CAUCHY TRANSFORMS OF SELF-SIMILAR MEASURES

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Abstract. The Cauchy transform of a measure in the plane,

$$F(z) = \frac{1}{2\pi i} \int_{\mathbb{C}} \frac{1}{z-w}\, d\mu(w),$$

is a useful tool for numerical studies of the measure, since the measure of any reasonable set may be obtained as the line integral of F around the boundary. We give an effective algorithm for computing F when μ is a self-similar measure, based on a Laurent expansion of F for large z and a transformation law (Theorem 2.2) for F that encodes the self-similarity of μ. Using this algorithm we compute F for the normalized Hausdorff measure on the Sierpiński gasket. Based on this experimental evidence, we formulate three conjectures concerning the mapping properties of F, which is a continuous function holomorphic on each component of the complement of the gasket.