GEORGIAN MATHEMATICAL JOURNAL: Vol. 6, No. 6, 1999, 591-600

ABSOLUTE SUMMABILITY FACTORS AND ABSOLUTE TAUBERIAN THEOREMS FOR DOUBLE SERIES AND SEQUENCES

SH. YANETZ (CHACHANASHVILI)

ABSTRACT. Let A and B be the linear methods of the summability of double series with fields of bounded summability A'_b and B'_b, respectively. Let T be certain set of double series. The condition $x \in T$ is called B_b-Tauberian for A if $A'_b \cap T \subset B'_b$.

Some theorems about summability factors enable one to find new B_b-Tauberian conditions for A from the already known B_b-Tauberian conditions for A.

The first paper on connections between summability factors and Tauberian theorems for simple series and sequences was by Kangro [1] who proved three theorems where conditions were formulated in terms of summability factors. The theorems of Kangro [1] and Baron [2, §27] are generalizations of the theorems of Meyer-König and Tietz [3]–[5] and Leviatan [6].

To prove absolute Tauberian theorems with the aid of absolute summability factors for double series and sequences we need to generalize some important theorems about absolute summability factors for simple series and sequences.

1. Basic Definitions and the Main Lemma

The following notation and definitions will be used in this section.

Let

$$\sum_{m,n=0}^{\infty} u_{mn} \tag{1.1}$$

1991 *Mathematics Subject Classification.* 40B05, 40D15, 40E05.

Key words and phrases. Double sequences and series, Tauberian conditions, summability factors, summability methods.

1072-947X/99/1100-0591$16.00/0

be an infinite double series of real or complex numbers with partial sums

$$U_{mn} = \sum_{k,\ell=0}^{m,n} u_{k\ell}.$$

Let $A = (a_{mnk\ell})$ be an infinite normal double matrix of real or complex numbers.

Let

$$U'_{mn} = \sum_{k,\ell=0}^{m,n} a_{mnk\ell} U_{k\ell}.$$

We say that (1.1) is summable by the method A, or, shortly, (1.1) is *A-summable* to the sum U' if there exists the limit

$$\lim_{m,n} U'_{mn} = U'. \tag{1.2}$$

In that case we call the matrix A a method of summability.

We call a double series (1.1) *boundedly A-summable*, or, shortly, *A_b-summable* to the sum U' if U'_{mn} satisfies conditions (1.2) and $U'_{mn} = O(1)$. In that case we also denote the method of summability A by A_b. Analogously, the B_b-summability will be defined with the aid of the double matrix $B = (b_{mnk\ell})$.

Let A'_b be the set of all A_b-summable double series. If (1.1) is A_b-summable to the sum U', we write

$$A_b\Big\{\sum_{m,n} u_{mn}\Big\} = U'.$$

The number U' is called the *A-sum* of the double series (1.1).

If for every absolutely convergent double series (1.1) is $|A|$-summable (see [7], p. 141), then we say that the method A *conserves absolute convergence.* If the method A conserves absolute convergence and

$$A_b\Big\{\sum_{m,n} u_{mn}\Big\} = \sum_{m,n} u_{mn}$$

for each absolute convergent double series (1.1), then the method A is called *absolute regular.*

The set of all $|A|$-summable double series is called the *field of absolute summability* of A and we denote it by A'_ℓ or $|A|'$. If

$$B_b\Big\{\sum_{m,n} u_{mn}\Big\} = A_b\Big\{\sum_{m,n} u_{mn}\Big\}$$

for each double series from $|B|' \cap |A|'$, then we say that A_ℓ is *consistent with B_ℓ.*

In the sequel we use the notation:

$$\begin{aligned}\overline{\Delta}_m x_{mn} &= x_{mn} - x_{m-1,n},\\ \overline{\Delta}_n x_{mn} &= x_{mn} - x_{m,n-1},\\ \overline{\Delta}_{mn} x_{mn} &= \overline{\Delta}_m(\overline{\Delta}_n x_{mn}) = \overline{\Delta}_n(\overline{\Delta}_m x_{mn}) =\\ &= x_{mn} - x_{m-1,n} - x_{m,n-1} + x_{m-1,n-1}.\end{aligned}$$

Let ρ be an arbitrary set of double series (1.1). We denote by r the set of double sequences $x = (U_{k\ell})$ associated with ρ, i.e.,

$$r = \Big\{x : \sum_{k,\ell} \overline{\Delta}_{k\ell} U_{k\ell} \in \rho\Big\}.$$

For example, if ρ is the set of bounded convergent double series, then $r = bc$, the set of bounded convergent double sequences; if ρ is the set $b\gamma_0$ of bounded double series converging to zero, then $r = bcn$, the set of bounded double sequences converging to zero; if ρ is the set μ of double series with bounded partial sums, then r is the set m of bounded double sequences; if ρ is the set ℓ of absolutely convergent double series, ℓ_0 absolutely convergent double series converging to zero, then r is the set a of absolutely converging double sequences, i.e., $a = \{x : U_{k\ell} = \Omega(1)\}$. Here $x_{mn} = \Omega(y_{mn})$ denotes $\sum\limits_{m,n} |\overline{\Delta}_{mn}(x_{mn}/y_{mn})| < \infty$. If ρ is the set ℓ_0 of double series absolutely converging to zero, then r is the set a_0 of double sequences x, for which $U_{k\ell} = \omega(1)$. Here $x_{mn} = \omega(y_{mn})$ denotes $x_{mn} = \Omega(y_{mn})$ and $x_{mn} = o(y_{mn})$.

We will use the notation:

$$\begin{aligned}x_{mn} &= \Omega_m(y_{mn}) \quad \text{means} \quad \sum_m |\overline{\Delta}_m(x_{mn}/y_{mn})| = O(1),\\ x_{mn} &= \Omega_n(y_{mn}) \quad \text{means} \quad \sum_n |\overline{\Delta}_n(x_{mn}/y_{mn})| = O(1),\\ x_{mn} &= \omega_m(y_{mn}) \quad \text{means} \quad x_{mn} = \Omega_m(y_{mn}), \quad x_{mn} = o(1),\\ x_{mn} &= \omega_n(y_{mn}) \quad \text{means} \quad x_{mn} = \Omega_n(y_{mn}), \quad x_{mn} = o(1).\end{aligned}$$

The numbers (ε_{mn}) are called *summability factors of type* (ρ, B_ℓ) (or (ρ, B_b)) if for every double series $\sum\limits_{m,n} u_{mn} \in \rho$ it always follows that

$$\sum_{m,n} \varepsilon_{mn} u_{mn} \in B'_\ell \quad \Big(\text{or} \quad \sum_{m,n} \varepsilon_{mn} u_{mn} \in B'_b\Big).$$

Let A and B be the methods of the summability of double series (1.1). Let T and T_0 be certain sets of double series (1.1). We denote three double series (1.1) by x, y and z.

The condition $x \in T$ will be called *B_ℓ-Tauberian* for A_ℓ if

$$A'_\ell \cap T \subset B'_\ell.$$

In particular, when the convergence method $E = C^{0,0}$ (see [7], p. 141), i.e., if $B'_\ell = E'_\ell = \ell$, instead of an E_ℓ-Tauberian condition, it will simply be called an *absolute Tauberian condition*.

Now we formulate the main lemma.

Lemma 1.1. *Let $A'_\ell \supset B'_\ell$ and A_ℓ be consistent with B_ℓ. If the condition $x \in T_0$ is B_ℓ-Tauberian for A_ℓ, then the condition $x \in T$ is also B_ℓ-Tauberian for A_ℓ if every element $x \in T$ can be represented by $x = y + z$, where $y \in T_0$ and $z \in B'_\ell$.*

Proof. Let $x \in A'_\ell \cap T$ and $x = y + z$ with $y \in T_0$ and $z \in B'_\ell$. Then clearly $y = x - z \in A'_\ell \cap T_0 \subset B'_\ell$. Hence $x \in B'_\ell$. $\square$

Let the numbers $\lambda_{mn} \neq 0$, $\mu_{mn} \neq 0$, where $\overline{\Delta}_{mn}\mu_{mn} \neq 0$ and $\mu_{-1,n} = \mu_{m,-1} = 0$.

Let us assume that μ_{mn} and λ_{mn} are factorizable, i.e.,

$$\mu_{mn} = \mu'_m \cdot \mu''_n, \quad \lambda_{mn} = \lambda'_m \cdot \lambda''_n,$$

and for $m, n > 0$

$$h' = \overline{\Delta}_m \mu'_m / \lambda'_m, \quad h'' = \overline{\Delta}_n \mu''_n / \lambda''_n.$$

Then

$$\frac{\overline{\Delta}_n \mu_{m-1,n}}{\lambda_{mn}} = \frac{\mu'_{m-1}}{\lambda'_m} \cdot \frac{\overline{\Delta}_n \mu''_n}{\lambda''_n}, \quad \frac{\overline{\Delta}_m \mu_{m,n-1}}{\lambda_{mn}} = \frac{\overline{\Delta}_m \mu_m}{\lambda'_m} \cdot \frac{\mu''_{n-1}}{\lambda''_n}.$$

2. Connections Between Absolute Summability Factors for Double Series and Absolute Tauberian Theorems

For every double series (1.1) a double sequence (V_{mn}) will be constructed by the formula

$$V_{mn} = \frac{1}{\mu_{mn}} \sum_{k,\ell=0}^{m,n} \lambda_{k\ell} u_{k\ell}. \tag{2.1}$$

Recall

$$\alpha_{mn} = \mu_{m-1,n-1}/\lambda_{mn}, \quad \gamma_{mn} = (\overline{\Delta}_{mn}\mu_{mn})/\lambda_{mn},$$
$$\beta'_{mn} = (\overline{\Delta}_n \mu_{m-1,n})/\lambda_{mn}, \quad \beta''_{mn} = (\overline{\Delta}_m \mu_{m,n-1})/\lambda_{mn}.$$

One can now state

Theorem 2.1. *Let $A'_\ell \supset B'_\ell$, where A_ℓ is consistent with B_ℓ. If the following conditions are fulfilled:*

(1) *the numbers α_{mn} are summability factors of type (ρ, B_ℓ),*

(2) *the double series $\omega' = \sum_{m,n} \beta'_{mn}\overline{\Delta}_m V_{mn}$ and $\omega'' = \sum_{m,n} \beta''_{mn}\overline{\Delta}_n V_{mn}$ are boundedly $|B|$-summable,*

(3) *condition $(u_{mn}/\gamma_{mn}) \in r$ is B_ℓ-Tauberian for A_ℓ,*

then the condition

$$(V_{mn}) \in r \tag{2.2}$$

is also B_ℓ-Tauberian for A.

Proof. From (2.1) we obtain

$$u_{mn} = \frac{1}{\lambda_{mn}}\overline{\Delta}_{mn}(\mu_{mn}V_{mn}).$$

We use the formula (see formula (15.31) in [2]) for the difference of products of double sequences and obtain

$$u_{mn} = \alpha_{mn}\overline{\Delta}_{mn}V_{mn} + \beta'_{mn}\overline{\Delta}_m V_{mn} + \beta''_{mn}\overline{\Delta}_n V_{mn} + \gamma_{mn}V_{mn}. \tag{2.3}$$

With the aid of the series

$$x = \sum_{m,n} u_{mn}, \quad y = \sum_{m,n} \gamma_{mn}V_{mn}, \quad z = \omega + \omega' + \omega'',$$

where

$$\omega = \sum_{m,n} \alpha_{mn}\overline{\Delta}_{mn}V_{mn}, \quad \omega' = \sum_{m,n} \beta'_{mn}\overline{\Delta}_m V_{mn}, \quad \omega'' = \sum_{m,n} \beta''_{mn}\overline{\Delta}_n V_{mn},$$

we get $x = y + z$ from (2.3).

If condition (2.2) is satisfied, then $\sum_{m,n} \overline{\Delta}_{mn}V_{mn} \in \rho$, and from conditions (1) and (2) it follows that $z \in B'_\ell$. We denote

$$T_0 = \Big\{ \sum_{m,n} u_{mn} : (u_{mn}/\gamma_{mn}) \in r \Big\}.$$

From (2.3) we conclude that $y \in T_0$. According to (3), the condition $x \in T_0$ is B_ℓ-Tauberian for A_ℓ. Thus the statement of Theorem 2.1 follows from Lemma 1.1 if one sets

$$T = \Big\{ \sum_{m,n} u_{mn} : (V_{mn}) \in r \Big\}. \qquad \square$$

If in Theorem 2.1 we take $B = E$ and $r = a$, then $(\rho, B_\ell) = (\ell, \ell)$. It is known (see [8], p. 131, Theorem 3) that $\varepsilon_{mn} \in (C_\ell^{\alpha,\beta}, C_\ell^{\alpha,\beta})$ for $\alpha, \beta \geq 0$ iff the following conditions

$$\Delta_{mn}^{\alpha,\beta}\varepsilon_{mn} = O[(m+1)^{-\alpha}(n+1)^{-\beta}], \quad \Delta_m^{\alpha}\varepsilon_{mn} = O[(m+1)^{-\alpha}],$$
$$\Delta_n^{\beta}\varepsilon_{mn} = O[(n+1)^{-\beta}], \quad \varepsilon_{mn} = O(1)$$

are fulfilled. From here, with $\alpha = \beta = 0$, we obtain following

Lemma 2.2. *For $\varepsilon_{mn} \in (|\ell|, |\ell|)$ it is necessary and sufficient that*

$$\varepsilon_{mn} = O(1).$$

The proof of the condition $\omega' = O(1)$ in the proof of Corollary 4.1 from [9], p. 160, and Lemma 2.2 imply

Corollary 2.3. *Let A be an absolutely regular method for the summability of double series. If the following conditions are fulfilled:*

(1) $\mu_{m-1,n-1} = O(\lambda_{mn})$,

(2) $\sum\limits_{m,n} |h_{mn}| < \infty$,

(3) $\sum\limits_{m,n} \left| \frac{\mu'_{m-1}}{\lambda'_m} \overline{\Delta}_m V_{mn} \right| = O(1)$, $\sum\limits_{mn} \left| \frac{\mu''_{n-1}}{\lambda''_n} \overline{\Delta}_n V_{mn} \right| = O(1)$,

(4) *condition $u_{mn} = \Omega(\gamma_{mn})$ is absolute Tauberian for A_ℓ,*
then $V_{mn} = \Omega(\gamma_{mn})$ is also absolute Tauberian for A_ℓ.

As we see by Lemma 2.2, $\varepsilon_{mn} \in (\ell, \ell)$ iff $\varepsilon_{mn} = O(1)$. Since $\ell_0 \subset \ell$, the conditions for $\varepsilon_{mn} \in (\ell, \ell)$ are necessary and sufficient for $\varepsilon_{mn} \in (\ell_0, \ell)$ as well. Therefore $\varepsilon_{mn} \in (\ell_0, \ell)$ iff $\varepsilon_{mn} = O(1)$. Thus analogously to Corollary 2.3 we obtain

Corollary 2.4. *Let A be an absolutely regular method of the summability for double series. If the conditions* (1), (2), *and* (3) *of Corollary* 2.3 *are fulfilled and*

(4) *the condition $u_{mn} = \omega(\gamma_{mn})$ is absolute Tauberian for A_ℓ,*
then $V'_{mn} = \omega(\gamma_{mn})$ is also absolute Tauberian for A_ℓ.

Analogous theorems for simple series were proved by S. Baron (see [2], p. 234, Corollary 27.3).

3. Connections Between Absolute Summability Factors for Double Sequences and Absolute Tauberian Theorems

In this section we shall find connections between summability factors for a double sequence and absolute Tauberian theorems.

In (2.1) let

$$\lambda_{mn} = a_{mn} b_{mn}.$$

With the notation used in Theorem 2.1 we get

Theorem 3.1. *Let $A'_\ell \supset B'_\ell$, where A_ℓ is consistent with B_ℓ. If the following conditions are fulfilled:*

(1) *the numbers α_{mn} are summability factors of type (ρ, B_ℓ),*

(2) *the numbers $\overline{\Delta}_{mn}\mu_{mn}/b_{mn}$ are summability factors for a double sequence of type (r, r),*

(3) *the double series ω' and ω'' are B_ℓ-summable,*

(4) *condition $(a_{mn}u_{mn}) \in r$ is B_ℓ-Tauberian for A_ℓ,*

then the condition

$$(V_{mn}) \in r \tag{3.1}$$

is also B_ℓ-Tauberian for A_ℓ.

Proof. With the aid of the formal series x, y, z as in the proof of Theorem 2.1, expansion (2.3) gives $x = y + z$. Then, as stated in the proof of Theorem 2.1, from condition (3.1) and conditions (1) and (3) it follows that $z \in B'_\ell$. In addition, from conditions (3.1) and (2) one gets

$$\left(\frac{\overline{\Delta}_{mn}\mu_{mn}}{b_{mn}} V_{mn} \right) \in r.$$

We set

$$T_0 = \Big\{ \sum_{m,n} u_{mn} : (a_{mn}u_{mn}) \in r \Big\}$$

and conclude that $y \in T_0$ if in T_0 we take $u_{mn} = \gamma_{mn} V_{mn}$. According to (4), the condition $x \in T_0$ is B_ℓ-Tauberian for A_ℓ. Thus the statement of Theorem 3.1 follows from Lemma 1.1 if one sets

$$T = \Big\{ \sum_{m,n} u_{mn} : (V_{mn}) \in r \Big\}.$$

In Theorem 3.1, if we put $B'_\ell = \ell$ and $r = a$, then $(\rho, B_\ell) = (\ell, \ell)$ and $(r, r) = (a, a)$. Then the convergence factors $\alpha_{mn} \in (\ell, \ell)$ and are characterized by Lemma 2.2. In condition (2) the convergence factors $\Delta_{mn}\mu_{mn}/b_{mn} \in (a, a)$ for the above-mentioned convergence factors, which we characterize by Lemma 3.2 below, are obtained from Theorem 1 of [7], p. 144.

If we examine $B = E$ in Theorem 1 of [7] and assume that $\overline{b}_{mnk\ell} = \overline{\Delta}_{mnk\ell}\delta_{mnk\ell}$, then

$$\sum_{\mu,\nu=k,\ell}^{m,n} \overline{b}_{mn\mu\nu}\overline{\xi}_{\mu\nu k\ell}\varepsilon_{\mu\nu} = \sum_{\mu,\nu=k,\ell}^{m,n} \overline{\Delta}_{mn}\delta_{mn\mu\nu}\varepsilon_{\mu\nu} =$$
$$= \overline{\Delta}_{mn} \sum_{\mu,\nu=k,\ell}^{m,n} \delta_{mnk\ell}\varepsilon_{\mu\nu} = \overline{\Delta}_{mn}\varepsilon_{mn}. \tag{3.2}$$

By substituting (3.2) into (4) of [7] we obtain

$$\sum_{m,n} |\overline{\Delta}_{mn}\varepsilon_{mn}| < \infty. \tag{3.3}$$

If in (5) of [7], p. 144, we take only the term with $k = m$ and put $\ell = 0$, we obtain

$$\sum_{n} \Big| \sum_{\nu=0}^{n} b_{mnm\nu}\varepsilon_{mn} \Big| = \sum_{n} |\overline{\Delta}_n\delta_{n\nu}\varepsilon_{m\nu}| = \sum_{n} |\overline{\Delta}_n\varepsilon_{m\nu}| = O(1).$$

Consequently, condition (5) of Theorem 1 in [7] implies the condition

$$\varepsilon_{mn} = \Omega_n(1) \tag{3.4}$$

and, analogously, the condition

$$\varepsilon_{mn} = \Omega_m(1). \tag{3.5}$$

Since by Proposition 1 of [7], p. 144, condition (5) is equivalent to conditions (6), (7), (8) of Proposition 1 in [7], the latter follow from (3.3), (3.4), and (3.5).

Indeed, from (3.4) we obtain

$$\sum_{m,n} \Big| \sum_{\mu,\nu=0}^{m,\ell-1} \overline{\Delta}_{mn}\delta_{mn\mu\nu}\varepsilon_{\mu\nu} \Big| = \sum_{m,n} \Big| \sum_{\nu=0}^{\ell-1} \overline{\Delta}_n\delta_{n\nu} \cdot \overline{\Delta}_m\varepsilon_m\nu \Big| =$$
$$= O(1) \sum_{n} \Big| \sum_{\nu=0}^{\ell-1} \overline{\Delta}_n\delta_{n\nu} \Big| = O(1). \qquad \square$$

Consequently, the above mentioned conditions (7) and (8) follow from (3.4) and (3.5), but in our case condition (6) is condition (3.3).

Thus we have proved

Lemma 3.2. *For* $\varepsilon_{mn} \in (|\mathfrak{C}|, |\mathfrak{C}|)$ *it is necessary and sufficient that conditions* (3.4) *and* (3.5) *and* $\varepsilon_{mn} = \Omega(1)$ *be fulfilled.*

If as above we combine Theorem 3.1 with Lemmas 2.2 and 3.2, and choose $\varepsilon_{mn} = \mu_{m-1,n-1}/\gamma_{mn}$ and $\varepsilon_{mn} = \overline{\Delta}_{mn}\mu_{mn}/b_{mn}$ (respectively), we obtain

Corollary 3.3. *Let A be an absolute regular method of the summability for double series. If the following conditions are fulfilled:*

(1) $\mu_{m-1,n-1} = O(\lambda_{mn})$,

(2) $\overline{\Delta}_m \mu_{mn} = \Omega(b_{mn})$, $\overline{\Delta}_{mn}\mu_{mn} = \Omega_m(b_{mn})$, $\overline{\Delta}_{mn}\mu_{mn} = \Omega_n(b_{mn})$,

(3) $\sum_{m,n} |h_{mn}| < \infty$,

(4) $\sum_m \left|\frac{\mu'_{m-1}}{\lambda'_m}\overline{\Delta}_m V_{mn}\right| = O(1)$ $\sum_n \left|\frac{\mu''_{n-1}}{\lambda''_n}\overline{\Delta}_n V_{mn}\right| = O(1)$

(5) *condition* $a_{mn}u_{mn} = \Omega(1)$ *is absolute Tauberian for* A_ℓ,

then the condition $V_{mn} = \Omega(1)$ *is also absolute Tauberian for* A_ℓ.

We use Theorem 3.1 with $r = a_0$ and show that the conditions of Lemma 3.2 are necessary and sufficient for $\varepsilon_{mn} \in (a_0, a)$. Indeed, from conditions (3.3)–(3.5) it follows that $\varepsilon_{mn} = O(1)$ by the formula (see [10], p. 10)

$$\varepsilon_{mn} = \varepsilon_{00} + \sum_{\mu,\nu=0}^{m-1,n-1} \Delta_{\mu\nu}\varepsilon_{\mu\nu} - \sum_{\mu=0}^{m-1} \Delta_\mu \varepsilon_{\mu 0} - \sum_{v=0}^{n-1} \Delta_\nu \varepsilon_{0\nu}, \quad m,n = 1,2,\ldots.$$

Therefore the conditions of Lemma 3.2 are necessary and sufficient for $\varepsilon_{mn} \in (a_0, a_0)$, since $\varepsilon_{mn}U_{mn} = O(1)U_{mn} = O(1)\cdot o(1) = o(1)$, and from (3.3)–(3.5) it follows that $(\varepsilon_{mn}U_{mn}) \in a$.

Theorem 3.1 and Lemma 3.2 imply

Corollary 3.4. *Let A be an absolute regular method for the summability of double series. If conditions* (1), (3), *and* (4) *of Corollary* 3.3 *and conditions*

(2′) $\overline{\Delta}_{mn}\mu_{mn} = \omega_m(b_{mn})$, $\overline{\Delta}_{mn}\mu_{mn} = \omega_n(b_{mn})$,

(5′) *condition* $a_{mn}u_{mn} = \omega(1)$ *is absolute Tauberian for* A_ℓ,

are fulfilled, then the condition $V_{mn} = \omega(1)$ *is also absolute Tauberian for* A_ℓ.

Analogous theorems for simple series were proved by H. Tietz ([11], p. 139, Theorems 3.1 and 3.3) and S. Baron ([2], p. 236, Corollary 27.4).

References

1. G. Kangro, The weakening of Tauber's condition. (Russian) *Izv. Akad. Nauk. Eston. SSR, Mat.* **19**(1970), 24–33.

2. S. Baron, Introduction to the Theory of Summability of Series. (Russian) *Second edition corrected and completed, Tallinn, Valgus,* 1977.

3. W. Meyer-König and H. Tietz, On Tauberian conditions of type σ. *Bull. Amer. Math. Soc.* **73**(1967), 926–927.

4. W. Meyer-König and H. Tietz, Über die Limitierungsumkehrsätze vom Typus σ. *Studia Math.* **31**(1968), 205–216.

5. W. Meyer-König and H. Tietz, Über Umkehrbedingungen in der Limitierungstheorie. *Arch. Mat.* (*Brno*) **5**(1969), 177–186.

6. D. Leviatan, Remarks on some Tauberian theorems of Meyer-König, Tietz and Stieglitz. *Proc. Amer. Math. Soc.* **29**(1971), 126–132.

7. S. Baron and S. Ianetz, Absolute convergence and summability factors in a double sequence. *Indian J. Math.* **28**(1986), No. 2, 141–153.

8. S. Baron, Summability factors and absoulte summability for double series which are absolutely Cesàro summable. (Russian) *Tartu Ül. Toimetised* No. 102(1961) 118–134.

9. S. Yanetz, Summability factors and Tauberian theorems for double series. *Tartu Ül. Toimetised* No. 960(1993), 151–165.

10. G. Kangro and S. Baron, Summability factors for Cesàro-summable and Cesàro-bounded double series. (Russian) *Tartu Ül. Toimetised* No. 73(1959), 3–49.

11. H. Tietz, Über absolute Tauber-Bedingungen. *Math. Z.* **113**(1970), 136–144.

(Received 26.06.1998)

Author's address:
Department of Mathematics and
Computer Science
Bar-Ilan University
52900 Ramat-Gan
Israel