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Evaluation of a Summation Formula Involving Recurrence Relation

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Abstract

The aim of the present paper is to derive a summation formula based on half argument involving recurrence relation of Gamma function.

Keywords: *Contiguous relation, Gauss second summation theorem ,Recurrence relation*

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1 Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{lc} a_1, a_2, \cdots, a_A & ; \\ & & z \\ b_1, b_2, \cdots, b_B & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{lc} (a_A) & ; \\ & & z \\ (b_B) & ; \end{array}\right] \equiv {}_AF_B\left[\begin{array}{lc} (a_j)_{j=1}^{A} & ; \\ & & z \\ (b_j)_{j=1}^{B} & ; \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \qquad (1) $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10), H.T. F. I p.103(32)]

$$(a-b)\ {}_2F_1\left[\begin{array}{cc} a,\ b\ ; & z \\ c\ \ ; & \end{array}\right] = a\ {}_2F_1\left[\begin{array}{cc} a+1,\ b\ ; & z \\ c\ \ ; & \end{array}\right] - b\ {}_2F_1\left[\begin{array}{cc} a,\ b+1\ ; & z \\ c\ ; & \end{array}\right] \tag{2}$$

Gauss second summation theorem is defined by [Prud., 491(7.3.7.5)]

$${}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \frac{1}{2} \\ \frac{a+b+1}{2}\ ; & \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud.,
p.491(7.3.7.3)]

$${}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \frac{1}{2} \\ \frac{a+b-1}{2}\ ; & \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5}$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function,
the above theorem can be written in the form

$${}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \frac{1}{2} \\ \frac{a+b-1}{2}\ ; & \end{array}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6}$$

Recurrence relation is defined by

$$\Gamma(z+1) = z\ \Gamma(z) \tag{7}$$

B. MAIN FORMULA :

$${}_2F_1\left[\begin{array}{cc} a,\ \ b\ ; & \frac{1}{2} \\ \frac{a+b+29}{2}\ ; & \end{array}\right] = \frac{2^b\ \Gamma(\frac{a+b+29}{2})}{(a-b)\ \Gamma(b)}\times$$

$$\times\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{8192a(-7905853580625+17901641997225a-15467069396610a^2+7198061846898a^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(-2078757113719a^4+401014719391a^5-53845005500a^6+5141534684a^7-351523887a^8)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(17085783a^9-576290a^{10}+12818a^{11}-169a^{12}+a^{13}+30623752512675b)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-21076873104060ab+48908173713858a^2b-10953594992484a^3b+5987537271801a^4b)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-653049221400a^5b+145268164044a^6b-8336836872a^7b+891863973a^8b-25765740a^9b)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(1356498a^{10}b-15444a^{11}b+351a^{12}b+3356967534030b^2+67380356527830ab^2)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-9754758898650a^2b^2+22117496003190a^3b^2-1934936201700a^4b^2+1070699106060a^5b^2)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-53143981380a^6b^2+11943338940a^7b^2-310617450a^8b^2+33011550a^9b^2-347490a^{10}b^2)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(17550a^{11}b^2+20687663696886b^3+3041140487100ab^3+28099549126290a^2b^3)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-1382567931600a^3b^3+3014416103100a^4b^3-118031308920a^5b^3+62708691780a^6b^3)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(-1424077200a^7b^3+306370350a^8b^3-2960100a^9b^3+296010a^10b^3+2468085362937b^4)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(12700187772795ab^4+631376019900a^2b^4+3670431326100a^3b^4-72878686650a^4b^4)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(151483053330a^5b^4-2653553700a^6b^4+1319397300a^7b^4-11100375a^8b^4+2220075a^9b^4)}{\left(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\right)\left(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\right)}+$$

$$+\frac{8192a(1633655568357b^5+718321271400ab^5+1966078891620a^2b^5+43378944840a^3b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(179763389190a^4b^5-1470155400a^5b^5+2895710580a^6b^5-18406440a^7b^5+8436285a^8b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(117627660900b^6+427868341380ab^6+56325846780a^2b^6+105898455180a^3b^6)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(1070955900a^4b^6+3379154940a^5b^6-9360540a^6b^6+17383860a^7b^6+28541690388b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(15830548920ab^7+29746525140a^2b^7+1483858800a^3b^7+2112414300a^4b^7+8023320a^5b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(20058300a^6b^7+1179623601b^8+3546891855ab^8+543716550a^2b^8+689837850a^3b^8)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(11504025a^4b^8+13037895a^5b^8+131329341b^9+72801300ab^9+110017050a^2b^9+4933500a^3b^9)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(4686825a^4b^9+2844270b^{10}+7425990ab^{10}+888030a^2b^{10}+888030a^3b^{10}+149526b^{11})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192a(63180ab^{11}+80730a^2b^{11}+1287b^{12}+2925ab^{12}+27b^{13})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192b(-7905853580625+30623752512675a+3356967534030a^2+20687663696886a^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(2468085362937a^4+1633655568357a^5+117627660900a^6+28541690388a^7+1179623601a^8)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(131329341a^9+2844270a^{10}+149526a^{11}+1287a^{12}+27a^{13}+17901641997225b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-21076873104060ab+67380356527830a^2b+3041140487100a^3b+12700187772795a^4b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(718321271400a^5b+427868341380a^6b+15830548920a^7b+3546891855a^8b+72801300a^9b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(7425990a^{10}b+63180a^{11}b+2925a^{12}b-15467069396610b^2+48908173713858ab^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-9754758898650a^2b^2+28099549126290a^3b^2+631376019900a^4b^2+1966078891620a^5b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(56325846780a^6b^2+29746525140a^7b^2+543716550a^8b^2+110017050a^9b^2+888030a^{10}b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(80730a^11b^2+71980618468898b^3-10953594992484ab^3+22117496003190a^2b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-1382567931600a^3b^3+3670431326100a^4b^3+43378944840a^5b^3+105898455180a^6b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(1483858800a^7b^3+689837850a^8b^3+4933500a^9b^3+888030a^{10}b^3-2078757113719b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(5987537271801ab^4-1934936201700a^2b^4+3014416103100a^3b^4-72878686650a^4b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(179763389190a^5b^4+1070955900a^6b^4+2112414300a^7b^4+11504025a^8b^4+4686825a^9b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(4010147193919b^5-653049221400ab^5+1070699106060a^2b^5-118031308920a^3b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(151483053330a^4b^5-1470155400a^5b^5+3379154940a^6b^5+8023320a^7b^5+13037895a^8b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-53845005500b^6+145268164044ab^6-53143981380a^2b^6+62708691780a^3b^6)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-2653553700a^4b^6+2895710580a^5b^6-9360540a^6b^6+20058300a^7b^6+5141534684b^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-8336836872ab^7+11943338940a^2b^7-1424077200a^3b^7+1319397300a^4b^7-18406440a^5b^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(17383860a^6b^7-351523887b^8+891863973ab^8-310617450a^2b^8+306370350a^3b^8)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-11100375a^4b^8+8436285a^5b^8+170085783b^9-25765740ab^9+33011550a^2b^9-2960100a^3b^9)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(2220075a^4b^9-576290b^{10}+1356498ab^{10}-347490a^2b^{10}+296010a^3b^{10}+12818b^{11})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192b(-15444ab^{11}+17550a^2b^{11}-169b^{12}+351ab^{12}+b^{13})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{16384(7905853580625+30623752512675a-3356967534030a^2+20687663696886a^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(-2468085362937a^4+1633655568357a^5-117627660900a^6+28541690388a^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(-1179623601a^8+131329341a^9-2844270a^{10}+149526a^{11}-1287a^{12}+27a^{13})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(17901641997225b+21076873104060ab+67380356527830a^2b-3041140487100a^3b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}\times$$

$$+\frac{16384(12700187772795a^4b-718321271400a^5b+427868341380a^6b-15830548920a^7b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(3546891855a^8b-72801300a^9b+7425990a^{10}b-63180a^{11}b+2925a^{12}b+15467069396610b^2)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(48908173713858ab^2+9754758898650a^2b^2+28099549126290a^3b^2-631376019900a^4b^2)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(1966078891620a^5b^2-56325846780a^6b^2+29746525140a^7b^2-543716550a^8b^2+110017050a^9b^2)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(-888030a^{10}b^2+80730a^{11}b^2+71980618846898b^3+10953594992484ab^3+22117496003190a^2b^3)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(1382567931600a^3b^3+3670431326100a^4b^3-43378944840a^5b^3+105898455180a^6b^3)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(-1483858800a^7b^3+689837850a^8b^3-4933500a^9b^3+888030a^10b^3+20787571137 19b^4)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(5987537271801ab^4+1934936201700a^2b^4+3014416103100a^3b^4+72878686650a^4b^4)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(179763389190a^5b^4-1070955900a^6b^4+2112414300a^7b^4-11504025a^8b^4+4686825a^9b^4)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(4010147193391b^5+653049221400ab^5+1070699106060a^2b^5+118031308920a^3b^5)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(151483053330a^4b^5+1470155400a^5b^5+3379154940a^6b^5-8023320a^7b^5+13037895a^8b^5)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(538450055500b^6+145268164044ab^6+53143981380a^2b^6+62708691780a^3b^6+2653553700a^4b^6)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(2895710580a^5b^6+9360540a^6b^6+20058300a^7b^6+51415534684b^7+8336836872ab^7)}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(11943338940a^2b^7+1424077200a^3b^7+1319397300a^4b^7+18406440a^5b^7+17383860a^6b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(351523887b^8+891863973ab^8+310617450a^2b^8+306370350a^3b^8+11100375a^4b^8)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(8436285a^5b^8+17085783b^9+25765740ab^9+33011550a^2b^9+2960100a^3b^9+2220075a^4b^9)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(576290b^{10}+1356498ab^{10}+347490a^2b^{10}+296010a^3b^{10}+12818b^{11}+15444ab^{11}+17550a^2b^{11})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(169b^{12}+351ab^{12}+b^{13})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{16384(7905853580625+17901641997225a+15467069396610a^2+7198061846898a^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(2078757113719a^4+401014719391a^5+53845005500a^6+5141534684a^7+351523887a^8)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(17085783a^9+576290a^{10}+12818a^{11}+169a^{12}+a^{13}+30623752512675b+21076873104060ab)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(48908173713858a^2b+10953594992484a^3b+5987537271801a^4b+653049221400a^5b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(145268164044a^6b+8336836872a^7b+891863973a^8b+25765740a^9b+1356498a^{10}b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(15444a^11b+351a^12b-3356967534030b^2+67380356527830ab^2+9754758898650a^2b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(22117496003190a^3b^2+1934936201700a^4b^2+1070699106060a^5b^2+53143981380a^6b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(11943338940a^7b^2+310617450a^8b^2+33011550a^9b^2+347490a^{10}b^2+17550a^{11}b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(20687663696886b^3-3041140487100ab^3+28099549126290a^2b^3+1382567931600a^3b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(3014416103100a^4b^3+118031308920a^5b^3+62708691780a^6b^3+1424077200a^7b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(306370350a^8b^3+2960100a^9b^3+296010a^{10}b^3-2468085362937b^4+12700187772795ab^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(-631376019900a^2b^4+3670431326100a^3b^4+72878686650a^4b^4+151483053330a^5b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(2653553700a^6b^4+1319397300a^7b^4+11100375a^8b^4+2220075a^9b^4+1633655568357b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(-718321271400ab^5+1966078891620a^2b^5-43378944840a^3b^5+179763389190a^4b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(1470155400a^5b^5+2895710580a^6b^5+18406440a^7b^5+8436285a^8b^5-117627660900b^6)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(427868341380ab^6-56325846780a^2b^6+105898455180a^3b^6-1070955900a^4b^6)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(3379154940a^5b^6+9360540a^6b^6+17383860a^7b^6+285416903888b^7-15830548920ab^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(29746525140a^2b^7-1483858800a^3b^7+2112414300a^4b^7-8023320a^5b^7+20058300a^6b^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(-11796236601b^8+3546891855ab^8-543716550a^2b^8+689837850a^3b^8-11504025a^4b^8)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(13037895a^5b^8+1313329341b^9-72801300ab^9+110017050a^2b^9-4933500a^3b^9+4686825a^4b^9)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(-2844270b^{10}+7425990ab^{10}-888030a^2b^{10}+888030a^3b^{10}+149526b^{11}-63180ab^{11})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{16384(80730a^2b^{11}-1287b^{12}+2925ab^{12}+27b^{13})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}\Bigg\}\Bigg] \qquad (8)$$

C. DERIVATION OF MAIN SUMMATION FORMULA (8) :

Substituting $c=\frac{a+b+29}{2}$ and $z=\frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b+29}{2} & ; & \end{matrix}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b & ; & \frac{1}{2}\\ \frac{a+b+29}{2} & ; & \end{matrix}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1 & ; & \frac{1}{2}\\ \frac{a+b+29}{2} & ; & \end{matrix}\right]$$

Now using Gauss second summation theorem, we get

$$L.H.S=a\,\frac{2^b\ \Gamma(\frac{a+b+29}{2})}{\Gamma(b)}\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{8192(-7905853580625+17901641997225a-15467069396610a^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(7198061846898a^3-2078757113719a^4+401014719391a^5-53845005500a^6+5141534684a^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-351523887a^8+17085783a^9-576290a^{10}+12818a^{11}-169a^{12}+a^{13}+30623752512675b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-21076873104060ab+48908173713858a^2b-10953594992484a^3b+5987537271801a^4b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-653049221400a^5b+145268164044a^6b-8336836872a^7b+891863973a^8b-25765740a^9b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(1356498a^{10}b-15444a^{11}b+351a^{12}b+3356967534030b^2+67380356527830ab^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-9754758898650a^2b^2+22117496003190a^3b^2-1934936201700a^4b^2+1070699106060a^5b^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-53143981380a^6b^2+11943338940a^7b^2-310617450a^8b^2+33011550a^9b^2-347490a^{10}b^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(17550a^{11}b^2+20687663696886b^3+3041140487100ab^3+28099549126290a^2b^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-1382567931600a^3b^3+3014416103100a^4b^3-118031308920a^5b^3+62708691780a^6b^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-1424077200a^7b^3+306370350a^8b^3-2960100a^9b^3+296010a^{10}b^3+2468085362937b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(12700187772795ab^4+631376019900a^2b^4+3670431326100a^3b^4-72878686650a^4b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(151483053330a^5b^4-2653553700a^6b^4+1319397300a^7b^4-11100375a^8b^4+2220075a^9b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(1633655568357b^5+718321271400ab^5+1966078891620a^2b^5+43378944840a^3b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(179763389190a^4b^5-1470155400a^5b^5+2895710580a^6b^5-18406440a^7b^5+8436285a^8b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(117627660900b^6+427868341380ab^6+56325846780a^2b^6+105898455180a^3b^6)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(1070955900a^4b^6+3379154940a^5b^6-9360540a^6b^6+17383860a^7b^6+285416903886b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(15830548920ab^7+29746525140a^2b^7+1483858800a^3b^7+2112414300a^4b^7+8023320a^5b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(20058300a^6b^7+1179623601b^8+3546891855ab^8+543716550a^2b^8+689837850a^3b^8)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(11504025a^4b^8+13037895a^5b^8+131329341b^9+72801300ab^9+110017050a^2b^9+4933500a^3b^9)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(4686825a^4b^9+2844270b^{10}+7425990ab^{10}+888030a^2b^{10}+888030a^3b^{10}+149526b^{11})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$\times\frac{8192(63180ab^{11}+80730a^2b^{11}+1287b^{12}+2925ab^{12}+27b^{13})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\Bigg\{\frac{8192(7905853580625+30623752512675a-3356967534030a^2+20687663696886a^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-2468085362937a^4+1633655568357a^5-117627660900a^6+28541690388a^7-1179623601a^8)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(131329341a^9-2844270a^{10}+149526a^{11}-1287a^{12}+27a^{13}+17901641997225b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(21076873104060ab+67380356527830a^2b-3041140487100a^3b+12700187772795a^4b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-718321271400a^5b+427868341380a^6b-15830548920a^7b+3546891855a^8b-72801300a^9b)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(7425990a^{10}b-63180a^{11}b+2925a^{12}b+15467069396610b^2+48908173713858ab^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(9754758898650a^2b^2+28099549126290a^3b^2-631376019900a^4b^2+1966078891620a^5b^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-56325846780a^6b^2+29746525140a^7b^2-543716550a^8b^2+110017050a^9b^2-888030a^{10}b^2)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(80730a^11b^2+71980618468898b^3+10953594992484ab^3+22117496003190a^2b^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(1382567931600a^3b^3+3670431326100a^4b^3-43378944840a^5b^3+105898455180a^6b^3)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(-1483858800a^7b^3+689837850a^8b^3-4933500a^9b^3+888030a^10b^3+20787571113719b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(5987537271801ab^4+1934936201700a^2b^4+3014416103100a^3b^4+72878686650a^4b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(179763389190a^5b^4-1070955900a^6b^4+2112414300a^7b^4-11504025a^8b^4+4686825a^9b^4)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(4010147193391b^5+653049221400ab^5+1070699106060a^2b^5+118031308920a^3b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(151483053330a^4b^5+1470155400a^5b^5+3379154940a^6b^5-8023320a^7b^5+13037895a^8b^5)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(53845005500b^6+145268164044ab^6+53143981380a^2b^6+62708691780a^3b^6+2653553700a^4b^6)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(2895710580a^5b^6+9360540a^6b^6+20058300a^7b^6+51415346684b^7+8336836872ab^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(11943338940a^2b^7+1424077200a^3b^7+1319397300a^4b^7+18406440a^5b^7+17383860a^6b^7)}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(351523887b^8+891863973ab^8+310617450a^2b^8+306370350a^3b^8+11100375a^4b^8+8436285a^5b^8)}{((\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(170857783b^9+25765740ab^9+33011550a^2b^9+2960100a^3b^9+2220075a^4b^9+576290b^{10})}{\Big(\prod_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(1356498ab^{10}+347490a^2b^{10}+296010a^3b^{10}+12818b^{11}+15444ab^{11}+17550a^2b^{11}+169b^{12})}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}+$$

$$+\frac{8192(351ab^{12}+b^{13})}{\Big(\prod\limits_{\zeta=1}^{13}\{a-b-(2\zeta-1)\}\Big)\Big(\prod\limits_{\eta=1}^{14}\{a-b+(2\eta-1)\}\Big)}\Bigg\}\Bigg]-$$

$$-b\,\frac{2^{b+1}\,\Gamma(\frac{a+b+29}{2})}{\Gamma(b+1)}\Bigg[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{8192(7905853580625+17901641997225a+15467069396610a^2)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(7198061846898a^3+2078757113719a^4+401014719391a^5+53845005500a^6+5141534684a^7)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(351523887a^8+17085783a^9+576290a^{10}+12818a^{11}+169a^{12}+a^{13}+30623752512675b)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(21076873104060ab+48908173713858a^2b+10953594992484a^3b+5987537271801a^4b)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(653049221400a^5b+145268164044a^6b+8336836872a^7b+891863973a^8b+25765740a^9b)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(1356498a^{10}b+15444a^{11}b+351a^{12}b-3356967534030b^2+67380356527830ab^2)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(9754758898650a^2b^2+22117496003190a^3b^2+1934936201700a^4b^2+1070699106060a^5b^2)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(53143981380a^6b^2+11943338940a^7b^2+310617450a^8b^2+33011550a^9b^2+347490a^{10}b^2)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(17550a^{11}b^2+20687663696886b^3-3041140487100ab^3+28099549126290a^2b^3)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(1382567931600a^3b^3+3014416103100a^4b^3+118031308920a^5b^3+62708691780a^6b^3)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(1424077200a^7b^3+306370350a^8b^3+2960100a^9b^3+296010a^{10}b^3-2468085362937b^4)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(12700187772795ab^4-631376019900a^2b^4+3670431326100a^3b^4+72878686650a^4b^4)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(151483053330a^5b^4+2653553700a^6b^4+1319397300a^7b^4+11100375a^8b^4+2220075a^9b^4)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(1633655568357b^5-718321271400ab^5+1966078891620a^2b^5-43378944840a^3b^5)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(179763389190a^4b^5+1470155400a^5b^5+2895710580a^6b^5+18406440a^7b^5+8436285a^8b^5)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-117627660900b^6+427868341380ab^6-56325846780a^2b^6+105898455180a^3b^6)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-1070955900a^4b^6+3379154940a^5b^6+9360540a^6b^6+17383860a^7b^6+285416903886b^7)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-15830548920ab^7+29746525140a^2b^7-1483858800a^3b^7+2112414300a^4b^7-8023320a^5b^7)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(20058300a^6b^7-11796236601b^8+3546891855ab^8-543716550a^2b^8+689837850a^3b^8)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-11504025a^4b^8+13037895a^5b^8+1313293341b^9-72801300ab^9+110017050a^2b^9-4933500a^3b^9)}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(4686825a^4b^9-2844270b^{10}+7425990ab^{10}-888030a^2b^{10}+888030a^3b^{10}+149526b^{11})}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-63180ab^{11}+80730a^2b^{11}-1287b^{12}+2925ab^{12}+27b^{13})}{\Big(\prod\limits_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod\limits_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{8192(-7905853580625+30623752512675a+3356967534030a^2+20687663696886a^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(2468085362937a^4+1633655568357a^5+117627660900a^6+28541690388a^7+1179623601a^8)}{((\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(131329341a^9+2844270a^{10}+149526a^{11}+1287a^{12}+27a^{13}+179016419997225b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-21076873104060ab+67380356527830a^2b+3041140487100a^3b+12700187772795a^4b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(718321271400a^5b+427868341380a^6b+15830548920a^7b+3546891855a^8b+72801300a^9b)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(7425990a^{10}b+63180a^{11}b+2925a^{12}b-15467069396610b^2+48908173713858ab^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-9754758898650a^2b^2+28099549126290a^3b^2+631376019900a^4b^2+1966078891620a^5b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(56325846780a^6b^2+29746525140a^7b^2+543716550a^8b^2+110017050a^9b^2+888030a^{10}b^2)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(80730a^{11}b^2+71980618468898b^3-10953594992484ab^3+22117496003190a^2b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-1382567931600a^3b^3+3670431326100a^4b^3+43378944840a^5b^3+105898455180a^6b^3)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(1483858800a^7b^3+689837850a^8b^3+4933500a^9b^3+888030a^10b^3-20787571137719b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(5987537271801ab^4-1934936201700a^2b^4+3014416103100a^3b^4-72878686650a^4b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(179763389190a^5b^4+1070955900a^6b^4+2112414300a^7b^4+11504025a^8b^4+4686825a^9b^4)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(401014719391b^5-653049221400ab^5+1070699106060a^2b^5-118031308920a^3b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(151483053330a^4b^5-1470155400a^5b^5+3379154940a^6b^5+8023320a^7b^5+13037895a^8b^5)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-53845005500b^6+145268164044ab^6-53143981380a^2b^6+62708691780a^3b^6-2653553700a^4b^6)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(2895710580a^5b^6-9360540a^6b^6+20058300a^7b^6+51415346684b^7-8336836872ab^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(11943338940a^2b^7-1424077200a^3b^7+1319397300a^4b^7-18406440a^5b^7+17383860a^6b^7)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-351523887b^8+891863973ab^8-310617450a^2b^8+306370350a^3b^8-11100375a^4b^8)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(8436285a^5b^8+170857783b^9-25765740ab^9+33011550a^2b^9-2960100a^3b^9+2220075a^4b^9)}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-576290b^{10}+1356498ab^{10}-347490a^2b^{10}+296010a^3b^{10}+12818b^{11}-15444ab^{11}+17550a^2b^{11})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}+$$

$$+\frac{8192(-169b^{12}+351ab^{12}+b^{13})}{\Big(\prod_{\xi=1}^{14}\{a-b-(2\xi-1)\}\Big)\Big(\prod_{\psi=1}^{13}\{a-b+(2\psi-1)\}\Big)}\Bigg\}\Bigg]$$

On simplification we get the result (8).

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