

A WIRSING-TYPE APPROACH TO SOME CONTINUED FRACTION EXPANSION

GABRIELA ILEANA SEBE

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Chan (2004) considered a certain continued fraction expansion and the corresponding Gauss-Kuzmin-Lévy problem. A Wirsing-type approach to the Perron-Frobenius operator of the associated transformation under its invariant measure allows us to obtain a near-optimal solution to this problem.

1. Introduction

The Gauss 1812 problem gave rise to an extended literature. In modern times, the so-called Gauss-Kuzmin-Lévy theorem is still one of the most important results in the metrical theory of regular continued fractions (RCFs). A recent survey of this topic is to be found in [10]. From the time of Gauss, a great number of such theorems followed. See, for example, [2, 6, 7, 8, 18].

Apart from the RCF expansion there are many other continued fraction expansions: the continued fraction expansion to the nearest integer, grotesque expansion, Nakada's α -expansions, Rosen expansions; in fact, there are too many to mention (see [4, 5, 11, 12, 13, 16, 17] for some background information). The Gauss-Kuzmin-Lévy problem has been generalized to the above continued fraction expansions (see [3, 14, 15, 19, 20, 21]).

Taking up a problem raised in [1], we consider another expansion of reals in the unit interval, different from the RCF expansion. In fact, in [1] Chan has studied the transformation related to this new continued fraction expansion and the asymptotic behaviour of its distribution function. Giving a solution to the Gauss-Kuzmin-Lévy problem, he showed in [1, Theorem 1] that the convergence rate involved is $O(q^n)$ as $n \rightarrow \infty$ with $0 < q < 1$. This unsurprising result can be easily obtained from well-known general results (see [9, pages 202 and 262–266] and [10, Section 2.1.2]) concerning the Perron-Frobenius operator of the transformation under the invariant measure induced by the limit distribution function.

Our aim here is to give a better estimation of the convergence rate discussed. First, in Section 2 we introduce equivalent, but much more concise and rigorous expressions than in [1] of the transformation involved and of the related incomplete quotients. Next, in Section 3, our strategy is to derive the Perron-Frobenius operator of this transformation

under its invariant measure. In Section 4, we use a Wirsing-type approach (see [22]) to study the optimality of the convergence rate. Actually, in Theorem 4.3 of Section 4 we obtain upper and lower bounds of the convergence rate which provide a near-optimal solution to the Gauss-Kuzmin-Lévy problem.

2. Another expansion of reals in the unit interval

In this section we describe another continued fraction expansion different from the regular continued fraction expansion for a number x in the unit interval $I = [0, 1]$, which has been actually considered in [1].

Define for any $x \in I$ the transformation

$$\tau(x) = 2^{\{(\log x^{-1})/\log 2\}} - 1, \quad x \neq 0; \quad \tau(0) = 0, \quad (2.1)$$

where $\{u\}$ denotes the fractionary part of a real u while \log stands for natural logarithm. (Nevertheless, the definition of τ is independent of the base of the logarithm used.) Putting

$$a_n(x) = a_1(\tau^{n-1}(x)), \quad n \in \mathbb{N}_+ = \{1, 2, \dots\}, \quad (2.2)$$

with $\tau^0(x) = x$ the identity map and

$$a_1(x) = \left[\frac{(\log x^{-1})}{\log 2} \right], \quad (2.3)$$

where $[u]$ denotes the integer part of a real u , one easily sees that every irrational $x \in (0, 1)$ has a unique infinite expansion

$$x = \frac{2^{-a_1}}{1 + \frac{2^{-a_2}}{1 + \dots}} = [a_1, a_2, \dots]. \quad (2.4)$$

Here, the incomplete quotients or digits $a_n(x)$, $n \in \mathbb{N}_+$ of $x \in (0, 1)$ are natural numbers.

Let \mathcal{B}_I be the σ -algebra of Borel subsets of I . There is a probability measure ν on \mathcal{B}_I defined by

$$\nu(A) = \frac{1}{\log(4/3)} \int_A \frac{dx}{(x+1)(x+2)}, \quad A \in \mathcal{B}_I, \quad (2.5)$$

such that $\nu(\tau^{-1}(A)) = \nu(A)$ for any $A \in \mathcal{B}_I$, that is, ν is τ -invariant.

3. An operator treatment

In the sequel we will derive the Perron-Frobenius operator of τ under the invariant measure ν .

Let μ be a probability measure on \mathcal{B}_I such that $\mu(\tau^{-1}(A)) = 0$ whenever $\mu(A) = 0$, $A \in \mathcal{B}_I$, where τ is the continued fraction transformation defined in Section 2. In particular,

this condition is satisfied if τ is μ -preserving, that is, $\mu\tau^{-1} = \mu$. It is known from [10, Section 2.1] that the Perron-Frobenius operator P_μ of τ under μ is defined as the bounded linear operator on $L_\mu^1 = \{f : I \rightarrow \mathbb{C} \mid \int_I |f| d\mu < \infty\}$ which takes $f \in L_\mu^1$ into $P_\mu f \in L_\mu^1$ with

$$\int_A P_\mu f d\mu = \int_{\tau^{-1}(A)} f d\mu, \quad A \in \mathcal{B}_I. \quad (3.1)$$

In particular the Perron-Frobenius operator P_λ of τ under the Lebesgue measure λ is

$$P_\lambda(x) = \frac{d}{dx} \int_{\tau^{-1}([0,x])} f d\lambda \quad \text{a.e. in } I. \quad (3.2)$$

PROPOSITION 3.1. *The Perron-Frobenius operator $P_\nu = U$ of τ under ν is given a.e. in I by the equation*

$$Uf(x) = \sum_{k \in \mathbb{N}} p_k(x) f(u_k(x)), \quad f \in L_\nu^1, \quad (3.3)$$

where

$$\begin{aligned} p_k(x) &= \frac{\gamma^{k+1}(x+1)(x+2)}{(\gamma^k + x + 1)(\gamma^{k+1} + x + 1)}, \quad x \in I, \\ u_k(x) &= \frac{\gamma^k}{x+1}, \quad x \in I, \end{aligned} \quad (3.4)$$

with $\gamma = 1/2$.

The *proof* is entirely similar to that of [10, Proposition 2.1.2].

An analogous result to [10, Proposition 2.1.5] is shown as follows.

PROPOSITION 3.2. *Let μ be a probability measure on \mathcal{B}_I . Assume that $\mu \ll \lambda$ and let $h = d\mu/d\lambda$. Then*

$$\mu(\tau^{-n}(A)) = \int_A \frac{U^n f(x)}{(x+1)(x+2)} dx \quad (3.5)$$

for any $n \in \mathbb{N}$ and $A \in \mathcal{B}_I$, where $f(x) = (x+1)(x+2)h(x)$, $x \in I$.

4. A Wirsing-type approach

Let μ be a probability measure on \mathcal{B}_I such that $\mu \ll \lambda$. For any $n \in \mathbb{N}$, put

$$F_n(x) = \mu(\tau^n < x), \quad x \in I, \quad (4.1)$$

where τ^0 is the identity map. As $(\tau^n < x) = \tau^{-n}((0, x))$, by Proposition 3.2 we have

$$F_n(x) = \int_0^x \frac{U^n f_0(u)}{(u+1)(u+2)} du, \quad n \in \mathbb{N}, x \in I, \quad (4.2)$$

with $f_0(x) = (x+1)(x+2)F'_0(x)$, $x \in I$, where $F'_0 = d\mu/d\lambda$.

In this section we will assume that $F'_0 \in C^1(I)$. So, we study the behaviour of U^n as $n \rightarrow \infty$, assuming that the domain of U is $C^1(I)$, the collection of all functions $f : I \rightarrow \mathbb{C}$ which have a continuous derivative.

Let $f \in C^1(I)$. Then the series (3.3) can be differentiated term-by-term, since the series of derivatives is uniformly convergent. Putting $\Delta_k = \gamma^k - \gamma^{2k}$, $k \in \mathbb{N}$ we get

$$\begin{aligned} p_k(x) &= \gamma^{k+1} + \frac{\Delta_k}{\gamma^k + x + 1} - \frac{\Delta_{k+1}}{\gamma^{k+1} + x + 1}, \\ (Uf)'(x) &= \sum_{k \in \mathbb{N}} \left[p'_k(x) f\left(\frac{\gamma^k}{x+1}\right) - p_k(x) \frac{\gamma^k}{(x+1)^2} f'\left(\frac{\gamma^k}{x+1}\right) \right] \\ &= \sum_{k \in \mathbb{N}} \left[\left(\frac{\Delta_{k+1}}{(\gamma^{k+1} + x + 1)^2} - \frac{\Delta_k}{(\gamma^k + x + 1)^2} \right) f\left(\frac{\gamma^k}{x+1}\right) - p_k(x) \frac{\gamma^k}{(x+1)^2} f'\left(\frac{\gamma^k}{x+1}\right) \right] \\ &= - \sum_{k \in \mathbb{N}} \left[\frac{\Delta_{k+1}}{(\gamma^{k+1} + x + 1)^2} \left(f\left(\frac{\gamma^{k+1}}{x+1}\right) - f\left(\frac{\gamma^k}{x+1}\right) \right) + p_k(x) \frac{\gamma^k}{(x+1)^2} f'\left(\frac{\gamma^k}{x+1}\right) \right], \end{aligned} \quad (4.3)$$

$x \in I$. Thus, we can write

$$(Uf)' = -Vf', \quad f \in C^1(I), \quad (4.4)$$

where $V : C(I) \rightarrow C(I)$ is defined by

$$Vg(x) = \sum_{k \in \mathbb{N}} \left(\frac{\Delta_{k+1}}{(\gamma^{k+1} + x + 1)^2} \int_{\gamma^k/(x+1)}^{\gamma^{k+1}/(x+1)} g(u) du + p_k(x) \frac{\gamma^k}{(x+1)^2} g\left(\frac{\gamma^k}{x+1}\right) \right), \quad (4.5)$$

$g \in C(I)$, $x \in I$. Clearly,

$$(U^n f)' = (-1)^n V^n f', \quad n \in \mathbb{N}_+, \quad f \in C^1(I). \quad (4.6)$$

We are going to show that V^n takes certain functions into functions with very small values when $n \in \mathbb{N}_+$ is large.

PROPOSITION 4.1. *There are positive constants $v > 0.206968896$ and $w < 0.209364308$, and a real-valued function $\varphi \in C(I)$ such that $v\varphi \leq V\varphi \leq w\varphi$.*

Proof. Let $h : \mathbb{R}_+ \rightarrow \mathbb{R}$ be a continuous bounded function such that $\lim_{x \rightarrow \infty} h(x) < \infty$. We look for a function $g : (0, 1] \rightarrow \mathbb{R}$ such that $Ug = h$, assuming that the equation

$$Ug(x) = \sum_{k \in \mathbb{N}} p_k(x) g\left(\frac{\gamma^k}{x+1}\right) = h(x) \quad (4.7)$$

holds for $x \in \mathbb{R}_+$. Then (4.7) yields

$$\frac{h(x)}{x+2} - \frac{h(2x+1)}{2x+3} = \frac{x+1}{(x+2)(2x+3)} g\left(\frac{1}{x+1}\right), \quad x \in \mathbb{R}_+. \quad (4.8)$$

Hence

$$g(u) = (u+2)h\left(\frac{1}{u} - 1\right) - (u+1)h\left(\frac{2}{u} - 1\right), \quad u \in (0, 1], \quad (4.9)$$

and we indeed have $Ug = h$ since

$$\begin{aligned} Ug(x) &= \sum_{k \in \mathbb{N}} p_k(x) \left[\left(\frac{\gamma^k}{x+1} + 2 \right) h\left(\frac{x+1}{\gamma^k} - 1\right) - \left(\frac{\gamma^k}{x+1} + 1 \right) h\left(\frac{2(x+1)}{\gamma^k} - 1\right) \right] \\ &= \frac{x+2}{2} \sum_{k \in \mathbb{N}} \frac{\gamma^{2k}}{(\gamma^k + x + 1)(\gamma^{k+1} + x + 1)} \\ &\quad \times \left[\left(\frac{x+1}{\gamma^{k+1}} + 1 \right) h\left(\frac{x+1}{\gamma^k} - 1\right) - \left(\frac{x+1}{\gamma^k} + 1 \right) h\left(\frac{x+1}{\gamma^{k+1}} - 1\right) \right] \\ &= h(x), \quad x \in \mathbb{R}_+. \end{aligned} \quad (4.10)$$

In particular, for any fixed $a \in I$ we consider the function $h_a : \mathbb{R}_+ \rightarrow \mathbb{R}$ defined by $h_a(x) = 1/(x+a+1)$, $x \in \mathbb{R}_+$. By the above, the function $g_a : (0, 1] \rightarrow \mathbb{R}$ defined as

$$\begin{aligned} g_a(x) &= (x+2)h_a\left(\frac{1}{x} - 1\right) - (x+1)h_a\left(\frac{2}{x} - 1\right) \\ &= \frac{x(x+2)}{ax+1} - \frac{x(x+1)}{ax+2}, \quad x \in (0, 1], \end{aligned} \quad (4.11)$$

satisfies $Ug_a(x) = h_a(x)$, $x \in I$. Setting

$$\varphi_a(x) = g'_a(x) = \frac{3ax^2 + 4(a+1)x + 6}{(ax+2)^2(ax+1)^2}, \quad (4.12)$$

we have

$$V\varphi_a(x) = -(Ug_a)'(x) = \frac{1}{(x+a+1)^2}, \quad x \in I. \quad (4.13)$$

We choose a by asking that $(\varphi_a/V\varphi_a)(0) = (\varphi_a/V\varphi_a)(1)$. This amounts to $3a^4 + 12a^3 + 18a^2 - 2a - 17 = 0$ which yields as unique acceptable solution $a = 0.794741181\dots$. For this value of a , the function $\varphi_a/V\varphi_a$ attains its maximum equal to $(3/2)(a+1)^2 = 4.83164386\dots$ at $x = 0$ and $x = 1$, and has a minimum $m(a) \simeq (\varphi_a/V\varphi_a)(0.39) = 4.776363306\dots$. It follows that for $\varphi = \varphi_a$ with $a = 0.794741181\dots$, we have

$$\frac{2\varphi}{3(a+1)^2} \leq V\varphi \leq \frac{\varphi}{m(a)}, \quad (4.14)$$

that is, $v\varphi \leq V\varphi \leq w\varphi$, where $v = 2/3(a+1)^2 > 0.206968896$, and $w = 1/m(a) < 0.209364308$. \square

COROLLARY 4.2. *Let $f_0 \in C^1(I)$ such that $f'_0 > 0$. Put $\alpha = \min_{x \in I} \varphi(x)/f'_0(x)$ and $\beta = \max_{x \in I} \varphi(x)/f'_0(x)$. Then*

$$\frac{\alpha}{\beta} v^n f'_0 \leq V^n f'_0 \leq \frac{\beta}{\alpha} w^n f'_0, \quad n \in \mathbb{N}_+. \quad (4.15)$$

Proof. Since V is a positive operator, we have

$$v^n \varphi \leq V^n \varphi \leq w^n \varphi, \quad n \in \mathbb{N}_+. \quad (4.16)$$

Noting that $\alpha f'_0 \leq \varphi \leq \beta f'_0$, we can write

$$\frac{\alpha}{\beta} v^n f'_0 \leq \frac{1}{\beta} v^n \varphi \leq \frac{1}{\beta} V^n \varphi \leq V^n f'_0 \leq \frac{1}{\alpha} V^n \varphi \leq \frac{1}{\alpha} w^n \varphi \leq \frac{\beta}{\alpha} w^n f'_0, \quad (4.17)$$

$n \in \mathbb{N}_+$, which shows that (4.15) holds. \square

THEOREM 4.3 (near-optimal solution to Gauss-Kuzmin-Lévy problem). *Let $f_0 \in C^1(I)$ such that $f'_0 > 0$. For any $n \in \mathbb{N}_+$ and $x \in I$,*

$$\begin{aligned} & \frac{(\log(4/3))^2 \alpha \min_{x \in I} f'_0(x)}{2\beta} v^n F(x)(1 - F(x)) \\ & \leq |\mu(\tau^n < x) - F(x)| \leq \frac{(\log(4/3))^2 \beta \max_{x \in I} f'_0(x)}{\alpha} w^n F(x)(1 - F(x)), \end{aligned} \quad (4.18)$$

where α, β, v and w are defined in Proposition 4.1 and Corollary 4.2 and $F(x) = (1/\log(4/3)) \log(2(x+1))/x+2$. In particular, for any $n \in \mathbb{N}_+$ and $x \in I$,

$$\begin{aligned} 0.01023923 v^n F(x)(1 - F(x)) & \leq |\lambda(\tau^n < x) - F(x)| \\ & \leq 0.334467468 w^n F(x)(1 - F(x)). \end{aligned} \quad (4.19)$$

Proof. For any $n \in \mathbb{N}$ and $x \in I$, set $d_n(F(x)) = \mu(\tau^n < x) - F(x)$. Then by (4.2) we have

$$d_n(F(x)) = \int_0^x \frac{U^n f_0(u)}{(u+1)(u+2)} du - F(x). \quad (4.20)$$

Differentiating twice with respect to x yields

$$\begin{aligned} d'_n(F(x)) \frac{1}{(\log(4/3))(x+1)(x+2)} &= \frac{U^n f_0(x)}{(x+1)(x+2)} - \frac{1}{(\log(4/3))(x+1)(x+2)}, \\ (U^n f_0(x))' &= \frac{1}{(\log(4/3))^2} \frac{d''_n(F(x))}{(x+1)(x+2)}, \quad n \in \mathbb{N}, x \in I. \end{aligned} \quad (4.21)$$

Hence by (4.6) we have

$$d''_n(F(x)) = (-1)^n \left(\log \left(\frac{4}{3} \right) \right)^2 (x+1)(x+2) V^n f'_0(x), \quad n \in \mathbb{N}, x \in I. \quad (4.22)$$

Since $d_n(0) = d_n(1) = 0$, it follows from a well-known interpolation formula that

$$d_n(x) = -\frac{x(1-x)}{2} d''_n(\theta), \quad n \in \mathbb{N}, x \in I \quad (4.23)$$

for a suitable $\theta = \theta(n, x) \in I$. Therefore

$$\mu(\tau^n < x) - F(x) = (-1)^{n+1} \left(\log \left(\frac{4}{3} \right) \right)^2 \frac{\theta + 1}{2} V^n f'_0(\theta) F(x) (1 - F(x)) \quad (4.24)$$

for any $n \in \mathbb{N}$ and $x \in I$, and another suitable $\theta = \theta(n, x) \in I$. The result stated follows now from Corollary 4.2. In the special case $\mu = \lambda$, we have $f_0(x) = (x+1)(x+2)$, $x \in I$. Then with $a = 0.794741181\dots$, we have

$$\begin{aligned} \alpha &= \min_{x \in I} \frac{\varphi(x)}{f'_0(x)} = \frac{7a+10}{5(a+2)^2(a+1)^2} = 0.123720515\dots, \\ \beta &= \max_{x \in I} \frac{\varphi(x)}{f'_0(x)} = 0.5, \end{aligned} \quad (4.25)$$

so that $(\log(4/3))^2 \alpha / 2\beta = 0.01023923\dots$ and $(\log 4/3)^2 \beta / \alpha = 0.334467468\dots$. The proof is complete. \square

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Gabriela Ileana Sebe: Department of Mathematics I, “Politehnica” University of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania
E-mail address: gisebe@mathem.pub.ro

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