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A NOTE ON WEAKLY QUASI CONTINUOUS FUNCTIONS

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ABSTRACT. The notion of weakly quasi continuous functions introduced by Popa and Stan [1]. In this paper, the authors obtain the further properties of such functions and introduce weak* quasi continuity which is weaker than semi continuity [2] but independent of weak quasi continuity.

KEY WORDS AND PHRASES. Weakly continuous, semi continuous, weakly quasi continuous, weakly* quasi continuous.
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1. INTRODUCTION

As weak forms of continuity in topological spaces, semi continuity, weak continuity [3], quasi continuity [4] and almost continuity in the sense of Husain [5] are well known. Neubrunnová [6] showed that semi continuity is equivalent to quasi continuity. Also, Noiri [7] showed that semi continuity, weak continuity and almost continuity are respectively independent. In 1973, Popa and Stan [1] introduced weak quasi continuity which is implied by both weak α-continuity [8] and semi continuity. It is shown in [7] that weak quasi continuity is equivalent to weak semi continuity due to Arya and Bhamini [9]. Recently, Noiri in [7,8] investigated fundamental properties of weakly quasi continuous functions and compared the interrelation among weak quasi continuity, weak α-continuity, semi continuity and almost continuity.

The purpose of this paper is to obtain some characterizations of weakly quasi continuous functions and investigate the relationships between such functions and some separation axioms. We also introduce weak* quasi continuity which is weaker than semi continuity but independent of weak quasi continuity.

2. PRELIMINARIES

Throughout the present paper, spaces always mean topological spaces and $f : X \to Y$ denotes a single valued function of a space X into a space Y. Let X be a space and A a subset of X. We denote the closure of A and the interior of A by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be semiopen [2] (resp. preopen [10], α-open [11]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$ (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$). We denote the family of semiopen (resp. preopen, α-open) sets of X by $SO(X)$ (resp. $PO(X)$, $\alpha(X)$). It is shown that $\alpha(X) = SO(X) \cap PO(X)$ [12]. The complement of a semiopen set is said to be semiclosed The intersection of all semiclosed sets containing A is called the semi-closure [13] of A and is denoted by s-Cl(A). The semi-interior [13] of A, denoted by s-Int(A), is defined by the union of all semiopen sets contained in A. A subset A of X is said to be regular open (resp regular closed) [14] if $A = \text{Int}(\text{Cl}(A))$ (resp $A = \text{Cl}(\text{Int}(A))$). A point $x \in X$ is in the θ-closure of A [15], denoted by $\text{Cl}_\theta(A)$, if $A \cap \text{Cl}(U) \neq \emptyset$ for each open set U containing x. A subset A is called θ-closed if $\text{Cl}_\theta(A) = A$

DEFINITION A. A function $f : X \to Y$ is said to be

(a) semi continuous [2] (briefly, s c) if $f^{-1}(V) \in SO(X)$ for each open set V of Y;

(b) almost continuous [5] if for each $x \in X$ and each open set V containing $f(x)$, $\text{Cl}(f^{-1}(V))$ is a neighborhood of x;

(c) weakly continuous [3] (resp. θ-continuous [16]) if for each $x \in X$ and each open set V containing $f(x)$, there exists an open set U containing x such that $f(U) \subset \text{Cl}(V)$ (resp $f(\text{Cl}(U)) \subset \text{Cl}(V)$);

(d) weakly α-continuous [8] (briefly, w.α.c.) if for each $x \in X$ and each open set V containing $f(x)$, there exists a $U \in \alpha(X)$ containing x such that $f(U) \subset \text{Cl}(V)$.

3. WEAKLY QUASI CONTINUOUS FUNCTIONS

DEFINITION 3.1. A function $f : X \to Y$ is said to be

(a) weakly quasi continuous [1] (briefly, w.q.c.) if for each $x \in X$, each open set G containing x and each open set V containing $f(x)$, there exists an open set U of X such that $\emptyset \neq U \subset G$ and $f(U) \subset \text{Cl}(V)$;

(b) weakly semi-continuous [9] (briefly, w.s.c.) if for each $x \in X$ and each open set V containing $f(x)$, there exists a $U \in SO(X)$ containing x such that $f(U) \subset \text{Cl}(V)$.

Noiri showed in [7, Theorem 4.1] that a function $f : X \to Y$ is w.q.c. if and only if for each $x \in X$ and each open set V containing $f(x)$, there exists a $U \in SO(X)$ containing x such that $f(U) \subset \text{Cl}(V)$. Hence we know that w.q.c. and w.s.c. are equivalent concepts.

The following is shown in [7, Theorem 4.2, 4.3] and [8, Lemma 5.3].

THEOREM 3.2. *For a function $f : X \to Y$, the following are equivalent:*

(a) *f is w.q.c.*

(b) *For each subset B of Y, s-$Cl(f^{-1}(Int(Cl(B)))) \subset f^{-1}(Cl(B))$.*

(c) *For each regular closed set F of Y, s-$Cl(f^{-1}(Int(F))) \subset f^{-1}(F)$.*

(d) *For each open set B of Y, s-$Cl(f^{-1}(B)) \subset f^{-1}(Cl(B))$.*

(e) *For each open set B of Y, $f^{-1}(B) \subset s$-$Int(f^{-1}(Cl(B)))$.*

(f) *For each regular closed set B of Y, $f^{-1}(B) \in SO(X)$.*

(g) *For each open set B of Y, $f^{-1}(B) \subset Cl(Int(f^{-1}(Cl(B))))$.*

THEOREM 3.3. *For a function $f : X \to Y$, the following are equivalent:*

(a) *f is w.q.c.*

(b) *For each subset B of Y, s-$Cl(f^{-1}(B)) \subset f^{-1}(Cl_\theta(B))$.*

(c) *For each subset A of X, $f(s$-$Cl(A)) \subset Cl_\theta(f(A))$.*

(d) *For each subset A of X, $f(Int(Cl(A))) \subset Cl_\theta(f(A))$.*

(e) *For each subset B of Y, $Int(Cl(f^{-1}(B))) \subset f^{-1}(Cl_\theta(B))$.*

(f) *For each open set B of Y, $Int(Cl(f^{-1}(B))) \subset f^{-1}(Cl(B))$.*

PROOF. It follows immediately from Theorem 3.2 and [17, Theorem 1.5].

THEOREM 3.4. *A function $f : X \to Y$ is a w.q.c. if and only if for each subset B of Y, s-$Cl(f^{-1}(Int(Cl_\theta(B)))) \subset f^{-1}(Cl_\theta(B))$.*

PROOF. Necessity. Let B be a subset of Y. Assume that $x \notin f^{-1}(\text{Cl}_\theta(B))$. Then $f(x) \notin \text{Cl}_\theta(B)$ and hence there exists an open set W containing $f(x)$ such that $B \cap \text{Cl}(W) = \emptyset$. This implies that $\mathrm{Cl}_\theta(B) \cap W = \emptyset$ and so $W \subset Y - \mathrm{Cl}_\theta(B)$, i e $\mathrm{Cl}(W) \subset \mathrm{Cl}(Y - \mathrm{Cl}_\theta(B))$ Since f is w q c, there exists a $U \in SO(X)$ containing x such that $f(U) \subset \mathrm{Cl}(W) \subset \mathrm{Cl}(Y - \mathrm{Cl}_\theta(B))$ This implies that $U \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B))) = \emptyset$ and hence $x \notin s\text{-}\mathrm{Cl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B))))$ Therefore, $s\text{-}\mathrm{Cl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) \subset f^{-1}(\mathrm{Cl}_\theta(B))$

Sufficiency Let B be an open set of Y Then clearly $\mathrm{Cl}(B) = \mathrm{Cl}_\theta(B)$ By hypothesis, we have $s\text{-}\mathrm{Cl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))) = s\text{-}\mathrm{Cl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) \subset f^{-1}(\mathrm{Cl}_\theta(B)) = f^{-1}(\mathrm{Cl}(B))$ Hence, by Theorem 3 2, f is w q c

The composition of two w q c functions may fail to be w q c [7] But Noiri showed in [7, Theorem 6 1 6] that under certain conditions the composition of two functions is w q c

THEOREM 3.5. *Let $f : X \to Y$ and $g : Y \to Z$ be functions.*

(a) If f is w.q.c. and g is θ-continuous, then $g \circ f$ is w.q.c.

(b) If f is s.c. and g is weakly continuous, then $g \circ f$ is w.q.c.

PROOF. (a) Let $x \in X$ and W be an open set of Z containing $g(f(x))$ Since g is θ-continuous, there exists an open set V of Y containing $f(x)$ such that $g(\mathrm{Cl}(V)) \subset \mathrm{Cl}(W)$ Since f is w q c, there exists a $U \in SO(X)$ containing x such that $f(U) \subset \mathrm{Cl}(V)$ Hence $g(f(U)) \subset g(\mathrm{Cl}(V)) \subset \mathrm{Cl}(W)$

(b) The proof is easy and hence omitted

COROLLARY 3.6 (Noiri [7]) *If $f : X \to Y$ is w.q.c. and $g : Y \to Z$ is continuous, then $g \circ f$ is w.q.c.*

LEMMA 3.7 (Noiri and Ahmad [18]) *Let A and B be subsets of X. If $A \in PO(X)$ and $B \in SO(X)$, then $A \cap B \in SO(X)$.*

THEOREM 3.8. *If $f : X \to Y$ is w.q.c. and $A \in PO(X)$, then the restriction $f|_A : A \to Y$ is w.q.c.*

PROOF. Let $x \in A$ and V be an open set of Y containing $f(x)$ Since f is w q c, there exists a $U \in SO(X)$ containing x such that $f(U) \subset \mathrm{Cl}(V)$ Since $A \in PO(X)$, by Lemma 3 7 $x \in A \cap U \in SO(X)$ and $(f|_A)(A \cap U) = f(A \cap U) \subset f(U) \subset \mathrm{Cl}(V)$ Hence $f|_A$ is w q c

COROLLARY 3.9 (Noiri [7]) *If $f : X \to Y$ is w.q.c. and A is open in X, then the restriction $f|_A : A \to Y$ is w.q.c.*

COROLLARY 3.10 (Arya and Bhamini [9]) *If $f : X \to Y$ is w.q.c. and $A \in \alpha(X)$, then the restriction $f|_A : A \to Y$ is w.q.c.*

Sufficient condition for a function to be w q c, when it is given to be so in some subspace, is given in the following

THEOREM 3.11. *Let $f : X \to Y$ be a function and $\{A_i | i \in I\}$ be a cover of X such that $A_i \in SO(X)$ for each $i \in I$. If $f|_{A_i} : A_i \to Y$ is w.q.c. for each $i \in I$, then f is w.q.c.*

PROOF. Let V be a regular closed set of Y. Then $(f|_{A_i})^{-1}(V) \in SO(A_i)$ Since $A_i \in SO(X)$, by Theorem 2 4 of [19], $(f|_{A_i})^{-1}(V) \in SO(X)$ for each $i \in I$ But $f^{-1}(V) = \cup_{i \in I}((f|_{A_i})^{-1}(V))$ Then $f^{-1}(V) \in SO(X)$ because the union of semiopen sets is semiopen [2] Hence, by Theorem 3 2 f is w q c

COROLLARY 3.12. *Let $f : X \to Y$ be a function and $\{A_i | i \in I\}$ be a cover of X such that $A_i \in \alpha(X)$ for each $i \in I$. If $f|_{A_i} : A_i \to Y$ is w.q.c. for each $i \in I$, then f is w.q.c.*

COROLLARY 3.13. *Let $f : X \to Y$ be a function and $\{A_i | i \in I\}$ be a cover of X such that A_i is open in X for each $i \in I$. If $f|_{A_i} : A_i \to Y$ is w.q.c. for each $i \in I$, then f is w.q.c.*

DEFINITION 3.14. Let A be a subset of X A function $f : X \to A$ is called a w q c retraction if f is w q c and $f|_A$ is the identity function on A

THEOREM 3.15. *Let A be a subset of X and $f : X \to A$ be a w.q.c. retraction. If X is T_2, then A is semiclosed in X.*

PROOF. Suppose that A is not semiclosed Then there exists a $x \in X$ such that $x \in s$-$\mathrm{Cl}(A) - A$ Since f is a w q c retraction, $f(x) \neq x$ By the T_2 property of X, there exist disjoint open sets U and V such that $x \in U$ and $f(x) \in V$ which implies $U \cap \mathrm{Cl}(V) = \emptyset$. Let $W \in SO(X)$ containing x. Then $U \cap W \in SO(X)$ and hence $(U \cap W) \cap A \neq \emptyset$ because $x \in s$-$\mathrm{Cl}(A)$. Let $y \in (U \cap W) \cap A$. Since $y \in A$, we have $f(y) = y \in U \cap W \cap A \subset U$ and hence $f(y) \notin \mathrm{Cl}(V)$ This implies that $f(W) \not\subset \mathrm{Cl}(V)$ because $y \in W$. This is contrary to the fact that f is w q c Hence A is semiclosed in X.

In [8], Noiri showed that if Y is T_2, $f_1 : X \to Y$ is s c., $f_2 : X \to Y$ is w α.c and $f_1 = f_2$ on a dense subset of X, then $f_1 = f_2$ on X Similarly, we have

THEOREM 3.16. *Let Y be T_2 and $f_1 : X \to Y$ be almost continuous. If $f_2 : X \to Y$ is w.q.c. and if $f_1 = f_2$ on a dense subset D of X, then $f_1 = f_2$ on X.*

PROOF. Similar to the proof of [8, Theorem 4 10] by using Lemma 3.7.

THEOREM 3.17. *Let Y be Urysohn and $f_1 : X \to Y$ be w.q.c. If $f_2 : X \to Y$ is w.α.c. and if $f_1 = f_2$ on a dense subset D of X, then $f_1 = f_2$ on X.*

PROOF. Similar to the proof of [8, Theorem 4.10].

4. GRAPHS OF FUNCTIONS

The graph of a function $f : X \to Y$, denoted by $G(f)$, is the subset $\{(x, f(x)) | x \in X\}$ of the product space $X \times Y$. Noiri [20] showed that if $f : X \to Y$ is weakly continuous and Y is T_2, then the graph $G(f)$ is closed. Using "w.q.c." and "semiclosed" instead of "weakly continuous" and "closed" respectively, we obtain the following.

THEOREM 4.1. *If $f : X \to Y$ is w.q.c. and Y is T_2, then for each $(x, y) \notin G(f)$, there exist $U \in SO(X)$ and open set V in X such that $x \in U$, $y \in V$ and $f(U) \cap Int(Cl(V)) = \emptyset$.*

PROOF. Let $(x, y) \notin G(f)$. Then $y \neq f(x)$. Since Y is T_2, there exist disjoint open sets V and W such that $y \in V$ and $f(x) \in W$. This implies that $\mathrm{Int}(\mathrm{Cl}(V)) \cap \mathrm{Cl}((W) = \emptyset$. Since f is w.q.c., there exists $U \in SO(X)$ containing x such that $f(U) \subset \mathrm{Cl}(W)$. Hence $f(U) \cap \mathrm{Int}(\mathrm{Cl}(V)) = \emptyset$.

COROLLARY 4.2. *If $f : X \to Y$ is w.q.c. and Y is T_2, then the graph $G(f)$ is semiclosed.*

PROOF. It follows from Theorem 4.1.

THEOREM 4.3. *If $f : X \to Y$ is a w.q.c. and S is θ-closed subset in $X \times Y$, then $p_1(S \cap G(f))$ is semiclosed in X, where p_1 is the projection of $X \times Y$ onto X.*

PROOF. Let $x \in s$-$\mathrm{Cl}(p_1(S \cap G(f)))$, where S is a θ-closed subset of $X \times Y$.

Let U and V be any open sets of X and Y containing x and $f(x)$, respectively. Since f is w.q.c., by Theorem 3.2 $x \in f^{-1}(V) \subset s$-$\mathrm{Int}(f^{-1}(\mathrm{Cl}(V)))$. Since $U \cap s$-$\mathrm{Int}(f^{-1}(\mathrm{Cl}(V))) \in SO(X)$ containing x, $(U \cap s\text{-}\mathrm{Int}(f^{-1}(\mathrm{Cl}(V)))) \cap p_1(S \cap G(f)) \neq \emptyset$. Let $x_0 \in (U \cap s\text{-}\mathrm{Int}(f^{-1}(\mathrm{Cl}(V)))) \cap p_1(S \cap G(f))$. This implies that $(x_0, f(x_0)) \in S$ and $f(x_0) \in \mathrm{Cl}(V)$. Therefore, $\phi \neq (U \times \mathrm{Cl}(V)) \cap S \subset \mathrm{Cl}(U \times V) \cap S$ and consequently, $(x, f(x)) \in \mathrm{Cl}_\theta(S)$. Since S is θ-closed, $(x, f(x)) \in S \cap G(f)$. Hence $x \in p_1(S \cap G(f))$. This shows that $p_1(S \cap G(f))$ is semiclosed in X.

COROLLARY 4.4. *If $f : X \to Y$ has a θ-closed graph $G(f)$ and $g : X \to Y$ is w.q.c., then $\{x \in X | f(x) = g(x)\}$ is semiclosed.*

PROOF. Since $\{x \in X | f(x) = g(x)\} = p_1(G(f) \cap G(g))$ and $G(f)$ is a θ-closed subset of $X \times Y$, it follows from Theorem 4.3 that $\{x \in X | f(x) = g(x)\}$ is semiclosed.

COROLLARY 4.5. *If $f : X \to Y$ is θ-continuous, $g : X \to Y$ is w.q.c. and Y is Urysohn, then $\{x \in X | f(x) = g(x)\}$ is semiclosed.*

PROOF. It follows from Theorem 7 of [21] and Corollary 4.4.

DEFINITION 4.6. Let $f : X \to Y$ be a function. The graph $G(f)$ is said to be strongly semiclosed if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in SO(X)$ and $V \in SO(Y)$ such that $x \in U, y \in V$ and $(U \times s\text{-}\mathrm{Cl}(V)) \cap G(f) = \emptyset$.

LEMMA 4.7. *If $f : X \to Y$ has a strongly semiclosed graph $G(f)$ if and only if for each $(x, y) \in X \times Y - G(f)$ there exist $U \in SO(X)$ and $V \in SO(Y)$ such that $x \in U, y \in V$ and $f(U) \cap s\text{-Cl}(V) = \emptyset$*

PROOF. It follows from Definition 4 6

THEOREM 4.8. *If $f : X \to Y$ is w.q.c. and Y is Urysohn, then $G(f)$ is strongly semiclosed in $X \times Y$.*

PROOF. Since $s\text{-Cl}(U) \subset \text{Cl}(U)$ for each subset U of X, it follows immediately from Lemma 4 7

5. WEAK* QUASI CONTINUITY

DEFINITION 5.1. A function $f \cdot X \to Y$ is weakly* quasi continuous (briefly, w* q c) if for each open set V of Y, $f^{-1}(Fr(V))$ is semiclosed in X, where $Fr(V)$ denotes the frontier of V

Every s c function is w* q c but the converse is not true as the following Example 5 2 shows Moreover, Example 5 2 and 5 3 show that w q c and w* q c are independent of each other

EXAMPLE 5.2. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, X, \{a\}, \{bc\}\}$ Let $f \ (X, \tau) \to (X, \sigma)$ be the identity function Then f is w* q c However, f is not s c and hence not w q c

EXAMPLE 5.3. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $\sigma = \{\phi, X, \{b\}\}$ Let $f : (X, \tau) \to (X\sigma)$ be the identity function Then f is w q c but f is not w* q c

The w q c functions are not generally s c [7] The next two theorems give conditions under which w q c and s c functions are equivalent A space X is said to be extremally disconnected if the closure of each open set is open in X

THEOREM 5.4. *Let $f : X \to Y$ be a function and X be extremally disconnected. Then f is s.c. if and only if f is w.q.c. and w*.q.c.*

PROOF. The necessity is clear

Sufficiency Let $x \in X$ and V be any open set containing $f(x)$ Since f is w q c , there exists a $U \in SO(X)$ containing x such that $f(U) \subset \text{Cl}(V)$ But since f is w* q c , $f^{-1}(Fr(V)) = f^{-1}(\text{Cl}(V) - V)$ is semiclosed and hence by Proposition of [22] $U - f^{-1}(Fr(V)) \in SO(X)$ Further $f(x) \notin Fr(V)$ implies $x \notin f^{-1}(Fr(V))$ The proof will be complete if we show that $f(x) \in f(U - f^{-1}(Fr(V))) \subset V$ Let $y \in U - f^{-1}(Fr(V))$ Then $f(y) \in \text{Cl}(V)$ But $y \notin f^{-1}(Fr(V))$ and so $f(y) \notin Fr(V) = \text{Cl}(V) - V$ which implies that $f(y) \in V$

In Theorem 5.4, we cannot drop the assumption that X is extremally disconnected as Example 5 5 shows

EXAMPLE 5.5. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{b\}, \{c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\sigma = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{a, b, c\}\}$ Let $f : (X, \tau) \to (X, \sigma)$ be the identity function Then f is w q c and w* q c but not s c

A space X is said to be rim-compact [14] if each point of X has a base of neighborhoods with compact frontiers

THEOREM 5.6. *If $f : X \to Y$ is w.q.c. with the closed graph $G(f)$ and Y is rim-compact, then f is s.c.*

PROOF. Let $x \in X$ and V be any open set containing $f(x)$ Since Y is rim-compact, there exists an open set W of Y such that $f(x) \in W \subset V$ and $Fr(W)$ is compact Because f is w q c , there exists a $U \in SO(X)$ containing x such that $F(U) \subset \text{Cl}(W)$ Let $y \in Fr(W)$ Since $f(x) \in W$ which is disjoint from $Fr(W)$, $(x, y) \notin G(f)$ Then since $G(f)$ is closed, there exist open sets U_y and V_y such that $x \in U_y$, $y \in V_y$ and $f(U_y) \cap V_y = \emptyset$ The collection $\{V_y | y \in Fr(W)\}$ is an open cover of $Fr(W)$ Since $Fr(W)$ is compact, there exist a finite number of points $y_1, y_2, ..., y_n$ in $Fr(W)$ such that $Fr(W) \subset \cup_{i=1}^{n} V_{y_i}$ Let $U_0 = U \cap (\cup_{i=1}^{n} U_{y_i})$ Then $U_0 \in SO(X)$ and

$$f(U_0) \subset f(\cap_{i=1}^n U_{y_i}) \subset \cap_{i=1}^n f(U_{y_i})$$

which is disjoint from $\cup_{i=1}^n V_{y_i}$ and hence disjoint from $Fr(W)$. Thus $f(U_0) \cap Fr(W) = \emptyset$. However $f(U_0) \subset f(U) \subset \mathrm{Cl}(W)$ Therefore, $f(U_0) \subset \mathrm{Cl}(W) - Fr(W) \subset W$ Hence f is s.c.

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