Computational Construction of W-graphs of Hecke algebras H(q, n) for n up to 15

Mitsuyuki Ochiai and Fujio Kako

CONTENTS

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We construct by computer all W-graphs corresponding to irreducible representations of Hecke algebras $H(q,n)$ for n up to 15, using a modification of a method proposed by Lascoux and Schützenberger (which fails for $n > 13$).

1. INTRODUCTION

V. Jones [1985] discovered a polynomial invariant in one variable for oriented knots and links, later generalized into the Homfly invariants in two variables [Freyd et al. 1985]. Jones [1987] also defined another two-variable invariant $X_L(q,\lambda)$ of an oriented link L, given by

$$X_L(q,\lambda) = \left(-\frac{1-\lambda q}{\sqrt{\lambda}(1-q)}\right)^{n-1} (\sqrt{\lambda})^e \operatorname{tr} \pi(\alpha),$$

where α is any element of the braid group B_n with $\hat{\alpha} = L$, e is the exponent sum of α, and π is the representation of B_n in the Hecke algebra $H(q,n)$ sending the standard generators of B_n to those of $H(q,n)$.

Ocneanu's trace $\operatorname{tr} g_i$ for each generator g_i is defined by

$$\operatorname{tr} g_i = \sum_Y W_Y(q,z) \operatorname{tr}_Y g_i,$$

where Y is a Young diagram associated with a partition of n, and tr_Y is the trace on the Hecke algebra obtained by evaluating the sum of the diagonal entries on the image of g_i in the matrix representation π_Y (see the precise definition in [Jones 1987]).

Two ways to compute $\operatorname{tr} g_i$ are known. One is due to P. Hoefsmit [1974] and H. Wenzl [1985], and is not well adapted to computer calculations,

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because it involves square roots of certain polynomials. The other, introduced by A. Lascoux and M. Schützenberger [1981], is combinatorial in nature and uses the W-graphs defined by Kazhdan and Lusztig [1979] for irreducible representations of the symmetric group S_n. Its explicit formula is given in [Gyoja 1986; 1987].

The difficulty with the Lascoux–Schützenberger method is the construction of the W-graphs. Those authors proposed an algorithm for this construction (Section 2), but did not give a proof of its validity. In an earlier version of the present article, we verified the validity of the Lascoux–Schützenberger algorithm for $n \leq 12$. However, after submission, the referee informed us that Tim Maclarnan had found, years before, an example with $n = 14$ where the W-graph is not correctly generated; in other words, the representation matrix obtained by the Lascoux–Schützenberger algorithm did not satisfy the defining relations of $H(q, 14)$ in that case.

We therefore extended our computations, and confirmed that the method fails for $n = 14$ and 15. By introducing certain modifications, we were able to overcome the incompleteness of the algorithm for these values of n, and constructed all W-graphs for irreducible representations of Hecke algebras $H(q, n)$ for n up to 15. This is described in Section 3, where we also give a table of cases where the original Lascoux–Schützenberger method fails.

The situation for $n \geq 16$ remains open.

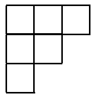
2. THE METHOD OF LASCOUX AND SCHÜTZENBERGER

Let $\Lambda(n)$ be the set of partitions of a positive integer n, a *partition* being a sequence $(\lambda_1, \lambda_2, \ldots, \lambda_k)$ of positive integers such that $\sum_i \lambda_i = n$ and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$. For example, $\Lambda(6)$ has 11 elements:

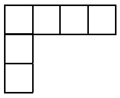
$$\{(6), (5,1), (4,2), (4,1,1), (3,3), (3,2,1), (3,1,1,1), (2,2,2), (2,2,1,1), (2,1,1,1,1), (1,1,1,1,1,1)\}.$$

An element of $\Lambda(n)$ can be pictorially expressed as a *Young diagram*, where the row lengths indicate the elements of the partition. Therefore a Young diagram is characterized by row lengths that are nonincreasing as we go down, and column lengths that are nonincreasing from left to right:

$(3,2,1)$



$(4,1,1)$



A *standard Young tableau* associated with a partition in $\Lambda(n)$ is an assignment of distinct integers $1, \ldots, n$ to the boxes in the Young diagram of the partition, in such a way that numbers within each row increase left to right, and numbers within each column increase top to bottom. For example, the partition $(3,2,1) \in \Lambda(6)$ admits the following standard Young tableaux:

$$\begin{array}{lll}1&4&6\\2&5\\3\end{array}\quad\begin{array}{lll}1&3&6\\2&5\\4\end{array}\quad\begin{array}{lll}1&2&6\\3&5\\4\end{array}\quad\begin{array}{lll}1&3&6\\2&4\\5\end{array}\quad\begin{array}{lll}1&2&6\\3&4\\5\end{array}\quad\begin{array}{lll}1&4&5\\2&6\\3\end{array}$$

$$\begin{array}{lll}1&3&5\\2&6\\4\end{array}\quad\begin{array}{lll}1&2&5\\3&6\\4\end{array}\quad\begin{array}{lll}1&3&4\\2&6\\5\end{array}\quad\begin{array}{lll}1&2&4\\3&6\\5\end{array}\quad\begin{array}{lll}1&2&3\\4&6\\5\end{array}\quad\begin{array}{lll}1&3&5\\2&4\\6\end{array}$$

$$\begin{array}{lll}1&2&5\\3&4\\6\end{array}\quad\begin{array}{lll}1&3&4\\2&5\\6\end{array}\quad\begin{array}{lll}1&2&4\\3&5\\6\end{array}\quad\begin{array}{lll}1&2&3\\4&5\\6\end{array}$$

Usually we denote a standard Young tableau by the associated *word*, which is the sequence of integers obtained by reading the entries row by row, from left to right, from bottom to top. Thus the words associated with the tableaux above are

325146 425136 435126 524136 534126 326145
426135 436125 526134 536124 546123 624135
634125 625134 635124 645123.

Let $X = \{x_1, x_2, \ldots, x_s\}$ be the collection of words associated with a Young diagram (or partition) Y. The following procedure associates with Y a graph $G(Y)$ with vertex set X (see also [Gyoja 1986; 1987]).

Algorithm [Lascoux and Schützenberger 1981]

1. Let $x = w_1 i w_2 j w_3$ and $x' = w_1 j w_2 i w_3$ be vertices in $G(Y)$, where w_1, w_2 and w_3 are subwords that may be empty and w_2 does not contain any number in the range $[i, j]$. Then x is adjacent to x' in $G(Y)$. In the example above, this makes 325146 and 425136 adjacent, but not 425136 and 625134.

2. Let x be a vertex in $G(Y)$. For each i with $1 \leq i \leq n-2$, define a vertex $x^{(i)}$ as follows:

pattern matched by x	value of $x^{(i)}$
$w_1 i w_2 (i+1) w_3 (i+2) w_4$	undefined
$w_1 (i+2) w_2 (i+1) w_3 i w_4$	undefined
$w_1 i w_2 (i+2) w_3 (i+1) w_4$	$w_1 (i+1) w_2 (i+2) w_3 i w_4$
$w_1 (i+1) w_2 i w_3 (i+2) w_4$	$w_1 (i+2) w_2 i w_3 (i+1) w_4$
$w_1 (i+1) w_2 (i+2) w_3 i w_4$	$w_1 i w_2 (i+2) w_3 (i+1) w_4$
$w_1 (i+2) w_2 i w_3 (i+1) w_4$	$w_1 (i+1) w_2 i w_3 (i+2) w_4$

(Here w_1, w_2, w_3 and w_4 are subwords of x, which may be empty.) Then, for any pair of vertices x and x' that are adjacent by the preceding step, we make $x^{(i)}$ and $x'^{(i)}$ adjacent as well. For example, $x_2 = 425136$ and $x_4 = 524136$ are adjacent in $G(Y)$ by step 1, so $x_2^{(2)} = 325146$ and $x_4^{(2)} = 534126$ are adjacent in $G(Y)$.

3. Apply step 2 repeatedly until no more adjacent pairs appear.

3. IRREDUCIBLE REPRESENTATIONS OF HECKE ALGEBRAS H(q, n)

Let $H(q,n)$ be the C-algebra on the generators $g_1, g_2, \ldots, g_{n-1}$ defined by the relations

$$g_i^2 = (q-1)g_i + q,$$
$$g_i g_{i+1} g_i = g_{i+1} g_i g_{i+1},$$
$$g_i g_j = g_j g_i \quad \text{if } |i-j| \geqq 2.$$

Then $H(q,n)$ is called a *Hecke algebra of type* A_{n-1}, and the g_i are its *standard generators.*

Let Y be a Young diagram for a partition in $\Lambda(n)$, and let $X = \{x_1, x_2, \ldots, x_s\}$ be the collection of words associated with Y. For each element x of X, define $I(x)$ as the set of $i \in \{1, \ldots, n-1\}$ such that the row containing i is above the one containing $i+1$ in x (where x is regarded as a standard Young tableau). For instance, if $x = 645123$ in our running example, we have $I(x) = \{3, 5\}$.

Given a triple $\{X, I, \mu\}$, where I is the function of x just introduced and μ is an arbitrary function $X \times X \to \{0, 1\}$, we define square matrices T_j of size s, for $j = 1, \ldots, n-1$. The (l,m)-entry of T_j is, by definition,

$$\begin{cases} -1 & \text{if } l = m \text{ and } j \in I(x_l); \\ q & \text{if } l = m \text{ and } j \notin I(x_l); \\ \sqrt{q} & \text{if } l \neq m,\ j \in I(x_l) \setminus I(x_m), \text{ and } \mu(x_l, x_m) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

We call $\{X, I, \mu\}$ a W-graph corresponding to Y if the matrices T_j satisfy the defining relations of Hecke algebras $H(q,n)$ under the representation π_Y with $\pi_Y(g_j) \equiv T_j$, for $j = 1, \ldots, n-1$ [Gyoja 1984; Kazdan and Lusztig 1979].

It was conjectured in [Lascoux and Schützenberger 1981] and [Gyoja 1986; 1987] that, if μ is the adjacency relation of the graph $G(Y)$ defined by the algorithm in Section 2, then $\{X, I, \mu\}$ is a W-graph. As detailed below, we have checked that this conjecture is true for n up to 13, but false for $n = 14$ and 15.

Moreover, we have introduced a modification in the definition of $G(Y)$ so that the conjecture for the modified $G(Y)$ remains valid for $n = 14, 15$.

To test the conjecture, we wrote software to construct the sets $I(x)$ and the graph $G(Y)$ for any Young diagram Y with $n \leq 15$. We performed direct matrix calculations to check whether the resulting matrices satisfy the defining relations of Hecke algebras $H(q,n)$, and we found that three of the 135 representations for $n = 14$ and twenty-one of the 176 representations for $n = 15$ do not satisfy the necessary relations (more specifically, they fail the conjugacy and commutation relations). This is summarized in Table 1.

(As mentioned in Section 1, it has been known for years that the algorithm of Section 2 sometimes fails, but to our knowledge the cases of failure have not previously been recorded in the literature.)

n	s	Y	e
14	48048	$\{5,4,3,2\}$	68
	68640	$\{5,4,2,2,1\}$	50
	48048	$\{4,4,3,2,1\}$	68
15	30030	$\{6,5,4\}$	8
	128700	$\{6,5,3,1\}$	68
	100100	$\{6,5,2,2\}$	48
	175175	$\{6,4,3,2\}$	322
	243243	$\{6,4,2,2,1\}$	250
	54054	$\{5,5,4,1\}$	48
	96525	$\{5,5,3,2\}$	232
	125125	$\{5,5,2,2,1\}$	110
	81081	$\{5,4,4,2\}$	80
	75075	$\{5,4,3,3\}$	100
	292864	$\{5,4,3,2,1\}$	1720
	125125	$\{5,4,2,2,2\}$	110
	243243	$\{5,4,2,2,1,1\}$	250
	75075	$\{4,4,4,2,1\}$	100
	81081	$\{4,4,3,3,1\}$	80
	96525	$\{4,4,3,2,2\}$	232
	175175	$\{4,4,3,2,1,1\}$	322
	100100	$\{4,4,2,2,2,1\}$	48
	54054	$\{4,3,3,3,2\}$	48
	128700	$\{4,3,3,2,2,1\}$	68
	30030	$\{3,3,3,3,2,1\}$	8

TABLE 1. Representations not accounted for by the Lascoux–Schützenberger method. The second column gives the size of the representation matrices T_j, and the last gives the number of edges missing from $G(Y)$ (see Table 2).

Very recently, Naruse [1994] found the W-graph associated with the Young diagram $\{4,4,3,2,1\}$ using Kazhdan–Lusztig polynomials and a computational construction. We compared his results with ours and found that there are 68 edges that the algorithm of Section 2 fails to detect. These edges can be generated from the following eight by repeated application of step 2 of the algorithm:

87C36B25AE149D–C8A36E25BD1479
87C36B25AE149D–C8E6AB279D1345
C4837B26AE159D–C8E4AB267D1359
C7B36A259E148D–CAE6BD27891345
76B5AE249D138C–EAB67C248D1359
B6A59E248D137C–EAB68C249D1357
D6A59E248C137B–DAE68C249B1357
A6E59D248C137B–EAC68D249B1357

Here A stands for 10, B for 11, and so on.

One may ask whether the Lascoux–Schützenberger algorithm can be salvaged so as to always yield a W-graph. This turns out to be possible, at least for $n = 14$ and 15, by adding to the graph $G(Y)$ edges suggested by failures in the commutation relations. The modified algorithm below allowed us to find the correct W-graphs for all Young diagrams with $n = 14$ and 15. (Unfortunately we do not have a proof that it works for higher values of n.)

Algorithm (modified Lascoux–Schützenberger)

1. Using the algorithm of Section 2, calculate the adjacency matrix and $I(x)$ for each word x.
2. Calculate T_j, for $j = 1, \dots, n-1$.
3. Form the commutator matrices $C_{i,j} = T_iT_j - T_jT_i$ for $i = 1, \dots, n-3$ and $j = i+2, \dots, n-1$. If there is a nonvanishing $C_{i,j}$, tentatively add an edge to the graph $G(Y)$ as follows. If the (l,m)-entry of $C_{i,j}$ is non zero, add to $G(Y)$ a pair (x_l, x_k) such that the (l,k)-entry of T_i or T_j is nonzero, or a pair (x_k, x_m) such that the (k,m)-entry of T_i or T_j is nonzero. After such an edge has been tentatively added, carry out step 2 of the algorithm of Section 2 and recompute the matrices T_j and their commutators. If the total number of nonzero entries in the commutators has decreased, accept the additional edge permanently; otherwise, discard it.
4. Repeat the preceding step as long as there are nonzero commutator matrices and it is possible to find acceptable edges.

The algorithm is successful if eventually all the commutator matrices are zero. In this case the matrices T_j obviously satisfy the defining relations of the Hecke algebras $H(q,n)$.

It is well known that the representation given by a W-graph corresponding to a Young diagram is irreducible [Gyoja 1984; Kazdan and Lusztig 1979]. Hence the matrices T_j give, in fact, irreducible representations of the Hecke algebras $H(q,n)$.

The results of our calculations are given in Tables 1 and 2. Table 1, as already mentioned, shows

$n = 14 \quad Y = \{5,4,3,2\}$

7B59D348C126AE–BC78D349E1256A
7D59C348B126AE–CD78E349A1256B
9D58C347B126AE–CD89E34AB12567
7B59C348E126AD–BC78E349A1256D
5948D37BE126AC–DE89A456B1237C
5D49B37AE1268C–DE9AB456C12378
9D57B36AE1248C–DE9AB56781234C
9D7BE456A1238C–DE9AB456C12378

$n = 14 \quad Y = \{5,4,2,2,1\}$

B6A59248D137CE–B9D5A24CE13678
87C6B35AE1249D–CBE78359A1246D

$n = 15 \quad Y = \{6,5,4\}$

47AD369CE1258BF–ACDE4678F12359B
58BE47ADF12369C–BDEF5789A12346C

$n = 15 \quad Y = \{6,5,3,1\}$

847B26ADF1359CE–B7DF289AC13456E
76AD459CF1238BE–D79F45ABC12368E
B7AE4569D1238CF–EABC456DF123789
C58B347AE1269DF–C8AE34BDF125679
958D347CF126ABE–D89F34ABC12567E
A59D348CF1267BE–D9AF34BCE125678

$n = 15 \quad Y = \{6,5,2,2\}$

7D6A459CF1238BE–DF7945ABC12368E
8C5B347AE1269DF–CE8A34BDF125679
9D58347CF126ABE–DF8934ABC12567E
AD59348CF1267BE–DF9A34BCE125678

$n = 15 \quad Y = \{6,4,3,2\}$

5948C37BE126ADF–CE89A456B1237DF
8C47B36AE1259DF–CE8AB46791235DF
5C48B37AE1269DF–CE8AB456D12379F
7B6AE459D1238CF–BD79E45AF12368C
7B6AD459F1238CE–BD79F45AC12368E
7E6AD459C1238BF–DE79F45AB12368C
AE69D348C1257BF–DE9AF34BC125678
9D58C347B126AEF–CD89E34AB12567F
8C7BE346A1259DF–CE8AB346D12579F
9E58D347C126ABF–DE89F34AB12567C
6A59D48CF1237BE–DF9AB567C12348E
9D58C47BF1236AE–DF9BC578A12346E
6D59C48BF1237AE–DF9BC567E12348A
5A49D38CF1267BE–DF9AB456C12378E
5D49C38BF1267AE–DF9BC456E12378A
9D8CF346B1257AE–DF9BC346E12578A
6A59D348C127BEF–AC68D349E1257BF
6A59C348E127BDF–AC68E349B1257DF
6D59C348B127AEF–CD68E349A1257BF
6E59D348C127ABF–DE68F349A1257BC

$n = 15 \quad Y = \{6,4,2,2,1\}$

B6A49258D137CEF–B9D4A25CE13678F
87C6B45AE1239DF–CBE78459A1236DF
C7B5A269E1348DF–CAE5B26DF134789
98D7C34BF1256AE–DCF8934AB12567E

$n = 15 \quad Y = \{5,5,4,1\}$

A369D258CF147BE–A6CDF289BE13457
B47AF369CE1258D–FABCD4678E12359
837BE26ADF1459C–B7DEF289AC13456
968CF257BE134AD–F9BCD5678E1234A

$n = 15 \quad Y = \{5,5,3,2\}$

5C48B37AEF1269D–CE8AB456DF12379
7B6AD459CF1238E–BD79F45ACE12368
7E6AD459CF1238B–DE79F45ABC12368
5A49D368CF127BE–DF9AB456CE12378
6A59D248CF137BE–DF9AB567CE12348
6D59C248BF137AE–DF9BC5678E1234A
9D58C347BF126AE–CD89E34ABF12567
8C7BE346AF1259D–CE8AB346DF12579
7B46A359DF128CE–BD79F34ACE12568
9E58D347CF126AB–DE89F34ABC12567
8C7BF256AE1349D–CF8AB256DE13479
6A59D248CF137BE–AC68D249EF1357B
6D59C248BF137AE–CD68E249AF1357B
9D58C247BF136AE–DF9BC578AE12346

$n = 15 \quad Y = \{5,5,2,2,1\}$

B6A49258DF137CE–B9D4A25CEF13678
D6A49258CF137BE–D9F4A25BCE13678
98D7C246BF135AE–DCF8926ABE13457
98D5C247BF136AE–DCF8924ABE13567

$n = 15 \quad Y = \{5,4,4,2\}$

7B36AE259D148CF–BD79EF356A1248C
5948CF37BE126AD–CE89AF456B1237D
8C47BF36AE1259D–CE8ABF46791235D
5C48BF37AE1269D–CE8ABF456D12379
7E36AD259C148BF–DE79AF356B1248C
AE369D258C147BF–DE9ABF356C12478
6A59DF248C137BE–DF9ABC56781234E
6D59CF248B137AE–DF9BCE56781234A
9D58CF347B126AE–CD89EF34AB12567
6A59CF248E137BD–AC68EF249B1357D
6D59CF248B137AE–CD68EF249A1357B
9D38CF257B146AE–DF9BCE35781246A

$n = 15 \quad Y = \{5,4,3,3\}$

7BE36A259D148CF–BDE79A356F1248C
7BD36A259F148CE–BDF79A356C1248E
59F48C37BE126AD–CEF89A456B1237D
8CF47B36AE1259D–CEF8AB46791235D
5CF48B37AE1269D–CEF8AB456D12379
7AE36D259C148BF–DEF79A356B1248C
59D48C27BF136AE–9DF5BC278A1346E
8CF7BE346A1259D–CEF8AB346D12579
6AE59D248C137BF–ACE68D249F1357B
6AE39D258C147BF–DEF9AB356C12478

$n = 15 \quad Y = \{5,4,3,2,1\}$

C7B36A259E148DF–CAE3BD267F14589
A5948D37CF126BE–D9F4AB357C1268E
98D67C25BF134AE–D9F6BC278A1345E
98D57C46BF123AE–DCF89A456B1237E
D5948C37BF126AE–D9F4BC357E1268A
76B5AE249D138CF–B7D59E24AF1368C
B6A59E248D137CF–B9D5AE24CF13678
76B5AD249F138CE–B7D59F24AC1368E
B6A59F248D137CE–B9D5AF24CE13678
87C6BF34AE1259D–CBE78F349A1256D
87C36B25AE149DF–C8A36E25BD1479F
D8C7BF456A1239E–DCF8AB456E12379
C4837B26AE159DF–C8E4AB267D1359F
87C36B25AE149DF–C8E6AB279D1345F
C7B36A259E148DF–CAE6BD27891345F
76B5AE349D128CF–EAB67C348D1259F
76B5AE349D128CF–BAD67E348F1259C
C4B37A269E158DF–CAE4BD267F13589
76E5AD349C128BF–EAC67D348F1259B
A9E58D347C126BF–EAC89D34BF12567
B6A59E348D127CF–BAE68D349C1257F
B6A59E348D127CF–EAB68C349D1257F
A6E59D348C127BF–EAC68D349B1257F
B6A59E348D127CF–BAD68E349F1257C
76E5AD249C138BF–D7E59F24AB1368C
E6A59D348C127BF–EAC68D349F1257B
B6A59D348F127CE–BAD68F349C1257E
A9E58D347C126BF–D9E8AF34BC12567
D6E59C348B127AF–DCE68F349A1257B
A6E59D248C137BF–D9E5AF24BC13678
C7B6AF349E1258D–CBF79E34AD12568
D8C37B26AF1459E–DBF3CE27891456A
87C6BF34AE1259D–FBC78D349E1256A
C7B6AF349E1258D–FBC79D34AE12568
87F6BE34AD1259C–FBD78E349A1256C
B7F6AE349D1258C–FBD79E34AC12568
D9E58C347B126AF–DCE89F34AB12567
E9D58C347B126AF–ECD89F34AB12567
A5948D37CF126BE–D9F8AB456C1237E
D5948C37BF126AE–DCF89A456B1237E
98D47C36BF125AE–DCF89A467B1235E
D8C47B36AF1259E–DCF8AB46791235E
D5C48B37AF1269E–DCF8AB456E12379
D7C36B25AF1489E–DBF6CE27891345A
A9E46D358C127BF–EAC46D358F1279B
E6D59C348B127AF–ECD68F349A1257B
76C5BF34AE1289D–FBC67D348E1259A
65A49D28CF137BE–D9F5AB267C1348E
65D49C28BF137AE–D9F5BC267E1348A
98D47C26BF135AE–D9F7BC28AE13456
D8C47B26AF1359E–DBF7CE289A13456
A5948D27CF136BE–A9D5CF278B1346E
98D57C46BF123AE–D9B57F46CE1238A
A5948D27CF136BE–D9F5AB278C1346E
D5C48B27AF1369E–DBF5CE27891346A
95D48C27BF136AE–D9F5BC278A1346E
98D4CF257B136AE–D9F4BC257E1368A
D5948C27BF136AE–D9F5BC278E1346A

TABLE 2. Additional edges needed to complete the graphs obtained by the Lascoux–Schützenberger method into W-graphs, in the cases $n = 14$ and 15. We use A for 10, B for 11, etc. See also text on next page.

the Young diagrams for which the original Lascoux–Schützenberger method fails to yield a W-graph. It also shows the number of additional edges needed. Table 2 lists some of the additional edges; the remaining ones are obtained by applying step 2 of the Lascoux–Schützenberger method. We take advantage of adjointness to omit certain Young diagrams: for instance, the Young diagram $\{4,4,3,2,1\}$ is adjoint to the diagram $\{5,4,3,2\}$, so the auxiliary edges necessary for $\{4,4,3,2,1\}$ are easily obtained from those of $\{5,4,3,2\}$.

On a Sun SPARCserver 1000 with 160 megabytes of main memory, our program HeckeRep.c needed about a week to compute all the $G(Y)$ for Young diagrams Y associated with $\Lambda(n)$, with $n \leq 15$, and the corresponding T_j. For $n = 15$ only, the calculations took about 137 hours of CPU time and 87 megabytes of main memory.

4. FINAL REMARKS

The first author and J. Murakami have established the three-parallel version of polynomial invariants of closed three- and four-braids associated with certain subspaces of representation matrices of the irreducible representation of $H(q,n)$, for $n = 9$ and $n = 12$. See [Ochiai and Murakami 1994].

We are now calculating three-parallel version of polynomial invariants of closed five-braids using irreducible representations of $H(q,15)$. The results will be published in a forthcoming article.

There is still no known effective algorithm to construct irreducible representations of $H(q,n)$ for large n.

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After finishing the first version of this article, the first author visited the Geometry Center at the University of Minnesota, where he fixed some bugs in the computer program. He would like to express his thanks to the Center's founding Director, Albert Marden, and to the Center staff for their hospitality.

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ELECTRONIC AVAILABILITY

The program HeckeRep.c described in Section 3 can be obtained by anonymous ftp from geom.umn.edu, in directory pub/contrib/expmath/ochiai.

Mitsuyuki Ochiai, Department of Information and Computer Sciences, Faculty of Science, Nara Women's University, Nara 630, Japan (ochiai@ics.nara-wu.ac.jp)

Fujio Kako, Department of Information and Computer Sciences, Faculty of Science, Nara Women's University, Nara 630, Japan (kako@ics.nara-wu.ac.jp)

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