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Research Article

Traveling Wave Solutions of the Nonlinear (3+1)-Dimensional Kadomtsev-Petviashvili Equation Using the Two Variables (G'/G, 1/G)-Expansion Method

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The two variables (G'/G, 1/G)-expansion method is proposed in this paper to construct new exact traveling wave solutions with parameters of the nonlinear (3 + 1)-dimensional Kadomtsev-Petviashvili equation. This method can be considered as an extension of the basic (G'/G)-expansion method obtained recently by Wang et al. When the parameters are replaced by special values, the well-known solitary wave solutions and the trigonometric periodic solutions of this equationwere rediscovered from the traveling waves.

1. Introduction

In the recent years, investigations of exact solutions to nonlinear PDEs play an important role in the study of nonlinear physical phenomena. Many powerful methods have been presented, such as the inverse scattering transform method [1], the Hirota method [2], the truncated Painlevé expansion method [3–6], the Backlund transform method [7, 8], the exp-function method [9–14], the tanh function method [15–18], the Jacobi elliptic function expansion method [19–21], the original (G'/G)-expansion method [22–33], the two variables (G'/G,1/G)-expansion method [34, 35], and the first integral method [36]. The key idea of the original (G'/G)-expansion method is that the exact solutions of nonlinear PDEs can be expressed by a polynomial in one variable (G'/G) in which $G = G(\xi)$ satisfies the second ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where λ and μ are constants. In this paper, we will use the two variables (G'/G,1/G)-expansion method, which can be considered as an extension of the original (G'/G)-expansion method. The key idea of the two variables (G'/G,1/G)-expansion method is that the exact traveling wave solutions of

nonlinear PDEs can be expressed by a polynomial in the two variables (G'/G) and (1/G), in which $G = G(\xi)$ satisfies a second order linear ODE, namely, $G''(\xi) + \lambda G(\xi) = \mu$, where λ and μ are constants. The degree of this polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms in the given nonlinear PDEs, while the coefficients of this polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method. According to Aslan [29], the two variables (G'/G, 1/G)-expansion method becomes the basic (G'/G)-expansion method if $\mu = 0$ in (2.1) and $b_i = 0$ in (2.12). Recently,Li et al. [34] have applied the two variables (G'/G, 1/G)-expansion method and determined the exact solutions of the Zakharov equations, while Zayed andabdelaziz [35] have applied this method to determine the exact solutions of the nonlinear KdV-mKdV equation.

The objective of this paper is to apply the two variables (G'/G, 1/G)-expansion method to find the exact traveling wave solutions of the following nonlinear (3+1)-dimensional Kadomtsev-Petviashvili equation:

$$u_{xt} + 6(u_x)^2 + 6uu_{xx} - u_{xxx} - u_{yy} - u_{zz} = 0. ag{1.1}$$

This equation describes the dynamics of solitons and nonlinear wave in plasma and superfluids. Recently, Zayed [24] has found the exact solutions of (1.1) using the original (G'/G)-expansion method, while Aslan [14] has discussed (1.1) using the exp-function method. Comparison between our results and that obtained in [14, 24] will be discussed later. The rest of this paper is organized as follows. In Section 2, the description of the two variables (G'/G, 1/G)-expansion method is given. In Section 3, we apply this method to (1.1). In Section 4, conclusions are obtained.

2. Description of the Two Variables (G'/G, 1/G)-Expansion Method

Before we describe the main steps of this method, we need the following remarks (see [34, 35]).

Remark 2.1. If we consider the second order linear ODE

$$G''(\xi) + \lambda G(\xi) = \mu, \tag{2.1}$$

and set $\phi = G'/G$ and $\psi = 1/G$, then we get

$$\phi' = -\phi^2 + \mu \psi - \lambda, \qquad \psi' = -\phi \psi. \tag{2.2}$$

Remark 2.2. If λ < 0, then the general solutions of (2.1) is

$$G(\xi) = A_1 \sinh(\xi \sqrt{-\lambda}) + A_2 \cosh(\xi \sqrt{-\lambda}) + \frac{\mu}{\lambda}, \tag{2.3}$$

where A_1 and A_2 are arbitrary constants. Consequently, we have

$$\psi^2 = \frac{-\lambda}{\lambda^2 \sigma + \mu^2} \Big(\phi^2 - 2\mu \psi + \lambda \Big), \tag{2.4}$$

where $\sigma = A_1^2 - A_2^2$.

Remark 2.3. If $\lambda > 0$, then the general solutions of (2.1) is

$$G(\xi) = A_1 \sin(\xi \sqrt{\lambda}) + A_2 \cos(\xi \sqrt{\lambda}) + \frac{\mu}{\lambda}, \tag{2.5}$$

and hence

$$\psi^2 = \frac{-\lambda}{\lambda^2 \sigma - \mu^2} \Big(\phi^2 - 2\mu \psi + \lambda \Big). \tag{2.6}$$

where $\sigma = A_1^2 + A_2^2$.

Remark 2.4. If $\lambda = 0$, then the general solutions of (2.1) is

$$G(\xi) = \frac{\mu}{2}\xi^2 + A_1\xi + A_2,\tag{2.7}$$

and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2} (\phi^2 - 2\mu\psi),\tag{2.8}$$

Suppose we have the following NLPDEs in the form:

$$F(u, u_t, u_x, u_{xx}, u_{tt}, ...) = 0,$$
 (2.9)

where F is a polynomial in u and its partial derivatives. In the following, we give the main steps of the two variables (G'/G, 1/G)-expansion method [34, 35].

Step 1. The traveling wave variable

$$u(x,t) = u(\xi), \quad \xi = x - Vt$$
 (2.10)

reduces (2.9) to an ODE in the form

$$P(u, u', u'', ...) = 0,$$
 (2.11)

where *V* is a constant and *P* is a polynomial in *u* and its total derivatives, while $\{\ \}' = d/d\xi$.

Step 2. Suppose that the solutions of (2.11) can be expressed by a polynomial in the two variables ϕ and ψ as follows:

$$u(\xi) = \sum_{i=0}^{i=N} a_i \phi^i + \sum_{i=1}^{i=N} b_i \phi^{i-1} \psi, \tag{2.12}$$

where a_i (i = 0, 1, ..., N) and b_i (i = 1, ..., N) are constants to be determined later.

Step 3. Determine the positive integer N in (2.12) by using the homogeneous balance between the highest order derivatives and the nonlinear terms in (2.11).

Step 4. Substitute (2.12) into (2.11) along with (2.2) and (2.4), the left-hand side of (2.11) can be converted into a polynomial in ϕ and ψ , in which the degree of ψ is not longer than 1. Equating each coefficients of this polynomial to zero yields a system of algebraic equations which can be solved by using the Maple or Mathematica to get the values of a_i , b_i , V, μ , A_1 , A_2 , and λ where λ < 0. Thus, we get the exact solutions in terms of the hyperbolic functions.

Step 5. Similar to Step 4, substitute (2.12) into (2.11) along with (2.2) and (2.6) for $\lambda > 0$ (or (2.2) and (2.8) for $\lambda = 0$), we obtain the exact solutions of (2.11) expressed by trigonometric functions (or by rational functions), respectively.

3. An Application

In this section, we apply the method described in Section 2, to find the exact traveling wave solutions of the nonlinear (3+1)-dimensional Kadomtsev-Petviashvili equation (1.1). To this end, we see that the traveling wave variables $\xi = x + y + z - Vt$ reduce (1.1) to the following ODE:

$$-(2+V)u'' + 6(u')^{2} + 6uu'' - u'''' = 0.$$
(3.1)

Balancing u'''' with uu'' in (3.1) we get N=2. Consequently, we get

$$u(\xi) = a_0 + a_1 \phi(\xi) + a_2 \phi^2(\xi) + b_1 \psi(\xi) + b_2 \phi(\xi) \psi(\xi), \tag{3.2}$$

where a_0 , a_1 , a_2 , b_1 , and b_2 are constants to be determined later. There are three cases to be discussed as follows.

Case 1. Hyperbolic function solutions (λ < 0).

If λ < 0, substituting (3.2) into (3.1) and using (2.2) and (2.4), the left-hand side of (3.1) becomes a polynomial in ϕ and ψ . Setting the coefficients of this polynomial to zero yields

a system of algebraic equations in a_0 , a_1 , a_2 , b_1 , b_2 , μ , σ , and λ which can be solved by using the Maple or Mathematica to find the following results:

$$a_0 = a_0,$$
 $a_1 = 0,$ $a_2 = 1,$ $b_1 = -\mu,$ $b_2 = \pm \sqrt{\frac{-(\lambda^2 \sigma + \mu^2)}{\lambda}},$ $V = 6a_0 - 5\lambda - 2.$ (3.3)

From (2.3), (3.2), and (3.3), we deduce the traveling wave solution of (1.1) as follows:

$$u(\xi) = a_0 - \frac{\mu}{A_1 \sinh(\xi\sqrt{-\lambda}) + A_2 \cosh(\xi\sqrt{-\lambda}) + (\mu/\lambda)}$$

$$- \frac{A_1 \cosh(\xi\sqrt{-\lambda}) + A_2 \sinh(\xi\sqrt{-\lambda})}{(A_1 \sinh(\xi\sqrt{-\lambda}) + A_2 \cosh(\xi\sqrt{-\lambda}) + \mu/\lambda)^2}$$

$$\times \left[A_1 \lambda \cosh(\xi\sqrt{-\lambda}) + A_2 \lambda \sinh(\xi\sqrt{-\lambda}) \mp \sqrt{\lambda^2 \sigma + \mu^2} \right],$$
(3.4)

where

$$\xi = x + y + z - (6a_0 - 5\lambda - 2)t. \tag{3.5}$$

In particular, by setting $A_1 = 0$, $A_2 > 0$ and $\mu = 0$ in (3.4), we have the solitary solution

$$u(\xi) = a_0 - \lambda \tanh(\xi \sqrt{-\lambda}) \left[\tanh(\xi \sqrt{-\lambda}) \mp i \operatorname{sech}(\xi \sqrt{-\lambda}) \right], \tag{3.6}$$

where $i = \sqrt{-1}$, while if $A_2 = 0$, $A_1 > 0$, and $\mu = 0$, then we have the solitary solution

$$u(\xi) = a_0 - \lambda \coth(\xi \sqrt{-\lambda}) \left[\coth(\xi \sqrt{-\lambda}) \mp \operatorname{csch}(\xi \sqrt{-\lambda}) \right]. \tag{3.7}$$

Case 2. Trigonometric function solutions ($\lambda > 0$).

If $\lambda > 0$, substituting (3.2) into (3.1) and using (2.2) and (2.6), we get a polynomial in ϕ and ψ . Vanishing each coefficient of this polynomial to get the algebraic equations which can be solved by using the Maple or Mathematica to find the following results:

$$a_0 = a_0,$$
 $a_1 = 0,$ $a_2 = 1,$ $b_1 = -\mu,$ $b_2 = \pm \sqrt{\frac{\lambda^2 \sigma - \mu^2}{\lambda}},$ $V = 6a_0 - 5\lambda - 2.$ (3.8)

From (2.5), (3.2), and (3.8), we deduce the traveling wave solution of (1.1) as follows:

$$u(\xi) = a_0 - \frac{\mu}{A_1 \sin(\xi\sqrt{\lambda}) + A_2 \cos(\xi\sqrt{\lambda}) + (\mu/\lambda)} + \frac{A_1 \cos(\xi\sqrt{\lambda}) - A_2 \sin(\xi\sqrt{\lambda})}{\left(A_1 \sin(\xi\sqrt{\lambda}) + A_2 \cos(\xi\sqrt{\lambda}) + \mu/\lambda\right)^2} \times \left[A_1\lambda \cos(\xi\sqrt{\lambda}) - A_2\lambda \sin(\xi\sqrt{\lambda}) \pm \sqrt{\lambda^2\sigma - \mu^2}\right],$$
(3.9)

where ξ has the same form (3.5).

In particular, by setting $A_1 = 0$, $A_2 > 0$, and $\mu = 0$ in (3.9), we have the periodic solution

$$u(\xi) = a_0 + \lambda \tan(\xi \sqrt{\lambda}) \left[\tan(\xi \sqrt{\lambda}) \mp \sec(\xi \sqrt{\lambda}) \right], \tag{3.10}$$

while if $A_2 = 0$, $A_1 > 0$, and $\mu = 0$, then we have the periodic solution

$$u(\xi) = a_0 + \lambda \cot(\xi \sqrt{\lambda}) \left[\cot(\xi \sqrt{\lambda}) \pm \csc(\xi \sqrt{\lambda})\right]. \tag{3.11}$$

Case 3. Rational function solutions ($\lambda = 0$).

If $\lambda = 0$, substituting (3.2) into (3.1) and using (2.2) and (2.8), we get a polynomial in ϕ and ψ . Setting each coefficients of this polynomial to be zero to get the algebraic equations which can be solved by using the Maple or Mathematica to find the following results:

$$a_0 = a_0,$$
 $a_1 = 0,$ $a_2 = 1,$ $b_1 = -\mu,$ $b_2 = \pm \sqrt{A_1^2 - 2\mu A_2},$ $V = 6a_0 - 2.$ (3.12)

From (2.7), (3.2), and (3.12), we deduce the traveling wave solution of (1.1) as follows:

$$u(\xi) = a_0 - \frac{\mu}{(\mu/2)\xi^2 + A_1\xi + A_2} + \frac{(\mu\xi + A_1)\left(\mu\xi + A_1 \pm \sqrt{A_1^2 - 2\mu A_2}\right)}{((\mu/2)\xi^2 + A_1\xi + A_2)^2},$$
(3.13)

where

$$\xi = x + y + z - (6a_0 - 2)t. \tag{3.14}$$

Remark 3.1. All solutions of this paper have been checked with Maple by putting them back into the original equation (1.1).

4. Conclusions

The two variables (G'/G,1/G)-expansion method has been used in this paper to discuss (1.1) and obtain the exact traveling wave solutions (3.4), (3.9), and (3.13) of Section 3. As the two parameters A_1 and A_2 take special values, we obtain the solitary wave solutions (3.6) and (3.7) and the trigonometric periodic solutions (3.10) and (3.11). On comparing these solutions with the result (11) of [14] obtained by Aslan using the exp-function method as well as the results (3.28)–(3.31) of [24] obtained by Zayed using the basic (G'/G)-expansion method, we conclude that all these solutions of (1.1) are different and satisfying that equation. The advantage of the two variables (G'/G,1/G)-expansion method over the basic (G'/G)-expansion method is that the first method is an extension of the second one.

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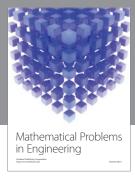
References

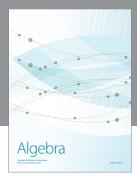
- [1] M. J. Ablowitz and P. A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, New York, NY, USA, 1991.
- [2] R. Hirota, "Exact solutions of the KdV equation for multiple collisions of solutions," *Physical Review Letters*, vol. 27, pp. 1192–1194, 1971.
- [3] J. Weiss, M. Tabor, and G. Carnevale, "The Painlevé property for partial differential equations," *Journal of Mathematical Physics*, vol. 24, no. 3, pp. 522–526, 1983.
- [4] N. A. Kudryashov, "Exact soliton solutions of a generalized evolution equation of wave dynamics," *Journal of Applied Mathematics and Mechanics*, vol. 52, pp. 361–365, 1988.
- [5] N. A. Kudryashov, "Exact solutions of the generalized Kuramoto-Sivashinsky equation," *Physics Letters A*, vol. 147, no. 5-6, pp. 287–291, 1990.
- [6] N. A. Kudryashov, "On types of nonlinear nonintegrable equations with exact solutions," *Physics Letters A*, vol. 155, no. 4-5, pp. 269–275, 1991.
- [7] M. R. Miura, Bäcklund Transformation, Springer, Berlin, Germany, 1978.
- [8] C. Rogers and W. F. Shadwick, *Bäcklund Transformations and Their Applications*, vol. 161, Academic Press, New York, NY, USA, 1982.
- [9] J.-H. He and X.-H. Wu, "Exp-function method for nonlinear wave equations," *Chaos, Solitons and Fractals*, vol. 30, no. 3, pp. 700–708, 2006.
- [10] E. Yusufoglu, "New solitary for the MBBM equations using Exp-function method," *Physics Letters A*, vol. 372, pp. 442–446, 2008.
- [11] S. Zhang, "Application of Exp-function method to high-dimensional nonlinear evolution equation," Chaos, Solitons and Fractals, vol. 38, no. 1, pp. 270–276, 2008.
- [12] A. Bekir, "The exp-function for Ostrovsky equation," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 10, pp. 735–739, 2009.
- [13] A. Bekir, "Application of the exp-function method for nonlinear differential-difference equations," *Applied Mathematics and Computation*, vol. 215, no. 11, pp. 4049–4053, 2010.
- [14] I. Aslan, "Comment on: "Application of Exp-function method for (3+1)-dimensional nonlinear evolution equations" [Comput. Math. Appl. 56 (2008) 1451–1456]," Computers and Mathematics with Applications, vol. 61, no. 6, pp. 1700–1703, 2011.
- [15] M. A. Abdou, "The extended tanh method and its applications for solving nonlinear physical models," *Applied Mathematics and Computation*, vol. 190, no. 1, pp. 988–996, 2007.
- [16] E. Fan, "Extended tanh-function method and its applications to nonlinear equations," *Physics Letters A*, vol. 277, no. 4-5, pp. 212–218, 2000.
- [17] S. Zhang and T.-c. Xia, "A further improved tanh function method exactly solving the (2 + 1)-dimensional dispersive long wave equations," *Applied Mathematics E-Notes*, vol. 8, pp. 58–66, 2008.

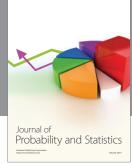
- [18] E. Yusufoğlu and A. Bekir, "Exact solutions of coupled nonlinear Klein-Gordon equations," *Mathematical and Computer Modelling*, vol. 48, no. 11-12, pp. 1694–1700, 2008.
- [19] Y. Chen and Q. Wang, "Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic function solutions to (1 + 1)-dimensional dispersive long wave equation," *Chaos, Solitons and Fractals*, vol. 24, no. 3, pp. 745–757, 2005.
- [20] S. Liu, Z. Fu, S. Liu, and Q. Zhao, "Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations," *Physics Letters A*, vol. 289, no. 1-2, pp. 69–74, 2001.
- [21] D. Lü, "Jacobi elliptic function solutions for two variant Boussinesq equations," *Chaos, Solitons and Fractals*, vol. 24, no. 5, pp. 1373–1385, 2005.
- [22] M. Wang, X. Li, and J. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters A*, vol. 372, no. 4, pp. 417– 423, 2008.
- [23] S. Zhang, J. L. Tong, and W. Wang, "A generalized (*G*′/*G*)-expansion method for the mKdV equation with variable coefficients," *Physics Letters A*, vol. 372, pp. 2254–2257, 2008.
- [24] E. M. E. Zayed, "Traveling wave solutions for higher-dimensional nonlinear evolution equations using the (G'/G)-expansion method," *Journal of Applied Mathematics & Informatics*, vol. 28, pp. 383–395, 2010.
- [25] E. M. E. Zayed, "The (*G*'/*G*)-expansion method and its applications to some nonlinear evolution equations in the mathematical physics," *Journal of Applied Mathematics and Computing*, vol. 30, no. 1-2, pp. 89–103, 2009.
- [26] A. Bekir, "Application of the (G'/G)-expansion method for nonlinear evolution equations," *Physics Letters A*, vol. 372, no. 19, pp. 3400–3406, 2008.
- [27] B. Ayhan and A. Bekir, "The (*G'/G*)-expansion method for the nonlinear lattice equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 9, pp. 3490–3498, 2012.
- [28] N. A. Kudryashov, "A note on the (G'/G)-expansion method," *Applied Mathematics and Computation*, vol. 217, no. 4, pp. 1755–1758, 2010.
- [29] I. Aslan, "A note on the (G'/G)-expansion method again," *Applied Mathematics and Computation*, vol. 217, no. 2, pp. 937–938, 2010.
- [30] N. A. Kudryashov, "Meromorphic solutions of nonlinear ordinary differential equations," Communications in Nonlinear Science and Numerical Simulation, vol. 15, no. 10, pp. 2778–2790, 2010.
- [31] I. Aslan, "Exact and explicit solutions to the discrete nonlinear Schrödinger equation with a saturable nonlinearity," *Physics Letters A*, vol. 375, no. 47, pp. 4214–4217, 2011.
- [32] I. Aslan, "Some exact solutions for Toda type lattice differential equations using the improved (*G*'/*G*)-expansion method," *Mathematical Methods in the Applied Sciences*, vol. 35, no. 4, pp. 474–481, 2012.
- [33] I. Aslan, "The discrete (*G*'/*G*)-expansion method applied to the differential-difference Burgers equation and the relativistic Toda lattice system," *Numerical Methods for Partial Differential Equations*. *An International Journal*, vol. 28, no. 1, pp. 127–137, 2012.
- [34] L.-x. Li, E.-q. Li, and M.-l. Wang, "The (G'/G, 1/G)-expansion method and its application to travelling wave solutions of the Zakharov equations," *Applied Mathematics B*, vol. 25, no. 4, pp. 454–462, 2010.
- [35] E. M. E. Zayed and M. A. M. Abdelaziz, "The two variables (G'/G,1/G) -expansion method for solving the nonlinear KdV-mKdV equation," *Mathematical Problems in Engineering*, vol. 2012, Article ID 725061, 14 pages, 2012.
- [36] F. Tascan and A. Bekir, "Applications of the first integral method to the nonlinear evolution equations," *Chinese Physics B*, vol. 19, Article ID 080201, 11 pages, 2010.













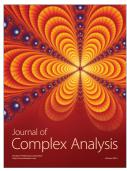




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