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Research Article

Characterization of Entire Sequences via Double Orlicz Space

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Let Γ denote the space of all entire sequences and $\wedge$ the space of all analytic sequences. This paper is a study of the characterization and general properties of entire sequences via double Orlicz space of Γ_M^2 of Γ^2 establishing some inclusion relations.

1. Introduction

Throughout, w, Γ, and $\wedge$ denote the classes of all, entire, and analytic scalar-valued single sequences, respectively.

We write w^2 for the set of all complex sequences (x_{mn}), where $m,n \in \mathbb{N}$ is the set of positive integers. Then, w^2 is a linear space under the coordinatewise addition and scalar multiplication.

Some initial works on double sequence spaces are found in Bromwich [1]. Later on, they were investigated by Hardy [2], Móricz [3], Móricz and Rhoades [4], Basarir and Sonalcan [5], Tripathy [6], Colak and Turkmenoglu [7], Turkmenoglu [8], and many others.

We need the following inequality in the sequel of the paper.

For $a,b \geq 0$ and $0 < p < 1$, we have

$$(a+b)^p \leq a^p + b^p. \tag{1.1}$$

The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (S_{mn}) is called convergent, where $S_{mn} = \sum_{i,j=1}^{m,n} x_{ij}$ $(m,n = 1,2,3,\dots)$ (see [9]).

A sequence $x = (x_{mn})$ is said to be double analytic if $\sup_{mn} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by $\wedge^2$. A sequence $x = (x_{mn})$is called double entire sequence if $|x_{mn}|^{1/m+n} \to 0$ as $m, n \to \infty$. The double entire sequences will be denoted by Γ^2. Let $\Phi = \{\text{all finite sequences}\}$.

Consider a double sequence $x = (x_{ij})$. The (m,n)th section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\, \delta_{ij}$ for all $m, n \in \mathbb{N}$,

$$\delta_{mn} = \begin{pmatrix} 0,0,\ldots,0,0,\ldots \\ 0,0,\ldots,0,0,\ldots \\ \vdots \\ 0,0,\ldots,1,0,\ldots \\ 0,0,\ldots,0,0,\ldots \end{pmatrix}, \tag{1.2}$$

with 1 in the (m,n)th position and zero otherwise. An FK-space (or a metric space) X is said to have AK property if (δ_{mn}) is a Schauder basis for X. Or equivalently, $x^{[m,n]} \to x$.

An FDK space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \to (x_{mn})$ $(m, n \in \mathbb{N})$ are also continuous.

Orlicz [10] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [11] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(1 \leq p < \infty)$. Subsequently, different classes of sequence spaces were defined by Parashar and Choudhary [12], Mursaleen et al. [13], Bektaş and Altin [14], Tripathy et al. [15], Rao and Subramanian [16], and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in [17].

Recalling [10] and [17], an Orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, nondecreasing, and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$, then this function is called modulus function, defined by Nakano [18] and further discussed by Ruckle [19], Maddox [20], and many others.

Let (Ω, Σ, μ) be a finite measure space. We denote by $E(\mu)$ the space of all (equivalence classes of) Σ-measurable functions x from Ω into $[0,\infty)$. Given an Orlicz function M, we define on $E(\mu)$a convex functional I_M by

$$I_M(x) = \int_\Omega M(x(t))\, d\mu, \tag{1.3}$$

and an Orlicz space $L^M(\mu)$ by $L^M(\mu) = \{x \in E(\mu) : I_M(\lambda x) < +\infty \text{ for some } \lambda > 0\}$ (for detail, see [10, 17]).

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct Orlicz sequence space:

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}, \tag{1.4}$$

where $w = \{\text{all complex sequences}\}$. The space ℓ_M with the norm

$$\|x\| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\} \tag{1.5}$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p$ $(1 \leq p < \infty)$, the spaces ℓ_M coincide with the classical sequence space ℓ_p.

If X is a sequence space, we give the following definitions:

(i) $X' =$ the continuous dual of X;

(ii) $X^{\alpha} = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn}x_{mn}| < \infty$, for each $x \in X\}$;

(iii) $X^{\beta} = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn}x_{mn}$ is convergent, for each $x \in X\}$;

(iv) $X^{\gamma} = \{a = (a_{mn}) : \sup_{m,n\geq 1} |\sum_{m,n=1}^{M,N} a_{mn}x_{mn}| < \infty$, for each $x \in X\}$;

(v) let X be an FK-space $\supset \Phi$, then $X^f = \{f(\delta_{mn}) : f \in X'\}$;

(vi) $X^{\wedge} = \{a = (a_{mn}) : \sup_{(mn)} |a_{mn}x_{mn}|^{1/m+n} < \infty$, for each $x \in X\}$;

(vii) X^{α}, X^{β}, X^{γ} are called α-(or Köthe-Toeplitz) dual of X, β-(or generalized Köthe-Toeplitz) dual of X, γ-dual of X, and $\wedge$-dual of X, respectively.

2. Definitions and preliminaries

Throughout the article, w^2 denote the spaces of all sequences. Γ_M^2 and $\wedge_M^2$ denote the Pringscheims of double Orlicz space of entire sequence and Pringscheims of double Orlicz space of bounded sequence, respectively

Let w^2 denote the set of all complex double sequences $x = (x_{mn})_{m,n=1}^{\infty}$ and $M : [0,\infty) \to [0,\infty)$ be an Orlicz function, or a modulus function. Given a double sequence, $x \in w^2$. Let t denote the double sequence with $t_{mn} = |x_{mn}|^{1/(m+n)}$ for all $m,n \in \mathbb{N}$. Define the sets

$$\begin{aligned} \Gamma_M^2 &= \left\{x \in w^2 : \left(M\left(\frac{t_{mn}}{\rho}\right)\right) \longrightarrow 0 \ (m,n \longrightarrow \infty) \text{ for some } \rho > 0\right\}, \\ \wedge_M^2 &= \left\{x \in w^2 : \sup_{(m,n)}\left(M\left(\frac{t_{mn}}{\rho}\right)\right) < \infty \text{ for some } \rho > 0\right\}. \end{aligned} \tag{2.1}$$

The space $\wedge_M^2$ is a metric space with the metric

$$\tilde{d}(x,y) = \inf\left\{\rho > 0 : \sup_{(m,n)}\left(M\left(\frac{|x_{mn} - y_{mn}|^{1/m+n}}{\rho}\right)\right) \leq 1\right\} \tag{2.2}$$

and the space Γ_M^2 is a metric space with the metric

$$d(x,y) = \left\{\rho > 0 : \sup_{(m,n)}\left(M\left(\frac{|x_{mn} - y_{mn}|^{1/m+n}}{\rho}\right)\right) : m,n = 1,2,3,\ldots\right\}. \tag{2.3}$$

3. Main results

Proposition 3.1. *If M is a modulus function, then Γ_M^2 is a linear set over the set of complex numbers $\mathbb{C}$.*

Proof. It is trivial. Therefore, the proof is omitted. □

Proposition 3.2. $(\Gamma_M^2)^\beta \subset_{\neq} \wedge^2$.

Proof. Let $y = \{y_{mn}\}$ be an arbitrary point in $(\Gamma_M^2)^\beta$. If y is not in $\wedge^2$, then for each natural number p, we can find an index $m_p n_p$ such that

$$M\left(\frac{|y_{m_p n_p}|^{1/m_p+n_p}}{\rho}\right) > p, \quad (p = 1,2,3,\dots). \tag{3.1}$$

Define $x = \{x_{mn}\}$ by

$$\begin{aligned} M\left(\frac{x_{mn}}{\rho}\right) &= \frac{1}{p^{m+n}} \quad \text{for } (m,n) = (m_p, n_p) \text{ for some } p \in N, \\ M\left(\frac{x_{mn}}{\rho}\right) &= 0 \quad \text{otherwise.} \end{aligned} \tag{3.2}$$

Then x is in Γ_M^2, but for infinitely mn,

$$M\left(\frac{|y_{mn}x_{mn}|}{\rho}\right) > 1. \tag{3.3}$$

Consider the sequence $z = \{z_{mn}\}$, where $M(z_{11}/\rho) = M(x_{11}/\rho) - s$ with

$$s = \sum M\left(\frac{x_{mn}}{\rho}\right), \quad M\left(\frac{z_{mn}}{\rho}\right) = M\left(\frac{x_{mn}}{\rho}\right) \quad (m,n = 1,2,3,\dots). \tag{3.4}$$

Then, z is a point of Γ_M^2. Also, $\sum M(z_{mn}/\rho) = 0$. Hence, z is in Γ_M^2; but, by (3.3), $\sum M(z_{mn}y_{mn}/\rho)$ does not converge:

$$\Longrightarrow \Sigma x_{mn}y_{mn} \text{ diverges.} \tag{3.5}$$

Thus, the sequence y would not be in $(\Gamma_M^2)^\beta$. This contradiction proves that

$$(\Gamma_M^2)^\beta \subset \wedge^2. \tag{3.6}$$

If we now choose $M = \text{id}$, where id is the identity and $y_{1n} = x_{1n} = 1$ and $y_{mn} = x_{mn} = 0$ $(m > 1)$ for all n, then obviously $x \in \Gamma_M^2$ and $y \in \wedge^2$, but

$$\sum_{m,n=1}^{\infty} x_{mn} y_{mn} = \infty. \text{ Hence, } y \notin \left(\Gamma_M^2\right)^{\beta}. \tag{3.7}$$

From (3.6) and (3.7), we are granted $(\Gamma_M^2)^{\beta} \subset_{\neq} \wedge^2$. This completes the proof. □

Proposition 3.3. *Γ_M^2 has AK, where M is a modulus function.*

Proof. Let $x = (x_{mn}) \in \Gamma_M^2$ and take $x^{[mn]} = \sum_{i,j=1}^{m,n} x_{ij}\delta_{ij}$ for all $m, n \in \mathbb{N}$. Hence,

$$d\left(x, x^{[rs]}\right) = \left\{\rho : \sup_{(m,n)} \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right) : m \geq r+1,\ n \geq s+1 \right\} \longrightarrow 0 \quad \text{as } m, n \longrightarrow \infty. \tag{3.8}$$

Therefore, $x^{[rs]} \to x$ as $r, s \to \infty$ in Γ_M^2. Thus, Γ_M^2 has AK. This completes the proof. □

Proposition 3.4. *Γ_M^2 is solid.*

Proof. Let $|x_{mn}| \leq |y_{mn}|$ and let $y = (y_{mn}) \in \Gamma_M^2 \cdot (M(|x_{mn}|^{1/m+n}/\rho)) \leq (M(|y_{mn}|^{1/m+n}/\rho))$, because M is nondecreasing. But $(M(|y_{mn}|^{1/m+n}/\rho)) \in \Gamma^2$ because $y \in \Gamma_M^2$. That is, $(M(|y_{mn}|^{1/m+n}/\rho)) \to 0$ as $m, n \to \infty$ and $(M(|x_{mn}|^{1/m+n}\rho)) \to 0$ as $m, n \to \infty$. Therefore, $x = \{x_{mn}\} \in \Gamma_M^2$. This completes the proof. □

Proposition 3.5. $(\Gamma_M^2)^{\wedge} \subset_{\neq} \wedge^2$.

Proof. Let $y \in \wedge$-dual of Γ_M^2. Then, $(M(|x_{mn}y_{mn}|/\rho)) \leq M^{m+n}$ for some constant $M > 0$ and for all $x \in \Gamma_M^2$. Therefore, $(M(|y_{mn}|/\rho)) \leq M^{m+n}$ for all m, n by taking $x = (\delta_{mn})$. This implies that $y \in \wedge^2$. Thus,

$$\left(\Gamma_M^2\right)^{\wedge} \subset \wedge^2. \tag{3.9}$$

We now choose $M = \text{id}$ and define the double sequences (y_{mn}) and (x_{mn}) by $y_{mn} = 1$ for all m and n, and by $x_{m1} = 2^{(m+1)^2}$ and $x_{mn} = 0$ $(n \geq 2)$ for all $m = 1, 2, \ldots$. Obviously, $y \in \wedge^2$ and since $x_{mn} = 0$ for all $m, n \geq 0$, (x_{mn}) converges to zero in the Pringsheim sense. Hence, $x \in \Gamma_M^2$. But,

$$\left| a_{m1} x_{m1} \right|^{1/(m+1)} = 2^{m+1} \longrightarrow \infty \quad \text{as } m \longrightarrow \infty, \text{ hence } x \notin \left(\Gamma_M^2\right)^{\wedge}. \tag{3.10}$$

From (3.9) and (3.10), we are granted $(\Gamma_M^2)^{\wedge} \subset_{\neq} \wedge^2$. This completes the proof. □

Proposition 3.6. *The dual space of* (Γ_M^2) *is* $\wedge^2$. *In other words,* $(\Gamma_M^2)^* = \wedge^2$.

Proof. We recall that

$$\delta_{mn} = \begin{pmatrix} 0,0,\ldots,0,0,\ldots \\ 0,0,\ldots,0,0,\ldots \\ \vdots \\ 0,0,\ldots,1,0,\ldots \\ 0,0,\ldots,0,0,\ldots \end{pmatrix} \tag{3.11}$$

has 1 in the (m,n)th position and zero otherwise, with

$$\begin{aligned} x = \delta_{mn}, &\left\{M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right\} \\ = &\begin{pmatrix} M((0)^{1/2}/\rho), M((0)^{1/3}/\rho), M((0)^{1/2}/\rho), \ldots \\ M((0)^{1/3}/\rho), M((0)^{1/4}/\rho), M((0)^{1/5}/\rho), \ldots \\ \vdots \\ M((0)/\rho), M((0)/\rho), \ldots, M((1)^{1/m+n}/\rho), M((0)/\rho), \ldots \\ M((0)/\rho), M((0)/\rho), \ldots, M((0)/\rho), M((0)/\rho), \ldots \\ \vdots \end{pmatrix} \end{aligned} \tag{3.12}$$

which is a double null sequence.

Hence, $\delta_{mn} \in \Gamma_M^2 \cdot f(x) = \sum_{m,n=1}^{\infty} x_{mn} y_{mn}$ with $x \in \Gamma_M^2$ and $f \in (\Gamma_M^2)^*$, where $(\Gamma_M^2)^*$ is the dual space of Γ_M^2. Take $x = (x_{mn}) = \delta_{mn} \in \Gamma_M^2$. Then,

$$|y_{mn}| \le \|f\| d(\delta_{mn}, 0) < \infty \quad \forall m, n. \tag{3.13}$$

Thus, (y_{mn}) is a double bounded sequence, and hence a double analytic sequence. In other words, $y \in \wedge^2$. Therefore, $(\Gamma_M^2)^* = \wedge^2$. This completes the proof. □

Proposition 3.7. $(\wedge_M^2)^\beta \subset_{\neq} \Gamma_M^2$.

Proof. Let $(x_{mn}) \in (\wedge_M^2)^\beta$,

$$\Longrightarrow \sum_{m,n=1}^{\infty} x_{mn} y_{mn} \text{ converges } \forall y \in \wedge_M^2. \tag{3.14}$$

Let us assume that $(x_{mn}) \notin \Gamma_M^2$. Then, there exist a sequence positive integers $(m_p + n_p)$ strictly increasing such that

$$\left(M\left(\frac{|x_{(m_p+n_p)}|}{\rho}\right)\right) > \frac{1}{2^{(m_p+n_p)}}, \quad (p = 1, 2, 3, \ldots). \tag{3.15}$$

Let

$$y_{(m_p,n_p)} = 2^{(m_p+n_p)} \quad (\text{for } p = 1,2,3,\ldots), \\ y_{m,n} = 0, \quad \text{otherwise}. \tag{3.16}$$

Then, $(y_{mn}) \in \wedge_M^2$.

However,

$$\sum_{m,n=1}^{\infty}\left(M\left(\frac{|x_{mn}y_{mn}|}{\rho}\right)\right) = \sum_{p=1}^{\infty}\left(M\left(\frac{|x_{(m_pn_p)}y_{(m_pn_p)}|}{\rho}\right)\right) > 1+1+1+\cdots. \tag{3.17}$$

We know that the infinite series $1+1+1+\cdots$ diverges. Hence, $\sum_{m,n=1}^{\infty}(M(|x_{mn}y_{mn}|/\rho))$ diverges. This contradicts (3.14). Hence, $(x_{mn}) \in \Gamma_M^2$. Therefore,

$$(\wedge_M^2)^{\beta} \subset \Gamma_M^2. \tag{3.18}$$

If we now choose $M = \text{id}$, where id is the identity and $y_{1n} = x_{1n} = 1$ and $y_{mn} = x_{mn} = 0$ $(m > 1)$ for all n, then obviously $x \in \Gamma_M^2$ and $y \in \wedge_M^2$, but

$$\sum_{m,n=1}^{\infty} x_{mn}y_{mn} = \infty. \text{ Hence, } y \notin (\wedge_M^2)^{\beta}. \tag{3.19}$$

From (3.18) and (3.19), we are granted $(\wedge_M^2)^{\beta} \subset_{\neq} \Gamma_M^2$. This completes the proof. □

Definition 3.8. Let $p = (p_{mn})$ be a double sequence of positive real numbers. Then,

$$\Gamma_M^2(p) = \left\{x = (x_{mn}) : \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{p_{mn}} \longrightarrow 0\ (m,n \longrightarrow \infty) \text{ for some } \rho > 0\right\}. \tag{3.20}$$

Suppose that p_{mn} is a constant for all m,n, then $\Gamma_M^2(p) = \Gamma_M^2$.

Proposition 3.9. *Let* $0 \le p_{mn} \le q_{mn}$ *and let* $\{q_{mn}/p_{mn}\}$ *be bounded. Then,* $\Gamma_M^2(q) \subset \Gamma_M^2(p)$.

Proof. Let

$$x \in \Gamma_M^2(q), \tag{3.21}$$

then

$$\left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{q_{mn}} \longrightarrow 0 \quad \text{as } m,n \longrightarrow \infty. \tag{3.22}$$

Let $t_{mn} = (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$, and let $\lambda_{mn} = p_{mn}/q_{mn}$. Since $p_{mn} \leq q_{mn}$, we have $0 \leq \lambda_{mn} \leq 1$. Let $0 < \lambda < \lambda_{mn}$, then

$$u_{mn} = \begin{cases} t_{mn} & (t_{mn} \geq 1), \\ 0 & (t_{mn} < 1), \end{cases}$$
$$v_{mn} = \begin{cases} 0 & (t_{mn} \geq 1), \\ t_{mn} & (t_{mn} < 1), \end{cases} \tag{3.23}$$
$$t_{mn} = u_{mn} + v_{mn}, \qquad t_{mn}^{\lambda_{mn}} = u_{mn}^{\lambda_{mn}} + v_{mn}^{\lambda_{mn}}.$$

Now, it follows that

$$u_{mn}^{\lambda_{mn}} \leq u_{mn} \leq t_{mn}, \qquad v_{mn}^{\lambda_{mn}} \leq v_{mn}^{\lambda}. \tag{3.24}$$

Since $t_{mn}^{\lambda_{mn}} = u_{mn}^{\lambda_{mn}} + v_{mn}^{\lambda_{mn}}$, we have $t_{mn}^{\lambda_{mn}} \leq t_{mn} + v_{mn}^{\lambda}$. Thus, $(M(|x_{mn}|^{1/m+n}/\rho)^{q_{mn}})^{\lambda_{mn}} \leq (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$ and

$$\left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)^{q_{mn}}\right)^{p_{mn}/q_{mn}} \leq \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{q_{mn}}, \tag{3.25}$$

which yields $(M(|x_{mn}|^{1/m+n}/\rho))^{p_{mn}} \leq (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$. However, $(M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}} \to 0$ (by (3.22)). Thus, $(M(|x_{mn}|^{1/m+n}/\rho))^{p_{mn}} \to 0$ as $m,n \to \infty$.

Hence,

$$x \in \Gamma_M^2(p). \tag{3.26}$$

From (3.21) and (3.26), we are granted

$$\Gamma_M^2(q) \subset \Gamma_M^2(p). \tag{3.27}$$

This completes the proof. □

Proposition 3.10. *(a) If $0 < \inf p_{mn} \leq p_{mn} \leq 1$, then $\Gamma_M^2(p) \subset \Gamma_M^2$.*
(b) If $1 \leq p_{mn} \leq \sup p_{mn} < \infty$, then $\Gamma_M^2 \subset \Gamma_M^2(p)$.

Proof. The above statements are special cases of Proposition 3.9. Therefore, it can be proved by similar arguments. □

Proposition 3.11. *If $0 < p_{mn} \leq q_{mn} < \infty$ for each m,n, then $\Gamma_M^2(p) \subseteq \Gamma_M^2(q)$.*

Proof. Let $x \in \Gamma_M^2(p)$, then

$$\left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{p_{mn}} \longrightarrow 0 \quad \text{as } m,n \longrightarrow \infty. \tag{3.28}$$

This implies that $(M(|x_{mn}|^{1/m+n}/\rho)) \leq 1$ for sufficiently large m,n. Since M is nondecreasing, we get

$$\left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{q_{mn}} \leq \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{p_{mn}}, \tag{3.29}$$

then $(M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}} \to 0$ as $m,n \to \infty$ (by using (3.28)). Let $x \in \Gamma_M^2(q)$. Hence, $\Gamma_M^2(p) \subseteq \Gamma_M^2(q)$. This completes the proof. □

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References

[1] T. J. I. Bromwich, *An Introduction to the Theory of Infinite Series*, Macmillan, New York, NY, USA, 1965.
[2] G. H. Hardy, "On the convergence of certain multiple series," *Proceedings of the Cambridge Philosophical Society*, vol. 19, pp. 86–95, 1917.
[3] F. Móricz, "Extensions of the spaces c and c_0 from single to double sequences," *Acta Mathematica Hungarica*, vol. 57, no. 1-2, pp. 129–136, 1991.
[4] F. Móricz and B. E. Rhoades, "Almost convergence of double sequences and strong regularity of summability matrices," *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 104, no. 2, pp. 283–294, 1988.
[5] M. Basarir and O. Sonalcan, "On some double sequence spaces," *Journal of the Indian Academy of Mathematics*, vol. 21, no. 2, pp. 193–200, 1999.
[6] B. C. Tripathy, "Statistically convergent double sequences," *Tamkang Journal of Mathematics*, vol. 34, no. 3, pp. 231–237, 2003.
[7] R. Colak and A. Turkmenoglu, "The double sequence spaces $\ell_\infty{}^2(p)$, $c_0{}^2(p)$ and $c^2(p)$," in preparation.
[8] A. Turkmenoglu, "Matrix transformation between some classes of double sequences," *Journal of Institute of Mathematics & Computer Sciences*, vol. 12, no. 1, pp. 23–31, 1999.
[9] T. Apostol, *Mathematical Analysis*, Addison-Wesley, London, UK, 1978.
[10] W. Orlicz, "Über Raume (L^M)," *Bulletin International de l'Académie Polonaise des Sciences et des Lettres, Série A*, pp. 93–107, 1936.
[11] J. Lindenstrauss and L. Tzafriri, "On Orlicz sequence spaces," *Israel Journal of Mathematics*, vol. 10, pp. 379–390, 1971.
[12] S. D. Parashar and B. Choudhary, "Sequence spaces defined by Orlicz functions," *Indian Journal of Pure and Applied Mathematics*, vol. 25, no. 4, pp. 419–428, 1994.
[13] Mursaleen, M. A. Khan, and Qamaruddin, "Difference sequence spaces defined by Orlicz functions," *Demonstratio Mathematica*, vol. 32, no. 1, pp. 145–150, 1999.
[14] Ç. A. Bektaş and Y. Altin, "The sequence space $l_M(p,q,s)$ on seminormed spaces," *Indian Journal of Pure and Applied Mathematics*, vol. 34, no. 4, pp. 529–534, 2003.
[15] B. C. Tripathy, M. Et, and Y. Altin, "Generalized difference sequence spaces defined by Orlicz function in a locally convex space," *Journal of Analysis and Applications*, vol. 1, no. 3, pp. 175–192, 2003.

[16] K. Chandrasekhara Rao and N. Subramanian, “The Orlicz space of entire sequences,” *International Journal of Mathematics and Mathematical Sciences*, vol. 2004, no. 68, pp. 3755–3764, 2004.
[17] M. A. Krasnoselskii and Y. B. Rutickii, *Convex Functions and Orlicz Spaces*, Noordhoff, Gorningen, The Netherlands, 1961.
[18] H. Nakano, “Concave modulars,” *Journal of the Mathematical Society of Japan*, vol. 5, no. 1, pp. 29–49, 1953.
[19] W. H. Ruckle, “FK spaces in which the sequence of coordinate vectors is bounded,” *Canadian Journal of Mathematics*, vol. 25, pp. 973–978, 1973.
[20] I. J. Maddox, “Sequence spaces defined by a modulus,” *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 100, no. 1, pp. 161–166, 1986.

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