier to implement numerical computation.

2. Integral Equation and Telegraph Equation of Buried Wires

Figure 1 illustrates a finite wire of length L and radius a, buried in a lossy ground at depth d, illuminated by a plane-wave from above ground. The upper half space is free-space, the lower half space is lossy ground, and the interface is at $z = 0$.

2.1. Electric Field Integral Equation

Taking into account the influence of interface via the plane-wave Fresnel reflection coefficient, we obtained the following integral equation for the unknown current distribution induced along the buried wire:

$$E_x^{\text{exc}} = -\frac{1}{j\omega\varepsilon}\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \gamma^2\right)\int_0^L I(x')g(x,x')\mathrm{d}x', \tag{2.1}$$

where ω is angular frequency of incidence plane-wave, and j is imaginary unit. ε is the complex permittivity of the lossy ground:

$$\varepsilon = \varepsilon_r\varepsilon_0 - j\frac{\sigma}{\omega}. \tag{2.2}$$

ε_r and σ are relative permittivity and conductivity of the ground. According to the image theory, the Green function $g(x,x')$ is given in [9] as

$$g(x,x') = \frac{e^{-\gamma R_1}}{4\pi R_1} - \Gamma_{\text{ref}}\frac{e^{-\gamma R_2}}{4\pi R_2}, \tag{2.3}$$

where γ is propagation constant of the ground:

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}, \tag{2.4}$$

and R_1, R_2 are given by

$$R_1 = \sqrt{(x - x')^2 + a^2} \qquad R_2 = \sqrt{(x - x')^2 + 4d^2}. \tag{2.5}$$

Γ_{ref} is Fresnel reflection coefficient (see [3]):

$$\Gamma_{\text{ref}} = \frac{1/n_0 \cos\theta - \sqrt{1/n_0 - \sin^2\theta}}{1/n_0 \cos\theta + \sqrt{1/n_0 - \sin^2\theta}},$$
$$\theta = \arctan\frac{|x - x'|}{2d}, \quad n_0 = \frac{\varepsilon}{\varepsilon_0}. \tag{2.6}$$

The excitation electric field on the wire for normal incidence plane-wave can be expressed as follows (see [7]):

$$E_x^{\text{exc}} = E_0 \Gamma_{\text{TM}} e^{-\gamma d}, \tag{2.7}$$

where Γ_{TM} denotes Fresnel transmission coefficient at the interface (see [7]):

$$\Gamma_{\text{TM}} = \frac{2}{1 + \sqrt{n_0}}. \tag{2.8}$$

2.2. Transmission Line Telegraph Equation

The transmission line telegraph equation of electromagnetic field coupling to buried wire can be expressed as follows:

$$\frac{\mathrm{d}V(x)}{\mathrm{d}x} + ZI(x) = E_x^{\text{exc}},$$
$$\frac{\mathrm{d}I(x)}{\mathrm{d}x} + YV(x) = 0, \tag{2.9}$$

where $V(x)$ and $I(x)$ are line voltage and current, and Z and Y are impedance and admittance of the ground, respectively (see [10]),

$$Z = \frac{j\omega\mu_0}{2\pi}\left[\ln\left(\frac{1 + \gamma a}{\gamma a}\right) + \frac{2e^{-2d|\gamma|}}{4 + \gamma^2 a^2}\right],$$
$$Y = \frac{\gamma^2}{Z}. \tag{2.10}$$

Eliminating $V(x)$ from (2.9), we have

$$\frac{d^2 I(x)}{dx^2} - \gamma^2 I(x) + \Upsilon E_x^{exc} = 0. \tag{2.11}$$

Actually the solution of (2.11) is an approximation of (2.1). The transmission line telegraph equation can be resolved analytically (see [1]).

3. The Iterative Solution of Integral Equation

A simple iterative approach to correct the results obtained using TL approximation for an overhead line of finite length is proposed in [11, 12], where two additional source terms representing the correction to the TL approximation are added to the classical transmission line telegrapher's equations. Resorting to the integral equation and transmission line telegraph equation, we will present a similar approach to obtain the iterative solution of (2.1).

Suggesting that $I(x)$ and $I_0(x)$ are solutions of (2.1) and (2.11), and $I'(x) = I(x) - I_0(x)$, the integral equation which $I'(x)$ satisfies can be derived from (2.1) and (2.11):

$$\frac{\Upsilon}{j\omega\varepsilon}\left(\frac{d^2}{dx^2} - \gamma^2\right)\int_0^L I'(x')g(x,x')dx' = \left(\frac{d^2}{dx^2} - \gamma^2\right)\left[I_0(x) - \frac{\Upsilon}{j\omega\varepsilon}\int_0^L I_0(x')g(x,x')dx'\right]. \tag{3.1}$$

We can get an approximate solution of this equation as before:

$$\left(\frac{d^2}{dx^2} - \gamma^2\right)I_1(x) = \left(\frac{d^2}{dx^2} - \gamma^2\right)\left[I_0(x) - \frac{\Upsilon}{j\omega\varepsilon}\int_0^L I_0(x')g(x,x')dx'\right]. \tag{3.2}$$

Repeating this process, an iterative solution can be established:

$$I(x) = I_0(x) + I_1(x) + \cdots, \tag{3.3}$$

$$\left(\frac{d^2}{dx^2} - \gamma^2\right)I_n(x) = \left(\frac{d^2}{dx^2} - \gamma^2\right)\left(I_{n-1}(x) - \frac{\Upsilon}{j\omega\varepsilon}\int_0^L I_{n-1}(x')g(x,x')dx'\right). \tag{3.4}$$

Denoting

$$F(x) = I_{n-1}(x) - \frac{\Upsilon}{j\omega\varepsilon}\int_0^L I_{n-1}(x')g(x,x')dx', \tag{3.5}$$

(3.4) is equivalent to

$$\left(\frac{d^2}{dx^2} - \gamma^2\right)(I_n(x) - F(x)) = 0. \tag{3.6}$$

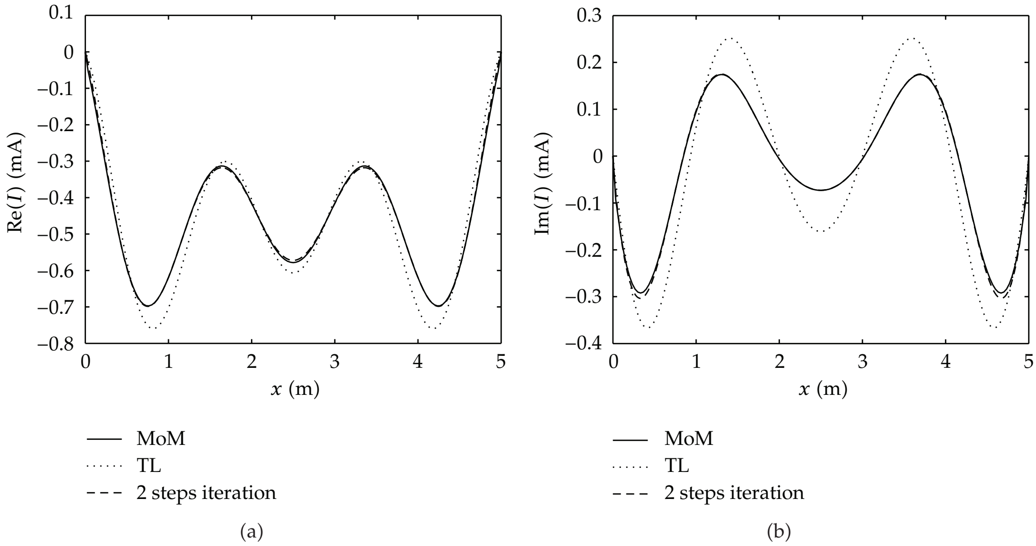


Figure 2: Induced current distribution along the thin wire ($L = 5$ m). (a) Real part. (b) Image part.

The solution of this equation with boundary condition $I_n(0) = I_n(L) = 0$ is given by [11]

$$I_n(x) = \frac{F(0)e^{-2\gamma L} - F(L)e^{-\gamma L}}{1 - e^{-2\gamma L}} e^{\gamma x} + \frac{F(L)e^{-\gamma L} - F(0)}{1 - e^{-2\gamma L}} e^{-\gamma x} + F(x). \tag{3.7}$$

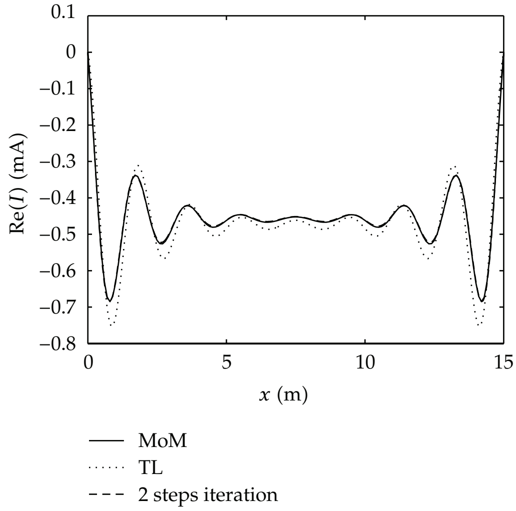
The convergence of the iterative solution is discussed in [13]. Since the solution of (2.11) is an approximation of (2.1), an excellent approximation to the solution of (2.1) can be obtained only after several iterations.

4. Numerical Results

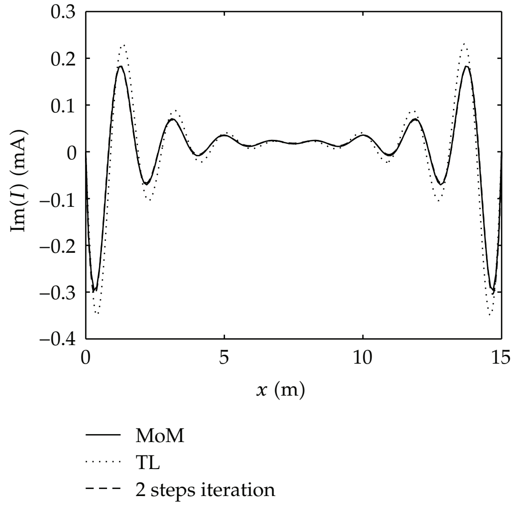
The parameters of our example are as follows: conductor radius $a = 1$ cm, depth $d = 2.5$ m, the excitation field is a normally incident plane wave with a frequency of 50 M and magnitude $E_0 = 1$ V/m, the relative dielectric constant of the ground is $\varepsilon_r = 10$, and the ground conductivity is $\sigma = 0.01$ S/m.

Figures 2, 3, and 4 show the real and imaginary parts of the current distribution induced along a wire with different lengths $L = 5$ m, $L = 15$ m, and $L = 25$ m. The results obtained by MoM, TL, and two steps iteration are given in these figures. Evidently the results obtained using the TL approximation are sufficient for longer wire, but it fails to give an accurate current distribution for relatively shorter wire lengths. However the results obtained after two-steps iteration are in good agreement with the results obtained by MoM.

Keeping other parameters as forenamed, Figure 5 shows the influence of the buried depths to the induced current distribution for $L = 5$ m; Figure 6 shows the influence of the ground conductivities to the induced current distribution for $L = 5$ m and $d = 0.5$ m.

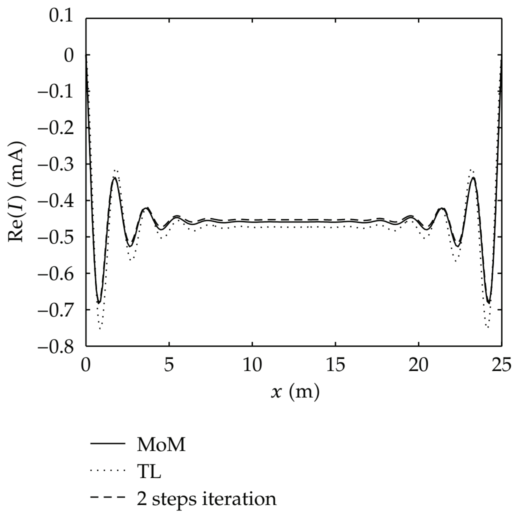


(a)

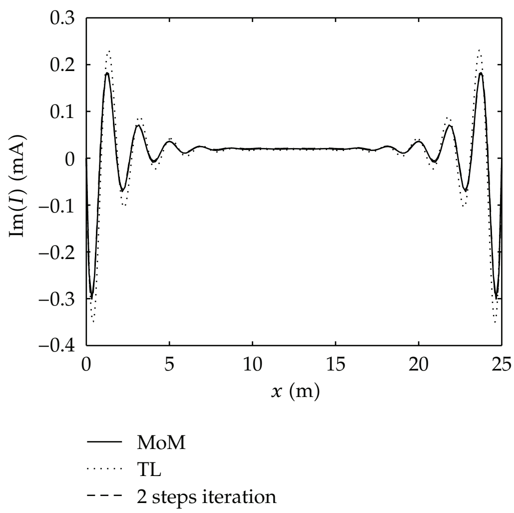


(b)

Figure 3: Induced current distribution along the thin wire ($L = 15$ m). (a) Real part. (b) Image part.



(a)



(b)

Figure 4: Induced current distribution along the thin wire ($L = 25$ m). (a) Real part. (b) Image part.

Obviously, induced current distribution magnitude decreases as the conductivity or buried depth increases, which is due to the increasing of the loss.

5. Conclusion

The problem of electromagnetic coupling on the buried thin wires is analyzed with the iterative method; the solution of the telegraph equation is used to approximate the solution

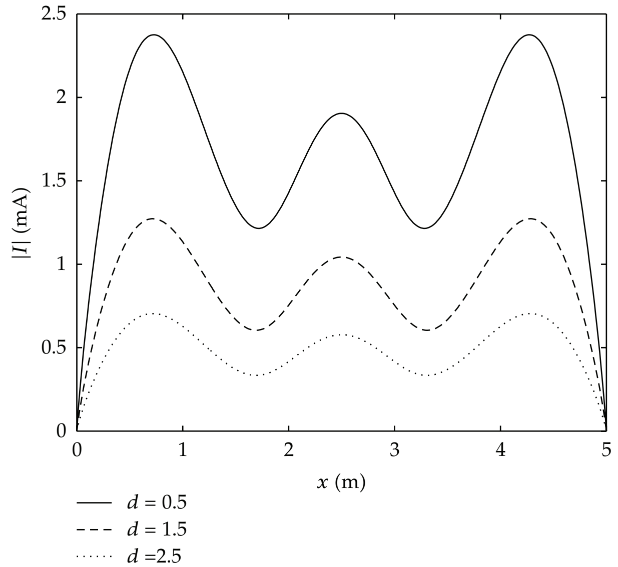


Figure 5: Induced current distribution magnitude along the thin wire for different depths.

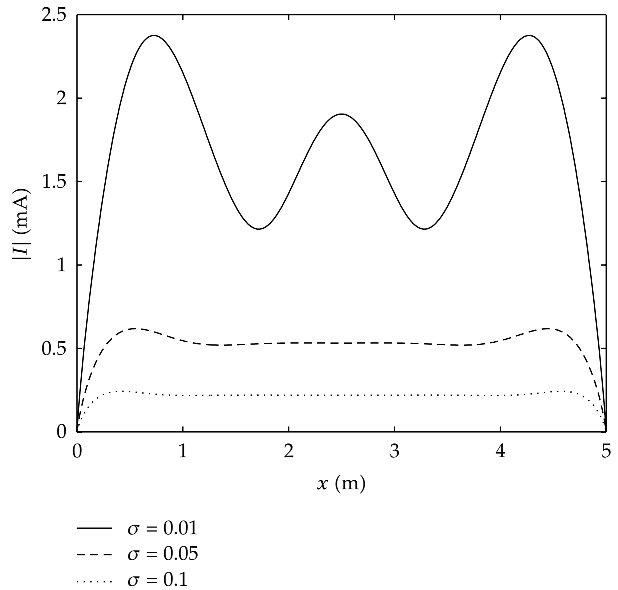


Figure 6: Induced current distribution magnitude along the thin wire for different conductivities.

of the integral equation in every step of iteration. And we can usually get satisfying result by just several iterations while taking the solution of the telegraph equation as initial value. This method may be applied to the multiple buried thin wires.

Acknowledgments

The authors would like to express their thanks to the reviewers for their valuable comments and suggestions. This work was supported by the National Natural Science Foundation of China (no. 10871231).

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