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Research Article

One-Signed Periodic Solutions of First-Order Functional Differential Equations with a Parameter

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We study one-signed periodic solutions of the first-order functional differential equation $u'(t) = -a(t)u(t) + \lambda b(t)f(u(t-\tau(t))), t \in \mathbb{R}$ by using global bifurcation techniques. Where $a, b \in C(\mathbb{R}, [0,\infty))$ are ω–periodic functions with $\int_0^\omega a(t)dt > 0$, $\int_0^\omega b(t)dt > 0$, τ is a continuous ω-periodic function, and $\lambda > 0$ is a parameter. $f \in C(\mathbb{R},\mathbb{R})$ and there exist two constants $s_2 < 0 < s_1$ such that $f(s_2) = f(0) = f(s_1) = 0$, $f(s) > 0$ for $s \in (0, s_1) \cup (s_1, \infty)$ and $f(s) < 0$ for $s \in (-\infty, s_2) \cup (s_2, 0)$.

1. Introduction

In recent years, there has been considerable interest in the existence of periodic solutions of the following equation:

$$u'(t) = -a(t)u(t) + \lambda b(t)f(u(t-\tau(t))), \quad (1.1)$$

where $a,\ b \in C(\mathbb{R}, [0,\infty))$ are ω-periodic functions, and $\int_0^\omega a(t)dt > 0, \int_0^\omega b(t)dt > 0$, τ is a continuous ω-periodic function, $\lambda > 0$ is a parameter. (1.1) has been proposed as a model for a variety of physiological processes and conditions including production of blood cells, respiration, and cardiac arrhythmias; see, for example, [1–12] and the references therein. Roughly speaking, $u(t)$ represents the number of adult (sexually mature) members in a population at time t, $a(t)$ is the per capita death rate, and $f(u(t-\tau(t)))$ is the rate at which new members are recruited into the population at time t (τ is the age at which members mature, and it is assumed that the birth rate at a given time depends only on the adult population size). The most famous models of this type are

(i) the Nicholson's blowflies equation proposed in [1] to explain the oscillatory population fluctuations observed by A. J. Nicholson in 1957 in his studies of the sheep blowfly *Lucilia cuprina*:

$$u'(t) = -au(t) + p \cdot u(t-h)e^{-\gamma u(t-h)}, \quad a, p, \gamma, h > 0; \tag{1.2}$$

(ii) the model for blood cell populations proposed by Mackey and Glass in [2]

$$u'(t) = -au(t) + p\frac{u(t-h)}{1 + [u(t-h)]^n}, \quad a, p, \gamma, h > 0, \ n > 1; \tag{1.3}$$

(iii) the model for the survival of red blood cells in an animal proposed by Wazewska-Czyzewska and Lasota in [3]

$$u'(t) = -au(t) + p \cdot e^{-\gamma u(t-h)}, \quad a, p, \gamma, h > 0. \tag{1.4}$$

Recently, Cheng and Zhang [7] studied the existence of positive ω-periodic solutions of the functional equation (1.1) under the assumptions:

(H1) $f \in C([0,\infty), [0,\infty))$, and $f(s) > 0$ for $s > 0$;

(H2) a, $b \in C(\mathbb{R}, [0,\infty))$ are ω–periodic functions, $\int_0^\omega a(t)dt > 0$, $\int_0^\omega b(t)dt > 0$, $\tau \in C(\mathbb{R}, \mathbb{R})$ is a ω-periodic function;

(H3) there exist $f_0, f_\infty \in (0,\infty)$ such that

$$f_0 = \lim_{|s|\to 0} \frac{f(s)}{s}, \qquad f_\infty = \lim_{|s|\to\infty} \frac{f(s)}{s}. \tag{1.5}$$

They proved the following.

Theorem A. *Assume* (H1)–(H3)*hold. Then for each* λ *satisfying*

$$\frac{1}{\sigma B f_\infty} < \lambda < \frac{1}{A f_0}, \quad \text{or} \ \frac{1}{\sigma A f_0} < \lambda < \frac{1}{B f_\infty}, \tag{1.6}$$

equation (1.1) *has a positive periodic solution, where*

$$A = \max_{t\in[0,\omega]} \int_0^\omega G(t,s)b(s)ds, \qquad B = \min_{t\in[0,\omega]} \int_0^\omega G(t,s)b(s)ds, \qquad \sigma = e^{\int_0^\omega a(t)dt}. \tag{1.7}$$

However, the condition used in [7] is not sharp, and the main results in [7] give no any information about the global structure of the set of positive periodic solutions. Moreover, f satisfied (H1) in [7], so a natural question is what would happen if f is allowed to have some zeros in $\mathbb{R}$? The purpose of this work is to study the global behavior of the components of one-signed solutions of (1.1) under the condition

(H4) $f \in C(\mathbb{R},\mathbb{R})$; there exist two constants $s_2 < 0 < s_1$ such that $f(s_2) = f(0) = f(s_1) = 0$, $f(s) > 0$ for $s \in (0,\ s_1) \cup (s_1,\infty)$, and $f(s) < 0$ for $s \in (-\infty, s_2) \cup (s_2, 0)$.

The rest of this paper is organized as follows. In Section 2, we give some notations and the main results. Section 3 is devoted to proving the main results.

2. Statement of the Main Results

Let $Y = \{u \in C(\mathbb{R},\mathbb{R}) : u(t) = u(t+\omega)\}$ with the norm

$$\|u\|_\infty = \max_{t\in[0,\omega]} |u(t)|. \tag{2.1}$$

Then $(Y,\ \|\cdot\|_\infty)$ is a Banach space. Let

$$E = \left\{u \in C^1(\mathbb{R},\mathbb{R}) : u(t) = u(t+\omega)\right\} \tag{2.2}$$

be the Banach space with the norm $\|u\| = \max\{\|u\|_\infty, \|u'\|_\infty\}$.

It is well known that (1.1) is equivalent to

$$u(t) = \lambda \int_t^{t+\omega} G(t,s)b(s)f(u(s-\tau(s)))ds := (Au)(t), \tag{2.3}$$

where

$$G(t,s) = \frac{e^{\int_t^s a(\theta)d\theta}}{e^{\int_0^\omega a(\theta)d\theta} - 1}, \quad s \in [t, t+\omega]. \tag{2.4}$$

Notice that $\int_0^\omega a(t)dt > 0$, we have

$$\frac{1}{\sigma - 1} \le G(t,s) \le \frac{\sigma}{\sigma - 1}, \tag{2.5}$$

where $\sigma = e^{\int_0^\omega a(t)dt}$, and $0 < 1/\sigma < 1$.

Define that K is a cone in Y by

$$K = \left\{u \in Y : u(t) \ge 0,\ u(t) \ge \frac{1}{\sigma}\|u\|\right\}. \tag{2.6}$$

It is not difficult to prove that $A(K) \subset K$ and $A : K \to K$ is completely continuous.

Let us consider the spectrum of the linear eigenvalue problem

$$u'(t) = -a(t)u(t) + \lambda b(t)u(t-\tau(t)), \quad t \in \mathbb{R}. \tag{2.7}$$

Lemma 2.1. *Assume that* (H2) *holds. Then the linear problem* (2.7) *has a unique eigenvalue* λ_1*, which is positive and simple, and the corresponding eigenfunction* φ *is of one sign.*

Proof. It is a direct consequence of the Krein-Rutman Theorem [13, Theorem 19.3]. □

In the rest of the paper, we always assume that

$$\|\varphi\| = 1, \quad \varphi(t) > 0, \ t \in \mathbb{R}. \tag{2.8}$$

Define $L : E \to Y$ by setting

$$Lu := u'(t) + a(t)u(t), \quad u \in E. \tag{2.9}$$

Then $L^{-1} : Y \to E$ is completely continuous.

Let $\zeta, \ \xi \in C(\mathbb{R}, \mathbb{R})$ be such that

$$f(s) = f_0 s + \zeta(s), \qquad f(s) = f_\infty s + \xi(s). \tag{2.10}$$

Clearly,

$$\lim_{|s|\to 0} \frac{\zeta(s)}{s} = 0, \qquad \lim_{|s|\to \infty} \frac{\xi(s)}{s} = 0. \tag{2.11}$$

Let us consider

$$Lu(t) - \lambda b(t) f_0 u(t - \tau(t)) = \lambda b(t)\zeta(u(t - \tau(t))) \tag{2.12}$$

as a bifurcation problem from the trivial solution $u \equiv 0$ and

$$Lu(t) - \lambda b(t) f_\infty u(t - \tau(t)) = \lambda b(t)\xi(u(t - \tau(t))) \tag{2.13}$$

as a bifurcation problem from infinity. We note that (2.12) and (2.13) are the same and each of them is equivalent to (1.1).

Let $\mathbb{E} = \mathbb{R} \times E$ under the product topology. We add the points $\{(\lambda, \infty) \mid \lambda \in \mathbb{R}\}$ to our space $\mathbb{E}$. Let S^+ denote the set of positive functions in E and $S^- = -S^+$, and $S = S^- \cup S^+$. They are disjoint and open in E. Finally, let $\Phi^\pm = \mathbb{R} \times S^\pm$ and $\Phi = \mathbb{R} \times S$.

Remark 2.2. It is worth remaking that if u is a nontrivial solution of (1.1) and a, b, and f satisfy (H2)–(H4), then $u \in S^\nu$ for some $\nu = \{+, -\}$. To see this, define

$$q(t) = \begin{cases} \dfrac{f(u(t))}{u(t)}, & u(t) \neq 0, \\ f_0, & u(t) = 0. \end{cases} \tag{2.14}$$

Thus (1.1) is equivalent to

$$u'(t) = -a(t)u(t) + \lambda b(t) q(t - \tau(t)) u(t - \tau(t)), \quad t \in \mathbb{R}. \tag{2.15}$$

Obviously, $b(\cdot)q(\cdot - \tau(\cdot))$ satisfies (H2). From Lemma 2.1, the nontrivial solution $u \in S^\nu$ for some $\nu \in \{+,-\}$.

The result of Rabinowitz [14] for (2.12) can be stated as follows: for each $\nu \in \{+,-\}$, there exists a continuum $\mathcal{C}^\nu$ of solutions of (2.12) joining $(\lambda_1/f_0, 0)$ to infinity, and $\mathcal{C}^\nu \setminus \{(\lambda_1/f_0, 0)\} \subset \Phi^\nu$.

The result of Rabinowitz [15] for (2.13) can be stated as follows: for each $\nu \in \{+,-\}$, there exists a continuum $\mathfrak{D}^\nu$ of solutions of (2.13) meeting $(\lambda_1/f_\infty, \infty)$, and $\mathfrak{D}^\nu \setminus \{(\lambda_1/f_\infty, \infty)\} \subset \Phi^\nu$.

Our main result is the following.

Theorem 2.3. *Assume* (H2)–(H4) *hold. Moreover, suppose that*

(H5) *f satisfies the Lipschitz condition in* $[s_2, s_1]$.

Then

(i) *for* $(\lambda, u) \in \mathcal{C}^+ \cup \mathcal{C}^-$,

$$s_2 < u(t) < s_1, \quad t \in [0,\ \omega]; \tag{2.16}$$

(ii) *for* $(\lambda, u) \in \mathfrak{D}^+ \cup \mathfrak{D}^-$, *we have that either*

$$\max_{t\in[0,\omega]} u(t) > s_1 \tag{2.17}$$

or

$$\min_{t\in[0,\omega]} u(t) < s_2. \tag{2.18}$$

Corollary 2.4. *Let* (H2)–(H5) *hold. Then*

(i) *if* $\lambda \in (\lambda_1/f_\infty, \lambda_1/f_0]$, *then* (1.1) *has at least two solutions* u_∞^+ *and* u_∞^-, *such that* u_∞^+ *is positive on* $[0,\ \omega]$ *and* u_∞^- *is negative on* $[0, \omega]$;

(ii) *if* $\lambda \in (\lambda_1/f_0, \infty)$, *then* (1.1) *has at least four solutions* u_∞^+, u_∞^-, u_0^+, *and* u_0^-, *such that* u_∞^+, u_0^+ *are positive on* $[0, \omega]$ *and* u_∞^-, u_0^- *are negative on* $[0, \omega]$.

Corollary 2.5. *Let* (H2)–(H5) *hold. Then*

(i) *if* $\lambda \in (\lambda_1/f_0, \lambda_1/f_\infty]$, *then* (1.1) *has at least two solutions* u_0^+ *and* u_0^-, *such that* u_0^+ *is positive on* $[0, \omega]$ *and* u_0^- *is negative on* $[0, \omega]$;

(ii) *if* $\lambda \in (\lambda_1/f_\infty, \infty)$, *then* (1.1) *has at least four solutions* u_∞^+, u_∞^-, u_0^+, *and* u_0^-, *such that* u_∞^+, u_0^+ *are positive on* $[0,\ \omega]$ *and* u_∞^-, u_0^- *are negative on* $[0, \omega]$.

3. Proof of the Main Results

To prove Theorem 2.3, we give a Proposition.

Proposition 3.1. (i) *The first-order boundary value problem*

$$u'(t) + a(t)u(t) = h(t), \quad u(0) = u(\omega) \tag{3.1}$$

has a unique solution for all $h \in L^1[0,\omega]$ *if and only if* $\int_0^\omega a(s)ds \neq 0$.

(ii) *Assume that* u *is a solution of* (3.1). *If* $h \geq 0$ *and* $h(\cdot) \not\equiv 0$ *on any subinterval of* $[0,\omega]$, *then* $u(t)\int_0^\omega a(s)ds > 0$ *on* $[0,\omega]$.

Proof. (i) The equation $u'(t) + a(t)u(t) = 0$ has a solution $u(t) = Ce^{-\int_0^t a(s)ds}$, where C is a constant. If $u(t)$ is a nontrivial solution, then by $u(0) = C$, $u(\omega) = Ce^{-\int_0^\omega a(s)ds}$, we can get that $\int_0^\omega a(s)ds = 0$.

On the other hand, from $\int_0^\omega a(s)ds = 0$, we can get that $u'(t) + a(t)u(t) = 0$ has a nontrivial solution $u(t) = Ce^{-\int_0^t a(s)ds}$, where $C \in \mathbb{R} \setminus \{0\}$.

(ii) We claim that $u(t) \neq 0, t \in [0,\omega]$. Suppose on the contrary that there exists $t_0 \in [0,\omega]$, such that $u(t_0) = 0$; it is not difficult to compute that

$$u'(t) + a(t)u(t) = h(t), \quad u(t_0) = u_0 \tag{3.2}$$

has a solution

$$u(t) = \int_{t_0}^t h(s)e^{\int_t^s a(\tau)d\tau}ds. \tag{3.3}$$

Since $h \geq 0$, we have

$$u(0) \leq u(t_0) \leq u(\omega). \tag{3.4}$$

If $h(\hat{t}) > 0, \hat{t} \in [0, \ t_0)$, then there exists a neighborhood $U(\hat{t}) \subset [0, \ t_0)$ of $\hat{t}$, such that $h(t) > 0$ on $U(\hat{t})$. Thus, $u(0) = \int_{t_0}^0 h(s)e^{\int_0^s a(\tau)d\tau}ds < 0$; this contradicts with $u(0) = u(\omega)$.

If $h(\bar{t}) > 0, \bar{t} \in (t_0,\omega]$, then there exists a neighborhood $U(\bar{t}) \subset (t_0,\omega]$ of $\bar{t}$, such that $h(t) > 0$ on $U(\hat{t})$. By using a similar way, we can prove that $u(\omega) > 0$, which also contradicts with $u(0) = u(\omega)$.

Hence $u(t) \neq 0$ on $[0, \ \omega]$. Moreover, it follows that

$$\int_0^\omega \frac{u'(t)}{u(t)}dt + \int_0^\omega a(t)dt = \int_0^\omega \frac{h(t)}{u(t)}dt, \tag{3.5}$$

that is,

$$(\ln u(t))|_0^{\omega} + \int_0^{\omega} a(t)dt = \int_0^{\omega} \frac{h(t)}{u(t)}dt. \quad (3.6)$$

Thus $\int_0^{\omega} a(t)dt = \int_0^{\omega} h(t)/u(t)dt$, that is $u \int_0^{\omega} a(s)ds > 0$. □

Next, we prove Theorem 2.3 and Corollaries 2.4 and 2.5.

Proof of Theorem 2.3. Suppose on the contrary that there exists $(\lambda, u) \in \mathcal{C}^{+} \cup \mathcal{C}^{-} \cup \mathfrak{D}^{+} \cup \mathfrak{D}^{-}$ such that either

$$\max\{u(t) \mid t \in [0, \omega]\} = s_1 \quad (3.7)$$

or

$$\min\{u(t) \mid \ t \in [0, \omega]\} = s_2. \quad (3.8)$$

We divide the proof into two cases.

Case 1 ($\max\{u(t) \mid t \in [0, \omega]\} = s_1$). In this case, we know that

$$0 \le u(t) \le s_1, \qquad 0 \le u(t - \tau(t)) \le s_1, \quad t \in [0, \omega]. \quad (3.9)$$

Let us consider the functional differential equation

$$u'(t) + a(t)u(t) = \lambda b(t) f(u(t - \tau(t))), \quad t \in \mathbb{R}. \quad (3.10)$$

By (H2), (H4) and (H5), there exists $m \ge 0$ such that $b(t)f(s) + ms$ is strictly increasing on s for $s \in [s_2, s_1]$. Then (3.10) can be rewritten to the form

$$Lu + \lambda m u(t - \tau(t)) = \lambda \left[b(t) f(u(t - \tau(t))) + m u(t - \tau(t)) \right], \quad (3.11)$$

and since $Ls_1 - a(t)s_1 = 0 = f(s_1)$,

$$Ls_1 - a(t)s_1 + \lambda m s_1 = \lambda \left[b(t) f(s_1) + m s_1 \right]. \quad (3.12)$$

Subtracting, we get

$$L(s_1 - u) + \lambda m (s_1 - u(t - \tau(t))) - a(t)s_1 \ge 0. \quad (3.13)$$

That is,

$$\begin{gathered} L(s_1 - u) + \lambda m s_1 \ge 0, \quad t \in [0, \omega], \\ s_1 - u(0) = s_1 - u(\omega) > 0. \end{gathered} \quad (3.14)$$

From Proposition 3.1, we deduce that $s_1 > u(t), t \in [0,\omega]$, which contradicts with that $\max\{u(t) \mid t \in [0,\omega]\} = s_1$. Hence,

$$u(t) < s_1, \quad t \in [0,\omega]. \tag{3.15}$$

Case 2 ($\min\{u(t) \mid t \in [0,\omega]\} = s_2$). In this case, we know that

$$s_2 \le u(t) \le 0, \quad s_2 \le u(t-\tau(t)) \le 0, \quad t \in [0,\omega]. \tag{3.16}$$

Let us consider (3.10); by (H2), (H4), and (H5), there exists $m \ge 0$ such that $b(t)f(s) + ms$ is strictly increasing in s for $s \in [s_2, s_1]$. Then

$$Lu + \lambda m u(t-\tau(t)) = \lambda\left[b(t)f(u(t-\tau(t))) + mu(t-\tau(t))\right] \tag{3.17}$$

and since $Ls_2 - a(t)s_2 = 0 = f(s_2)$,

$$Ls_2 - a(t)s_2 + \lambda m s_2 = \lambda\left[b(t)f(s_2) + ms_2\right]. \tag{3.18}$$

Subtracting, we get

$$L(s_2 - u) + \lambda m(s_2 - u(t-\tau(t))) - a(t)s_2 \le 0. \tag{3.19}$$

That is

$$\begin{gathered} L(s_2 - u) + \lambda m s_2 \le 0, \quad t \in [0,\omega], \\ s_2 - u(0) = s_2 - u(\omega) < 0. \end{gathered} \tag{3.20}$$

From Proposition 3.1, we deduce that $s_2 - u(t) < 0, t \in [0,\ \omega]$, this contradicts with that $\min\{u(t) \mid t \in [0,\ \omega]\} = s_2$. Therefore,

$$s_2 < u(t), \quad t \in [0,\omega]. \tag{3.21}$$

□

Proof of Corollaries 2.4 and 2.5. Since boundary value problem

$$u'(t) + a(t)u(t) = 0, \quad u(0) = u(\omega) \tag{3.22}$$

has a unique solution $u \equiv 0$, we get

$$\left(\mathcal{C}^+ \cup \mathcal{C}^- \cup \mathfrak{D}^+ \cup \mathfrak{D}^-\right) \subset \{(\lambda, u) \in \mathbb{R} \times E \mid \lambda \ge 0\}. \tag{3.23}$$

Take $\Lambda \in \mathbb{R}$ as an interval such that $\Lambda \cap \{\lambda_1/f_\infty\} = \{\lambda_1/f_\infty\}$ and $\mathcal{M}$ as a neighborhood of $(\lambda_1/f_\infty, \infty)$ whose projection on $\mathbb{R}$ lies in Λ and whose projection on E is bounded away from 0. Then by [15, Theorem 1.6, and Corollary 1.8], we have that for each $\nu \in \{+,-\}$, either

(1) $\mathfrak{D}^\nu \setminus \mathcal{M}$ is bounded in $\mathbb{R} \times E$ in which case $\mathfrak{D}^\nu \setminus \mathcal{M}$ meets $\{(\lambda, 0) \mid \lambda \in \mathbb{R}\}$, or

(2) $\mathfrak{D}^\nu \setminus \mathcal{M}$ is unbounded.

Moreover, if (1) occurs and $\mathfrak{D}^\nu \setminus \mathcal{M}$ has a bounded projection on $\mathbb{R}$, then $\mathfrak{D}^\nu \setminus \mathcal{M}$ meets $(\lambda_k/f_\infty, \infty)$, where $\lambda_k \neq \lambda_1$ is another eigenvalue of (2.7).

Obviously, Theorem 2.3 (ii) implies that (1) does not occur. So $\mathfrak{D}^+ \setminus \mathcal{M}$ is unbounded.

Remark 2.2 guarantees that $\mathfrak{D}^+$ is a component of solutions of (2.12) in S^+ which meets $(\lambda_1/f_\infty, \infty)$, and consequently $\mathrm{Proj}_{\mathbb{R}}(\mathfrak{D}^+ \setminus \mathcal{M})$ is unbounded. Thus

$$\mathrm{Proj}_{\mathbb{R}}(\mathfrak{D}^+) \supset \left(\frac{\lambda_1}{f_\infty}, +\infty\right). \tag{3.24}$$

Similarly, we get

$$\mathrm{Proj}_{\mathbb{R}}(\mathfrak{D}^-) \supset \left(\frac{\lambda_1}{f_\infty}, +\infty\right). \tag{3.25}$$

By Theorem 2.3, for any $(\lambda, u) \in (\mathcal{C}^+ \cup \mathcal{C}^-)$,

$$\|u\|_\infty < \max\{s_1, \ |s_2|\} := s^*. \tag{3.26}$$

(3.26) and (2.12) imply that

$$\|u\| < \max\left\{s^*, \ \|a\|_\infty s^* + \lambda \|b\|_\infty \max_{|s| \le s^*} \left| f(s) \right| \right\}, \tag{3.27}$$

which means that the sets $\{(\lambda, u) \in \mathcal{C}^+ \mid \ \lambda \in [0, d]\}$ and $\{(\lambda, u) \in \mathcal{C}^- \mid \ \lambda \in [0, d]\}$ are bounded for any fixed $d \in (0, \infty)$. This together with the fact that $\mathcal{C}^+$ and $\mathcal{C}^-$ join $(\lambda_1/f_0, 0)$ to infinity yields, respectively, that

$$\begin{gathered} \mathrm{Proj}_{\mathbb{R}}(\mathcal{C}^+) \supset \left(\frac{\lambda_1}{f_0}, +\infty\right), \\ \mathrm{Proj}_{\mathbb{R}}(\mathcal{C}^-) \supset \left(\frac{\lambda_1}{f_0}, +\infty\right). \end{gathered} \tag{3.28}$$

Combining (3.24), (3.25), and (3.28), we conclude the desired results. □

Remark 3.2. The methods used in the proof of Theorem 2.3, Corollaries 2.4, and 2.5 have been used in the study of other kinds of boundary value problems; see [16–18] and the references therein.

Remark 3.3. The conditions in Corollaries 2.4 and 2.5 are sharp. Let us take

$$a(t) \equiv a > 0, \qquad \lambda = a, \qquad b(t) = 1, \qquad f(s) = s + h(s), \qquad \tau(t) \equiv 0. \tag{3.29}$$

Let

$$h(s) = \begin{cases} -\dfrac{2s}{s^2+1}, & s \in (-\infty, -1) \cup (1, +\infty), \\ -\dfrac{2s^3}{s^2+1}, & s \in [-1, 1], \end{cases} \tag{3.30}$$

and consider problem

$$u'(t) = -a(t)u(t) + a[u(t) + h(u(t))], \quad t \in [0, \omega], \ u(0) = u(\omega). \tag{3.31}$$

It is easy to see that $\lambda_1 = a$, $f_0 = f_\infty = 1$. Since

$$\frac{\lambda_1}{f_\infty} = a = \frac{\lambda_1}{f_0}, \tag{3.32}$$

the conditions of Corollaries 2.4 and 2.5 are not valid. In this case, (3.31) has no nontrivial solution. In fact, if u is a nontrivial solution of (3.31), then

$$0 = \int_0^\omega u'(t)dt = a\int_0^\omega h(u(t))dt \neq 0, \tag{3.33}$$

which is a contradiction.

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