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ANALYTIC SMOOTHING EFFECT FOR THE CUBIC HYPERBOLIC SCHRÖDINGER EQUATION IN TWO SPACE DIMENSIONS

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Abstract. We study the Cauchy problem for the cubic hyperbolic Schrödinger equation in two space dimensions. We prove existence of analytic global solutions for sufficiently small and exponential decaying data. The method of proof depends on the generalized Leibniz rule for the generator of pseudo-conformal transform acting on pseudo-conformally invariant nonlinearity.

1. Introduction

We study the Cauchy problem for the hyperbolic Schrödinger equations in two space dimensions

$$i\partial_t u + \Box u = \lambda |u|^2 u, \ (t,x) \in \mathbb{R} \times \mathbb{R}^2, \tag{1.1}$$

where $i = \sqrt{-1}$, $u : \mathbb{R} \times \mathbb{R}^2 \ni (t,x) \mapsto u(t,x) \in \mathbb{C}$, $\partial_t = \partial/\partial t$, $\Box = \partial_1^2 - \partial_2^2$, $\partial_j = \partial/\partial x_j$, $x = (x_1, x_2)$, and $\lambda \in \mathbb{C}$.

The two dimensional cubic hyperbolic Schrödinger equation describes the gravity waves on liquid surface and ion-cyclotron waves in plasma (see for instance [1, 29, 30] and references therein). We refer the reader to [4, 20, 21, 33] and reference therein for recent study on the Cauchy problem for non elliptic nonlinear Schrödinger equations. Especially, analyticity of solutions to non elliptic nonlinear Schrödinger equations is studied in [4].

Space-time analytic smoothing effect for the local solutions to the nonlinear Schrödinger equations in n space dimensions is studied in [6, 25]. Space-time analyticity is characterized by the Galilei generator $J(t) = x + it\nabla$ and the pseudo-conformal generator $K(t) = |x|^2 + nit + 2it(t\partial_t + x \cdot \nabla)$. Since the following equality holds ([7])

$$\Big(i\partial_t + \frac{1}{2}\Delta\Big)K = (K + 4it)\Big(i\partial_t + \frac{1}{2}\Delta\Big),$$

the following inequality plays an important role in construct analytic solutions

$$\sum_{l \geq 0} \frac{a^l}{l!} \big\| (K + \mu t)^l f; L^p(0,T;X) \big\| \leq C \frac{1}{1 - abT} \sum_{l \geq 0} \frac{a^l}{l!} \| K^l f; L^p(0,T;X) \|,$$

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where $\mu \in \mathbb{C}$, $abT < 1$, $a, T > 0$, $b > 2$, $1 \le p \le \infty$ and X is an appropriate Banach space of functions on $\mathbb{R}^n$. In [14, 15, 16], we prove the space-time analytic smoothing effect for the global solutions to the nonlinear Schrödinger equations with sufficiently small data by using the Leibniz rule for the pseudo-conformal generator such as:

$$\big(K + 4it\big)|u|^{4/n}u = \Big(1 + \frac{2}{n}\Big)|u|^{4/n}Ku - \frac{2}{n}u^2|u|^{4/n-2}\overline{Ku}.$$

The hyperbolic Galilei transform $\mathcal{G}_v$ and the pseudo-conformal transform $\mathcal{P}_\theta$ are defined by

$$(\mathcal{G}_v u)(t,x) = e^{-iv\cdot x - it(v_1^2 - v_2^2)}u(t, x_1 + 2v_1t, x_2 - 2v_2t), \quad v \in \mathbb{R}^2,$$

$$(\mathcal{P}_\theta u)(t,x) = (1-\theta t)^{-1}e^{-i\frac{\theta}{4(1-\theta t)}(x_1^2 - x_2^2)}u\Big(\frac{t}{1-\theta t}, \frac{x_1}{1-\theta t}, \frac{-x_2}{1-\theta t}\Big), \quad \theta \in \mathbb{R},$$

respectively. We see that (1.1) is invariant under the transforms $\mathcal{G}_v$ and $\mathcal{P}_\theta$ (see [29, 30]).

In this paper, we consider the analyticity in both space-time variables of solutions to (1.1). We prove the Leibniz rule for the pseudo-conformal generator $K_h(t) = x_1^2 - x_2^2 + 4it(t\partial_t + x\cdot\nabla) + 4it$ holds even for (1.1) (see Lemma 2.3 below).

For stating our main result precisely, we introduce the following notation. L^p denotes the usual Lebesgue space $L^p(\mathbb{R}^2)$, $1 \le p \le \infty$. The Fourier transform $\mathcal{F}$ is defined by

$$\mathcal{F}[\varphi](\xi) = (2\pi)^{-1}\int_{\mathbb{R}^2} e^{-i\xi\cdot x}\varphi(x)dx$$

and $\mathcal{F}^{-1}$ is its inverse. We denote the linear part of (1.1) by $\mathcal{L} = i\partial_t + \Box$. The free propagator of hyperbolic Schrödinger equation is defined by $U_h(t) = e^{it\Box}$, $t \in \mathbb{R}$. We use the notation such as $\big(U_h\phi\big)(t) = U_h(t)\phi$. We put $U_1(t) = e^{it\partial_1^2}$, $U_2(t) = e^{-it\partial_2^2}$. Then

$$U_h(t) = (U_1(t)\otimes I)(I \otimes U_2(t)) = (I\otimes U_2(t))(U_1(t)\otimes I),$$

where I is the identity in $L^2(\mathbb{R})$ and $\otimes$ denotes the tensor product. The relations are abbreviated as $U_h(t) = U_1(t)U_2(t) = U_2(t)U_1(t)$. The Galilei generators are defined by

$$J_h(t) = (J_1(t), J_2(t)) = (x_1, x_2) + 2it(\partial_1, -\partial_2) = U_h(t)(x_1, x_2)U_h(-t), \ t \in \mathbb{R}.$$

For $t \neq 0$, we put $M_h(t) = e^{i\frac{x_1^2 - x_2^2}{4t}}$ and we use the notation such as $(M_h^{-1})(t) = M_h(-t)$. For $t \neq 0$, J_h is represented as:

$$J_h(t) = M_h(t)2it(\partial_1, -\partial_2)M_h^{-1}(t).$$

According to [8, 13, 22], we define

$$A_\delta(t) = U_h(t)e^{\delta\cdot x}U_h(-t), \quad t\in\mathbb{R},\ \delta\in\mathbb{R}^2,$$

where $\delta\cdot x = \delta_1x_1 + \delta_2x_2$. For $t \neq 0$, A_δ is represented as:

$$A_\delta(t) = M_h(t)e^{2it\delta\cdot(\partial_1, -\partial_2)}M_h^{-1}(t),$$

where $e^{2it\delta\cdot(\partial_1,-\partial_2)} = \mathcal{F}^{-1}e^{-2t\delta\cdot(\xi_1,-\xi_2)}\mathcal{F}$. We define the generator of dilations by

$$P(t) = t\partial_t + x\cdot\nabla, \quad t\in\mathbb{R}.$$

We define

$$\tilde{K}_h(t) = x_1^2 - x_2^2 + 4itP(t), \quad t\in\mathbb{R}.$$

For $t \neq 0$, $\tilde{K}_h$ is represented as:

$$\tilde{K}_h(t) = 4itM_h(t)P(t)M_h^{-1}(t).$$

We define the pseudo-conformal generator by

$$K_h(t) = \tilde{K}_h(t) + 4it = U_h(t)\big(x_1^2 - x_2^2 + 4it^2\partial_t\big)U_h(-t), \quad t \in \mathbb{R}.$$

We introduce the following basic function space:

$$\mathcal{X} = L^\infty(\mathbb{R}; L^2) \cap L^4(\mathbb{R}; L^4),$$

with the norm

$$\|u; \mathcal{X}\| = \|u; L^\infty(\mathbb{R}; L^2)\| + \|u; L^4(\mathbb{R}; L^4)\|.$$

Let $D \subset \mathbb{R}^n$, $a > 0$ and w be a real-valued function on $\mathbb{R}^2$. We define the following function spaces:

$$G^D(x; L^2) \equiv \Big\{\phi \in L^2; \|\phi; G^D(x; L^2)\| < \infty\Big\},$$
$$\|\phi; G^D(x; L^2)\| \equiv \sup_{\delta \in D} \|e^{\delta \cdot x}\phi; L^2\|,$$

$$G^D(J_h; \mathcal{X}) \equiv \Big\{u \in \mathcal{X}; \|u; G^D(J_h; \mathcal{X})\| < \infty\Big\},$$
$$\|u; G^D(J_h; \mathcal{X})\| \equiv \sup_{\delta \in D} \|A_\delta u; \mathcal{X}\|,$$

$$G^{D,a}(x, w; L^2) \equiv \Big\{\phi \in G^D(x; L^2); \|\phi; G^{D,a}(x, w; L^2)\| < \infty\Big\},$$
$$\|\phi; G^{D,a}(x, w; L^2)\| \equiv \sum_{l \geq 0} \frac{a^l}{l!} \|w^l \phi; G^D(x; L^2)\|,$$

$$G^{D,a}(J_h, K_h; \mathcal{X}) \equiv \Big\{u \in G^D(J; \mathcal{X}); \|u; G^{D,a}(J_h, K_h; \mathcal{X})\| < \infty\Big\},$$
$$\|u; G^{D,a}(J_h, K_h; \mathcal{X})\| \equiv \sum_{l \geq 0} \frac{a^l}{l!} \|K_h^l u; G^D(J_h; \mathcal{X})\|.$$

For any $r > 0$ and any Banach space $\mathscr{X}$, we put

$$B_r(\mathscr{X}) = \Big\{u \in \mathscr{X}; \|u; \mathscr{X}\| \leq r\Big\}.$$

We consider the following integral equation associated with the Cauchy problem (1.1) with data ϕ:

$$u = U_h\phi - i\lambda F_h\big(|u|^2 u\big)$$

where $\big(F_h f\big)(t) = \int_0^t U_h(t-s)f(s)ds$, $t \in \mathbb{R}$.

Since the free propagator U_h is written as a product of one dimensional free Schrödinger propagators, U_h has the same properties as those of the free Schrödinger propagator (see Lemma 2.1 below). Especially, we have by the representation

$$U_h(t)\phi = (4\pi t)^{-1} \iint_{\mathbb{R}\times\mathbb{R}} e^{\frac{|x_1 - y_1| - |x_2 - y_2|}{4it}} \phi(y_1, y_2)dy_1dy_2, \quad t \neq 0,$$
$$\|U_h(t)\phi; L^2\| = \|\phi; L^2\|, \quad t \in \mathbb{R},$$
$$\|U_h(t)\phi; L^\infty\| \leq C|t|^{-1}\|\phi; L^1\|, \quad t \neq 0,$$

and

$$\|U_h(t)\phi; L^p\| \le C|t|^{-1+\frac{2}{p}}\|\phi; L^{p'}\|, \quad t \neq 0$$

for $2 \le p \le \infty$. There are many papers on the Cauchy problem for the nonlinear Schrödinger equations and on the analyticity of solutions to the nonlinear evolution equations we refer the reader to [2, 3, 5, 18, 30, 32] for the former and to [4, 6, 11, 12, 13, 17, 19, 22, 23, 24, 25, 26, 27, 28, 31] for the latter.

We say that a domain $D \subset \mathbb{R}^2$ is symmetric if D satisfies the following conditions: $0 \in D$ and for any $\delta = (\delta_1, \delta_2) \in D$ we have $(-\delta_1, -\delta_2)$, $(\delta_1, -\delta_2) \in D$. We state our main result:

Theorem 1.1. *There exists an $\varepsilon > 0$ such that for any $a > 0$, any symmetric domain $D \subset \mathbb{R}^2$ and any $\phi \in B_\varepsilon(G^{D,a}(x, x_1^2 - x_2^2; L^2))$, (1.1) has a unique solution $u \in G^{D,a}(J_h, K_h; \mathcal{X})$.*

Remark 1.2. Since $|x_1^2 - x_2^2| \le x_1^2 + x_2^2$, the following inequality holds:

$$\|\phi; G^{D,a}(x, x_1^2 - x_2^2; L^2)\| \le \|\phi; G^{D,a}(x, x_1^2 + x_2^2; L^2)\|.$$

Remark 1.3. Regarding analyticity, the operators J_h and K_h correspond analyticity in space and in time, respectively.

Remark 1.4. As stated in the theorem, a and D may be taken independent of ε.

2. Preliminaries

In this section, we introduce the some basic lemmas.

Lemma 2.1 ([2, 30, 32])**.** *For any (r_j, q_j) satisfying $2/r_j = 1 - 2/q_j$, with $q_j \in [2, \infty)$, $j = 1, 2$, the following inequalities hold:*

$$\|U_h\phi; L^{r_1}(\mathbb{R}; L^{q_1})\| \le C\|\phi; L^2\|,$$

$$\|F_h f; L^{r_1}(\mathbb{R}; L^{q_1})\| \le C\|f; L^{r_2'}(\mathbb{R}; L^{q_2'})\|,$$

where p' denotes the Hölder conjugate of p defined by $1/p + 1/p' = 1$.

The following result is similar to the previous results in [7, 13], where we can find commutation relation between pseudo-conformal generator $K(t)$ (not hyperbolic $K_h(t)$) and the linear operator $i\partial_t + \frac{1}{2}\Delta$ (not $\mathcal{L} = i\partial_t + \Box$).

Lemma 2.2. *Let $t \in \mathbb{R}$. We have*

$$[K_h(t), \mathcal{L}] = -8it\mathcal{L}, \quad [A_\delta(t), \mathcal{L}] = 0,$$

where $[A, B] = AB - BA$ is the commutator.

Proof. Since $\mathcal{L} = U_h(t) i\partial_t U_h(-t)$, we have

$$\begin{aligned} [K_h(t), \mathcal{L}] &= U_h(t)\big[x_1^2 - x_2^2 + 4it^2\partial_t, i\partial_t\big]U_h(-t) \\ &= U_h(t)\big[4it^2\partial_t, i\partial_t\big]U_h(-t) \\ &= -8it\mathcal{L} \end{aligned}$$

and

$$[A_\delta(t), \mathcal{L}] = U_h(t)\big[e^{\delta\cdot x}, i\partial_t\big]U_h(-t) = 0.$$

□

Lemma 2.3. *Let $t \in \mathbb{R}$. We have*

$$\big(K_h(t) + 8it\big)u_1\overline{u_2}u_3 = (K_h(t)u_1)\overline{u_2}u_3 - u_1(\overline{K_h(t)u_2})u_3 + u_1\overline{u_2}(K_h(t)u_3).$$

Proof. By the Leibniz rule for $P(t) = t\partial_t + x\cdot\nabla$, we have

$$\begin{aligned}
\big(K_h(t)+8it\big)u_1\overline{u_2}u_3 &= \big(\tilde K_h(t)+12it\big)u_1\overline{u_2}u_3\\
&= M_h(t)\big(4itP(t)+12it\big)M_h^{-1}(t)\big(u_1\overline{u_2}u_3\big)\\
&= M_h(t)\big(4itP(t)+12it\big)M_h^{-1}(t)u_1\overline{M_h^{-1}(t)u_2}M_h^{-1}(t)u_3\\
&= \Big((\tilde K_h(t)+4it)u_1\Big)\overline{u_2}u_3 - u_1\Big(\overline{(\tilde K_h(t)+4it)u_2}\Big)u_3\\
&\quad + u_1\overline{u_2}\Big(\big(\tilde K_h(t)+4it\big)u_3\Big)\\
&= (K_h(t)u_1)\overline{u_2}u_3 - u_1(\overline{K_h(t)u_2})u_3 + u_1\overline{u_2}(K_h(t)u_3).
\end{aligned}$$

□

Lemma 2.4. *Let $t\in\mathbb{R}$. We have*

$$\big(K_h(t)+8it\big)^l(u_1\overline{u_2}u_3) = \sum_{l_1+l_2+l_3=l}\frac{(-1)^{l_2}l!}{l_1!l_2!l_3!}K_h^{l_1}(t)u_1\overline{K_h^{l_2}(t)u_2}K_h^{l_3}(t)u_3,$$

for all $l\in\mathbb{Z}_{\ge 0}$.

The above lemma follows immediately by Lemma 2.3.

3. Proof of Theorem 1.1

Let $\phi\in B_\varepsilon(G^{D,a}(x,x_1^2-x_2^2;L^2))$, $u\in G^{D,a}(J_h,K_h;\mathcal{X})$. We define $\Phi: u\mapsto \Phi u$ by

$$\Phi u = U_h\phi - i\lambda F_h\big(|u|^2u\big).$$

Let $r>0$, we define a metric space $(X(r),d)$ by

$$\begin{gathered}
X(r) = B_r(G^{D,a}(J_h,K_h;\mathcal{X})),\\
d(u,v) = \|u-v;G^{D,a}(J_h,K_h;\mathcal{X})\|.
\end{gathered}$$

We see that $(X(r),d)$ is a complete metric space. We show that Φ is a contraction mapping in $(X(r),d)$. By Lemma 2.1, we have

$$\begin{aligned}
\|\Phi u;\mathcal{X}\| &\le C\|\phi;L^2\| + C\|u^3;L^{4/3}(\mathbb{R};L^{4/3})\|\\
&\le C\|\phi;L^2\| + C\|u;L^4(\mathbb{R};L^4)\|^3.
\end{aligned}$$

Hence Φ is a mapping in $\mathcal{X}$. Since $M_h^{-1}A_\delta$ gives an analytic continuation

$$(M_h^{-1}A_\delta\psi)(t,x) = M_h^{-1}(t,x_1+2it\delta_1,x_2-2it\delta_2)\psi(t,x_1+2it\delta_1,x_2-2it\delta_2),$$

for $t\neq 0$, $x+2it\delta\in\mathbb{R}^2+2itD$ and $\psi\in G^D(J_h;L^\infty(\mathbb{R};L^2))$, by Lemmas 2.2 and 2.4, we have

$$\begin{aligned}
A_\delta K_h^l\Phi u &= U_h e^{\delta\cdot x}(x_1^2-x_2^2)^l\phi - i\lambda F_h\Big(A_\delta\big(K_h+8is\big)^l|u|^2u\Big)\\
&= U_h e^{\delta\cdot x}(x_1^2-x_2^2)^l\phi - i\lambda F_h\Big(A_\delta\sum_{l_1+l_2+l_3=l}\frac{(-1)^{l_2}l!}{l_1!l_2!l_3!}K_h^{l_1}u\overline{K_h^{l_2}u}K_h^{l_3}u\Big)\\
&= U_h e^{\delta\cdot x}(x_1^2-x_2^2)^l\phi - i\lambda\sum_{l_1+l_2+l_3=l}\frac{(-1)^{l_2}l!}{l_1!l_2!l_3!}F_h\Big(A_\delta K_h^{l_1}u\overline{A_{-\delta}K_h^{l_2}u}A_\delta K_h^{l_3}u\Big)
\end{aligned}$$

for all $l \in \mathbb{Z}_{\geq 0}$. In the same way as above, we have

$$\begin{aligned}
&\|A_\delta K_h^l \Phi u; \mathcal{X}\| \\
&\leq C\|e^{\delta\cdot x}(x_1^2 - x_2^2)^l \phi; L^2\| \\
&\quad + C \sum_{l_1+l_2+l_3=l} \frac{l!}{l_1! l_2! l_3!} \Big\| A_\delta K_h^{l_1} u \overline{A_{-\delta} K_h^{l_2} u} A_\delta K_h^{l_3} u; L^{4/3}(\mathbb{R}; L^{4/3}) \Big\| \\
&\leq C\|e^{\delta\cdot x}(x_1^2 - x_2^2)^l \phi; L^2\| + C \sum_{l_1+l_2+l_3=l} \frac{l!}{l_1! l_2! l_3!} \|A_\delta K_h^{l_1} u; L^4(\mathbb{R}; L^4)\| \\
&\quad \times \|A_{-\delta} K_h^{l_2} u; L^4(\mathbb{R}; L^4)\| \, \|A_\delta K_h^{l_3} u; L^4(\mathbb{R}; L^4)\|.
\end{aligned}$$

By the assumption $-D = D$, we have

$$\begin{aligned}
&\frac{a^l}{l!} \sup_{\delta\in D} \|A_\delta K_h^l \Phi u; \mathcal{X}\| \\
&\leq C\frac{a^l}{l!} \sup_{\delta\in D} \|e^{\delta\cdot x}(x_1^2 - x_2^2)^l \phi; L^2\| + C \sum_{l_1+l_2+l_3=l} \prod_{j=1}^{3} \frac{a^{l_j}}{l_j!} \sup_{\delta\in D} \|A_\delta K_h^{l_j} u; \mathcal{X}\|.
\end{aligned}$$

Therefore, we obtain

$$\|\Phi u; G^{D,a}(J_h, K_h; \mathcal{X})\| \leq C\|\phi; G^{D,a}(z, x_1^2 - x_2^2; L^2)\| + C\|u; G^{D,a}(J_h, K_h; \mathcal{X})\|^3.$$

Similarly, we have

$$\begin{aligned}
&\|\Phi u - \Phi v; G^{D,a}(J_h, K_h; \mathcal{X})\| \\
&\leq C\Big(\|u; G^{D,a}(J_h, K_h; \mathcal{X})\|^2 + \|v; G^{D,a}(J_h, K_h; \mathcal{X})\|^2\Big) \|u - v; G^{D,a}(J_h, K_h; \mathcal{X})\|
\end{aligned}$$

for all $u, v \in G^{D,a}(J_h, K_h; \mathcal{X})$. Hence, Φ is a mapping in $G^{D,a}(J_h, K_h; \mathcal{X})$. We take $\varepsilon, r > 0$ satisfying

$$\begin{gathered}
C\varepsilon + Cr^3 \leq r, \\
Cr^2 < 1.
\end{gathered}$$

Then Φ is a contraction mapping in $(X(r), d)$. This completes the proof of Theorem 1.1.

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