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LATTICE BOLTZMANN METHOD FOR COUPLED BURGERS EQUATIONS

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Abstract. In this paper, we propose a lattice Boltzmann model for coupled Burgers equations (CBEs). With a proper time-space scale and the Chapman-Enskog expansion, the governing equations are recovered successfully from the lattice Boltzmann equations, and the resulting local equilibrium distribution functions are also obtained. The partial derivative $\partial(uv)/\partial x$ in the model is treated as a source term and discretized with a 2nd-order central difference scheme. Numerical experiments show that the numerical results by the Lattice Boltzmann Method (LBM) either agree well with the corresponding exact solutions or are quite comparable with those available numerical results in the literature.

1. Introduction

In this article, we are concerned with the lattice Boltzmann numerical solutions of the coupled Burgers equations (CBEs) which were first derived by Esipov [7] to study the model of polydispersive sedimentation. The coupled Burgers equations can be written as the nonlinear partial differential equations

$$\frac{\partial u}{\partial t} + \eta u \frac{\partial u}{\partial x} + \alpha \frac{\partial (uv)}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0, \quad x \in [a,b],\ t \in [0,T], \tag{1.1}$$

$$\frac{\partial v}{\partial t} + \xi v \frac{\partial v}{\partial x} + \beta \frac{\partial (uv)}{\partial x} - \frac{\partial^2 v}{\partial x^2} = 0, \quad x \in [a,b],\ t \in [0,T] \tag{1.2}$$

with initial conditions

$$u(x,0) = \phi(x), \quad v(x,0) = \varphi(x), \tag{1.3}$$

and boundary conditions

$$u(a,t) = h_1(a,t), \quad u(b,t) = h_2(b,t), \quad v(a,t) = l_1(a,t), \quad v(b,t) = l_2(b,t), \tag{1.4}$$

where η, ξ are real constants, α, β are arbitrary constants depending on the system parameters such as the Péclet number, the Stokes velocity of particles due to gravity, and the Brownian diffusivity. This system is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or collision under the effect of gravity.

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Note that exact solutions of (1.1)–(1.4) are generally not available. Numerically, (1.1)–(1.4) have been studied by many authors with different methods, see [7]-[14]. Esipov [7] provided some numerical simulations for the CBEs and compared them with available experimental results. Kaya [9] used the decomposition method to successfully obtain some solutions of both homogenous and inhomogeneous CBEs in the form of a convergent power series. In order to solve the CBEs, Abdou and Soliman [1] used a variational iteration method, while Dehghan et al. [5] developed and used the Adomian-Páde technique. Khater et al. [10] used the Chebyshev spectral collocation method to solve the CBEs and successfully obtained some approximate solutions. Rashid et al. [17] proposed a Fourier pseudo-spectral method. Based on a Crank-Nicolson formulation for time integration and cubic B-spline functions for space integration, Mittal et al. [14] also proposed a useful numerical scheme for the CBEs.

Recently, the lattice Boltzmann method (LBM) has been developed as an alternative and promising numerical scheme for simulating fluid flows [2, 16, 4, 12] and for solving various mathematical physics problems [15, 21, 22, 6, 19, 18, 11]. The LBM can be either regarded as an extension of the lattice gas automaton [13] or as a special discrete form of the Boltzmann equation for the kinetic theory [8]. Unlike conventional numerical schemes which are on a basis of a discretization of partial differential equations under macroscopic conservation laws, the LBM is based on microscopic models and mesoscopic kinetic equations. Thanks to its advantages in geometrical flexibility, natural parallelity, simplicity of programming and numerical efficiency, the LBM has made great success in many fields and been extensively applied to solve numerous problems. It also shows some potentials to simulate nonlinear systems, such as reaction-diffusion equations [15], convection-diffusion equations [19], wave equations [21] and Burgers equations [6, 18]. In this paper, a lattice Boltzmann model for the CBEs is developed. We treat the partial derivative $\partial(uv)/\partial x$ as a source term and then use the second order central difference scheme to approximate such term. With a proper time-space scale and the Chapman-Enskog expansion, the resulting governing equations can be recovered from the lattice Boltzmann equations and the resulting local equilibrium distribution functions can also be obtained. Numerical results are presented and are compared with those available ones in the literature. Numerical experiments show that the LBM method is reliable and efficient for solving the CBEs.

This article is organized as follows. In Section 1, a brief introduction is given. In Section 2, a lattice Boltzmann model for the CBEs is developed. The application of the LBM to the coupled Burgers equations is also presented in the section. In Section 3, three numerical examples are given to illustrate the efficiency and accuracy of the LBM method numerically. Some conclusions are made in the final section.

2. Lattice Boltzmann model

To propose a lattice Boltzmann model for the coupled Burgers equations, we rewrite (1.1)–(1.2) in the form

$$\frac{\partial u}{\partial t} + \eta u \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = -\alpha \frac{\partial (uv)}{\partial x}, \tag{2.1}$$

$$\frac{\partial v}{\partial t} + \xi v \frac{\partial v}{\partial x} - \frac{\partial^2 v}{\partial x^2} = -\beta \frac{\partial (uv)}{\partial x}. \tag{2.2}$$

According to the theory of the lattice Boltzmann method, it consists of two steps: (1) colliding, which occurs when particles arrive at an interaction node from different directions and possibly change their moving directions in the light of scattering rules; (2) streaming, which occurs when each particle moves to the nearest node along its moving direction. Usually, with the single-relaxation-time and the Bhatnagar-Gross-Krook (BGK) approximation [3], these two steps can be combined together and converted into the lattice Boltzmann equations. The corresponding evolution equations of the distribution functions in the model read as

$$f_i(x+e_i\Delta t, t+\Delta t) - f_i(x,t) = -\frac{\Delta t}{\tau}(f_i(x,t) - f_i^{eq}(x,t)) + \Delta t \frac{F_i(x,t)}{b}, \tag{2.3}$$

$$g_i(x+e_i\Delta t, t+\Delta t) - g_i(x,t) = -\frac{\Delta t}{\tau}(g_i(x,t) - g_i^{eq}(x,t)) + \Delta t \frac{G_i(x,t)}{b}, \tag{2.4}$$

where $f_i(x,t)$ and $g_i(x,t)$ are the distribution functions of particles; $f_i^{eq}(x,t)$ and $g_i^{eq}(x,t)$ are the local equilibrium distribution functions of particles; Δx and Δt are the lattice space and time increments, respectively; $c = \Delta x/\Delta t$ is the particle speed and τ is the dimensionless relaxation time; $F_i(x,t)$ and $G_i(x,t)$ depend only on the right hand side of (2.1)–(2.2), respectively; $\{e_i, i = 0, 1, \dots, b-1\}$ is the set of discrete velocity directions, for the 3-bit model in the one-dimensional case, $\{e_0, e_1, e_2\} = \{0, c, -c\}$. The macroscopic variables, u and v are defined in terms of the distribution functions as

$$u = \sum_i f_i = \sum_i f_i^{eq}, \ v = \sum_i g_i = \sum_i g_i^{eq}. \tag{2.5}$$

To derive the macroscopic equations (2.1)–(2.2) (or (1.1)–(1.2)) from the lattice BGK model, the Chapman-Enskog expansion is performed under the assumption that the Kundsen number $\epsilon = l/L$ is small, where l is the mean free path and L is the characteristic length. The Chapman-Enskog expansion is then applied to $f_i(x,t)$ and $g_i(x,t)$, i.e.,

$$f_i = f_i^{eq} + \sum_{n=1}^{\infty} \epsilon^n f_i^{(n)} = f_i^{eq} + \epsilon f_i^{(1)} + \epsilon^2 f_i^{(2)} + \dots \tag{2.6}$$

$$g_i = g_i^{eq} + \sum_{n=1}^{\infty} \epsilon^n g_i^{(n)} = g_i^{eq} + \epsilon g_i^{(1)} + \epsilon^2 g_i^{(2)} + \dots \tag{2.7}$$

With time scale $t_1 = \epsilon t, t_2 = \epsilon^2 t$ and space scale $x_1 = \epsilon x$, then the time derivation and the space derivation can be expanded formally as:

$$\partial_t = \epsilon \partial_{t_1} + \epsilon^2 \partial_{t_2}, \quad \partial_x = \epsilon \partial_{x_1}. \tag{2.8}$$

In (2.3)–(2.4), assume further that the second order terms $F_i(x,t)$ and $G_i(x,t)$ take the form

$$F_i(x,t) = \epsilon^2 \tilde{F}_i(x,t), \quad G_i(x,t) = \epsilon^2 \tilde{G}_i(x,t) \tag{2.9}$$

for some functions $\tilde{F}_i(x,t), \tilde{G}_i(x,t)$.

For simplicity, we only consider how to recover (2.1) from (2.3), and (2.2) from (2.4) through a similar derivation. Performing the Taylor expansion on (2.3) up to terms with order of $O(\epsilon^3)$ and using (2.6), (2.8)–(2.9), one can get the partial differential equations in order of ϵ, ϵ^2 as

$$(\partial_{t_1} + e_i \partial_{x_1}) f_i^{eq} = -\frac{1}{\tau \Delta t} f_i^{(1)}, \tag{2.10}$$

$$\partial_{t_2} f_i^{eq} + (\partial_{t_1} + e_i\partial_{x_1}) f_i^{(1)} + \frac{\Delta t}{2}(\partial_{t_1} + e_i\partial_{x_1})^2 f_i^{eq} = -\frac{1}{\tau\Delta t} f_i^{(2)} + \frac{\tilde{F}_i(\mathbf{x}, t)}{b}. \quad (2.11)$$

Taking (2.10) times ϵ plus (2.11) times ϵ^2, summing over i and using (2.5), (2.8), (2.10) and

$$\sum_i f_i^{(k)} = 0 \quad (k \geq 1), \quad (2.12)$$

we have

$$\partial_t u + \partial_x(\sum_{i=0}^{b-1} e_i f_i^{eq}) + \epsilon^2 \Delta t(\frac{1}{2} - \tau) \sum_{i=0}^{b-1} (\partial_{t_1} + e_i\partial_{x_1})^2 f_i^{eq} = \sum_{i=0}^{b-1} \epsilon^2 \frac{\tilde{F}_i(x, t)}{b} + O(\epsilon^3). \quad (2.13)$$

Now, in view of (2.1), one can set

$$\sum_{i=0}^{b-1} e_i f_i^{eq} = \frac{\eta}{2} u^2, \quad (2.14)$$

$$\epsilon^2 \sum_{i=0}^{b-1} (\partial_{t_1} + e_i\partial_{x_1})^2 f_i^{eq} = \lambda \frac{\partial^2 u}{\partial x^2}, \quad (2.15)$$

$$\sum_{i=0}^{b-1} \epsilon^2 \frac{\tilde{F}_i(x, t)}{b} = \sum_{i=0}^{b-1} \frac{F_i(x, t)}{b} = -\alpha \frac{\partial(uv)}{\partial x}, \quad (2.16)$$

where

$$\lambda = \frac{1}{\Delta t(\tau - 0.5)}. \quad (2.17)$$

The CBEs with the second order accuracy of truncation error is hence obtained.

In addition, if summing (2.10) over i and using (2.5), (2.12) and (2.14), we obtain

$$\partial_{t_1} u + \eta u \partial_{x_1} u = 0. \quad (2.18)$$

On the other hand, using (2.10), (2.15) and (2.18), we get the 2nd-order moment equation of the resulting local equilibrium distribution function

$$\sum_{i=0}^{b-1} e_i e_i f_i^{eq} = \lambda u + \frac{\eta^2}{3} u^3. \quad (2.19)$$

In view of (2.5), (2.14) and (2.19), the equilibrium distribution functions $\{f_i^{eq}\}$ of 3-bit model can be obtained as

$$\begin{aligned} f_0^{eq} &= u - \frac{\eta^2}{3c^2} u^3 - \frac{\lambda u}{c^2}, \\ f_1^{eq} &= \frac{\lambda u}{2c^2} + \frac{\eta^2}{6c^2} u^3 + \frac{\eta}{4c} u^2, \\ f_2^{eq} &= \frac{\lambda u}{2c^2} + \frac{\eta^2}{6c^2} u^3 - \frac{\eta}{4c} u^2. \end{aligned} \quad (2.20)$$

Similarly, the equilibrium distribution functions for $\{g_i^{eq}\}$ can be achieved as

$$\begin{aligned} g_0^{eq} &= v - \frac{\xi^2}{3c^2}v^3 - \frac{\lambda v}{c^2}, \\ g_1^{eq} &= \frac{\lambda v}{2c^2} + \frac{\xi^2}{6c^2}v^3 + \frac{\xi}{4c}v^2, \\ g_2^{eq} &= \frac{\lambda v}{2c^2} + \frac{\xi^2}{6c^2}v^3 - \frac{\xi}{4c}v^2. \end{aligned} \tag{2.21}$$

Next, we use the second order central difference scheme to approximate partial derivatives $\partial(uv)/\partial x$ in (2.1)–(2.2), i.e., the source terms

$$\begin{aligned} F(x,t) &= -\alpha\frac{\partial(uv)}{\partial x} \\ &= -\alpha u\frac{\partial v}{\partial x} - \alpha v\frac{\partial u}{\partial x} \\ &\approx -\alpha\Big[u\frac{v(x+\Delta x,t)-v(x-\Delta x,t)}{2\Delta x} \\ &\quad + v\frac{u(x+\Delta x,t)-u(x-\Delta x,t)}{2\Delta x})\Big], \end{aligned} \tag{2.22}$$

$$\begin{aligned} G(x,t) &= -\beta\frac{\partial(uv)}{\partial x} \\ &= -\beta u\frac{\partial v}{\partial x} - \beta v\frac{\partial u}{\partial x} \\ &\approx -\beta\Big[u\frac{v(x+\Delta x,t)-v(x-\Delta x,t)}{2\Delta x} \\ &\quad + v\frac{u(x+\Delta x,t)-u(x-\Delta x,t)}{2\Delta x})\Big]. \end{aligned} \tag{2.23}$$

Finally, in order not to make large errors on the boundary which can influence the global evolution, we use the three-adjacent-point difference scheme (second order accuracy) to approximate partial derivatives $v\frac{\partial u}{\partial x}$, $u\frac{\partial v}{\partial x}$ on the boundary, i.e.,

$$v\frac{\partial u}{\partial x}|_{x=a} = v(a,t)\frac{-3u(a,t)+4u(a+\Delta x,t)-u(a+2\Delta x,t)}{2\Delta x}, \tag{2.24}$$

$$u\frac{\partial v}{\partial x}|_{x=a} = u(a,t)\frac{-3v(a,t)+4v(a+\Delta x,t)-v(a+2\Delta x,t)}{2\Delta x}, \tag{2.25}$$

$$v\frac{\partial u}{\partial x}|_{x=b} = v(b,t)\frac{u(b-2\Delta x,t)-4u(b-\Delta x,t)+3u(b,t)}{2\Delta x}, \tag{2.26}$$

$$u\frac{\partial v}{\partial x}|_{x=b} = u(b,t)\frac{v(b-2\Delta x,t)-4v(b-\Delta x,t)+3v(b,t)}{2\Delta x}. \tag{2.27}$$

3. Numerical results and analysis

In this section, numerical results of the 3-bit LBM for the coupled Burgers equations are given. The efficiency and accuracy of the LBM are demonstrated through three numerical examples. In particular, we compare the LBM numerical simulations with the exact solutions and/or some available numerical results [10, 14] in the literature. Here, we use the discrete L_∞-norm error and the relative discrete L_2-norm error as follows:

$$\|E(u)\|_\infty = \max_{0\le i\le N}\{|u^{\text{exact}}(x_i,t) - u^{\text{num}}(x_i,t)|\},$$

$$\|E(u)\|_2 = \Big(\sum_{i=0}^{N} |u^{\rm exact}(x_i,t) - u^{\rm num}(x_i,t)|^2\Big)^{1/2} \Big/ \Big(\sum_{i=0}^{N} |u^{\rm exact}(x_i,t)|^2\Big)^{1/2},$$

where N is the number of the lattice, $u^{\rm num}(x_i,t)$ and $u^{\rm exact}(x_i,t)$ represent the numerical solution and the exact solution, respectively.

Example 3.1. To examine the performance of the 3-bit LBM for solving Burgers equations, we set the parameters $\eta = \xi = -2$ and $\alpha = \beta = 1$. With initial condition

$$u(x,0) = v(x,0) = \sin(x),$$

Equations (1.1)–(1.2) read as follows

$$\frac{\partial u}{\partial t} - 2u\frac{\partial u}{\partial x} + \frac{\partial (uv)}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0, \tag{3.1}$$

$$\frac{\partial v}{\partial t} - 2v\frac{\partial v}{\partial x} + \frac{\partial (uv)}{\partial x} - \frac{\partial^2 v}{\partial x^2} = 0, \tag{3.2}$$

whose exact solution is [9]

$$u(x,t) = v(x,t) = e^{-t}\sin(x).$$

To obtain the LBM numerical solution, we choose $\Delta x = 0.04$, $\Delta t = 0.001$, $\lambda = c^2/2$, and the domain $\{x, x \in [-\pi,\pi]\}$. As before, $c = \Delta x/\Delta t$ is the particle speed. To test the computational efficiency and accuracy of the LBM, we use both L_2-norm and L_∞-norm errors for u, v as shown in Table 1. The numerical solution $u(x,t)$ is depicted in Figure 1 for $t = 0.1, 0.5, 1$. Because of the symmetry of u and v, it is also true for $v(x,t)$.

Table 1. L_∞, L_2 errors for u, v at different time stages

	u		v	
t	L_∞	L_2	L_∞	L_2
0.1	8.6936e–4	6.1750e–4	8.6936e–4	6.1750e–4
0.5	5.4963e–4	3.1121e–4	5.4963e–4	3.1121e–4
1	2.3350e–4	1.3417e–4	2.3350e–4	1.3417e–4

Example 3.2. Choosing $\eta = \xi = 2$, the CBEs in (1.1)–(1.2) are

$$\frac{\partial u}{\partial t} + 2u\frac{\partial u}{\partial x} + \alpha\frac{\partial (uv)}{\partial x} - \frac{\partial^2 u}{\partial x^2} = 0, \tag{3.3}$$

$$\frac{\partial v}{\partial t} + 2v\frac{\partial v}{\partial x} + \beta\frac{\partial (uv)}{\partial x} - \frac{\partial^2 v}{\partial x^2} = 0. \tag{3.4}$$

If we use the initial conditions

$$u(x,0) = a_0(1 - \tanh(Dx)), \quad v(x,0) = a_0[\frac{2\beta - 1}{2\alpha - 1} - \tanh(Dx)],$$

then the corresponding exact solution (u,v) for (3.3)–(3.4) is [20]

$$u(x,t) = a_0(1 - \tanh(D(x - 2Dt))),$$
$$v(x,t) = a_0[\frac{2\beta - 1}{2\alpha - 1} - \tanh(D(x - 2Dt))],$$

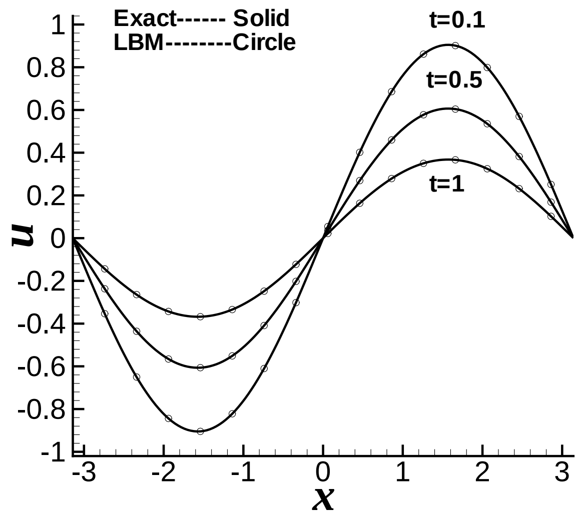

FIGURE 1. A comparison of numerical and exact solutions in Example 3.1

where $D = a_0(4\alpha\beta - 1)/(4\alpha - 2)$, a_0 is an arbitrary constant. The LBM numerical results of this example are showed in Tables 2–3 by using the domain $[-10, 10]$, the L_∞ error and the relative L_2 error with $\Delta t = 0.01$, $\Delta x = 1$, $c = \Delta x/\Delta t$, $\lambda = c^2/3$. In both tables, we compare the LBM results with those available ones in the literature [10, 14]. As is seen from the tables, the 3-bit lattice Boltzmann method is quite comparable with those methods used in [10, 14].

TABLE 2. Comparison of L_∞, L_2 errors for $u(x,t)$ in Example 3.2.

			Khater [10]		Mittal [14]		LBM	
t	α	β	$\lVert E(u)\rVert_\infty$	$\lVert E(u)\rVert_2$	$\lVert E(u)\rVert_\infty$	$\lVert E(u)\rVert_2$	$\lVert E(u)\rVert_\infty$	$\lVert E(u)\rVert_2$
0.5	0.1	0.3	4.38e–5	1.44e–3	4.167e–5	6.736e–4	4.028e–5	6.589e–4
	0.3	0.03	4.58e–5	6.68e–4	4.590e–5	7.326e–4	4.936e–5	7.335e–4
1	0.1	0.3	8.66e–5	1.27e–3	8.258e–5	1.325e–3	8.140e–5	1.298e–3
	0.3	0.03	9.16e–5	1.30e–3	9.182e–5	1.452e–3	9.014e–5	1.422e–3

Example 3.3. Consider the CBEs (1.1)–(1.2) with initial condition [14]

$$u(x,0) = \begin{cases} \sin(2\pi x), & 0 \le x \le 0.5, \\ 0, & 0.5 < x \le 1, \end{cases}$$

$$v(x,0) = \begin{cases} 0, & 0 \le x \le 0.5, \\ -\sin(2\pi x), & 0.5 < x \le 1, \end{cases}$$

TABLE 3. Comparison of L_∞, L_2 errors for $v(x,t)$ in Example 3.2.

			Khater [10]		Mittal [14]		LBM	
t	α	β	$\|E(v)\|_\infty$	$\|E(v)\|_2$	$\|E(v)\|_\infty$	$\|E(v)\|_2$	$\|E(v)\|_\infty$	$\|E(v)\|_2$
0.5	0.1	0.3	4.99e–5	5.42e–4	1.480e–4	9.057e–4	2.021e–5	4.705e–4
	0.3	0.03	1.81e–4	1.20e–3	5.729e–4	1.591e–3	1.862e–4	1.301e–3
1	0.1	0.3	9.92e–5	1.29e–3	4.770e–5	1.252e–3	4.029e–5	9.389e–4
	0.3	0.03	3.62e–4	2.35e–3	3.617e–4	2.250e–3	3.657e–4	2.557e–3

and zero boundary conditions. The numerical solution is computed on the domain $[0,1]$. In Tables 4 and 5, we present a comparison of the 3-bit LBM numerical results and those obtained by the cubic B-spline collocation scheme [14] in term of maximum values of u,v for different times ($i.e., t = 0.1, 0.2, 0.3, 0.4$) and different values of α,β. Here, we choose $\eta=\xi=2$, $N=50, \Delta t = 0.0001$ and $\lambda = c^2/2$. The numerical solutions (u,v) are also depicted in Figures 2–3 with $\alpha=\beta=10$ and $\alpha=\beta=100$, respectively. A sharp decay is clearly displayed in Figure 3 when α,β are large. Note that the 3-bit LBM numerical results also agree well with those in [14].

TABLE 4. Comparison of maximum values of $u(x,t), v(x,t)$ for $\alpha = \beta = 10$.

	Mittal [14]		LBM		Mittal [14]		LBM	
t	$u^{\max}$	x	$u^{\max}$	x	$v^{\max}$	x	$v^{\max}$	x
0.1	0.14456	0.58	0.14464	0.58	0.14306	0.66	0.14327	0.66
0.2	0.05237	0.54	0.05241	0.54	0.04697	0.56	0.04704	0.56
0.3	0.01932	0.52	0.01934	0.52	0.01725	0.52	0.01727	0.52
0.4	0.00718	0.50	0.00719	0.50	0.00641	0.50	0.00642	0.50

TABLE 5. Comparison of maximum values of $u(x,t), v(x,t)$ for $\alpha = \beta = 100$.

	Mittal [14]		LBM		Mittal [14]		LBM	
t	$u^{\max}$	x	$u^{\max}$	x	$v^{\max}$	x	$v^{\max}$	x
0.1	0.04175	0.46	0.04167	0.46	0.05065	0.76	0.05080	0.76
0.2	0.01479	0.58	0.01478	0.58	0.01033	0.64	0.001035	0.64
0.3	0.00534	0.54	0.00534	0.54	0.00350	0.56	0.00351	0.56
0.4	0.00198	0.52	0.00198	0.52	0.00129	0.52	0.00129	0.52

To the best of our knowledge, there is no exact solution available in the literature for general values of η,ξ. Hence, numerical experiments are important in study of

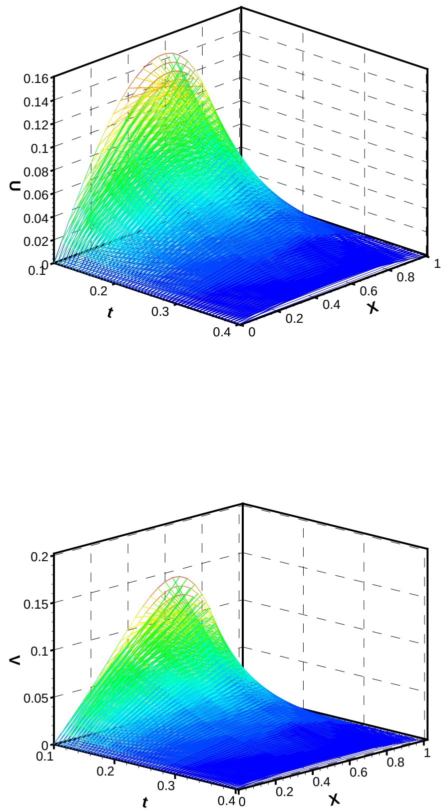

FIGURE 2. Numerical solution $(u(x,t), v(x,t))$ of Example 3.3 for $\alpha = \beta = 10$.

the behavior of solutions for larger values of η, ξ. With $\alpha = \beta = 10$, numerical solutions (u, v) in Example 3.3 are plotted in Figures 4–5 to illustrate the effect of different values of η, ξ, i.e., $\eta = \xi = 20, 200$. From the figures it can be seen that the solution (u, v) decays to zero as time t or the value of η, ξ increases. It can also be observed that the LBM is capable of finding numerical solutions for larger values of η, ξ. As a comparison, if taking the numerical solution on a fine mesh of 500 lattices as the "exact" solution, then one can see from Figures 4–5 that the numerical solution on a much coarser mesh (containing 50 lattices) agrees well with such "exact" solution.

Conclusions. In this article, a 3-bit lattice Boltzmann model for the CBEs is proposed. The partial derivative $\partial(uv)/\partial x$ is treated as a source term and discretized with a second-order central difference scheme. By choosing an appropriate time-space scale and applying the Chapman-Enskog expansion, the resulting governing

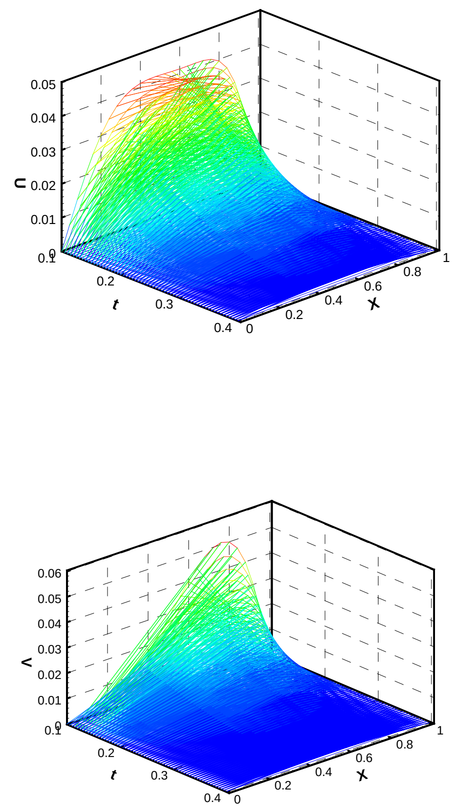

FIGURE 3. Numerical solution $(u(x,t), v(x,t))$ of Example 3.3 for $\alpha = \beta = 100$.

equations are recovered from the lattice Boltzmann equations and the resulting local equilibrium distribution functions are also obtained. Then, three numerical examples are presented to illustrate the efficiency of the LBM. Numerical experiments show that LBM results agree well with either the exact solutions or other available results. Certain important behaviors of general CBEs are also verified numerically. Undoubtedly, it is worth further investigating the LBM in solving other types of nonlinear partial differential equations in future.

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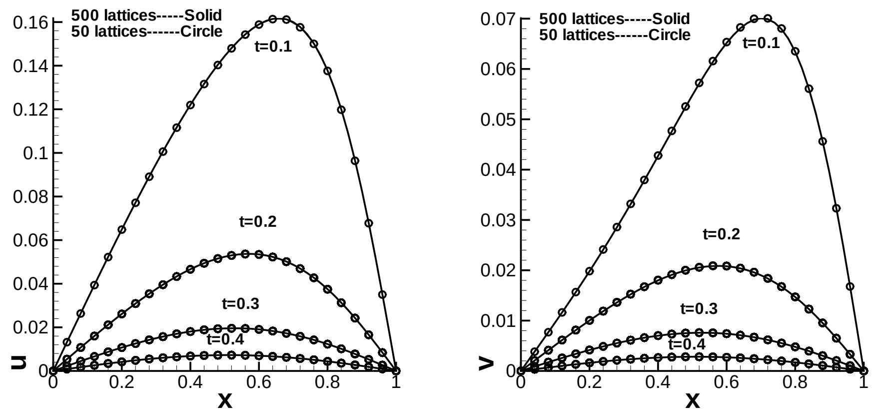

FIGURE 4. Numerical solution (u, v) at different time levels for $\alpha = \beta = 10$ and $\eta = \xi = 20$.

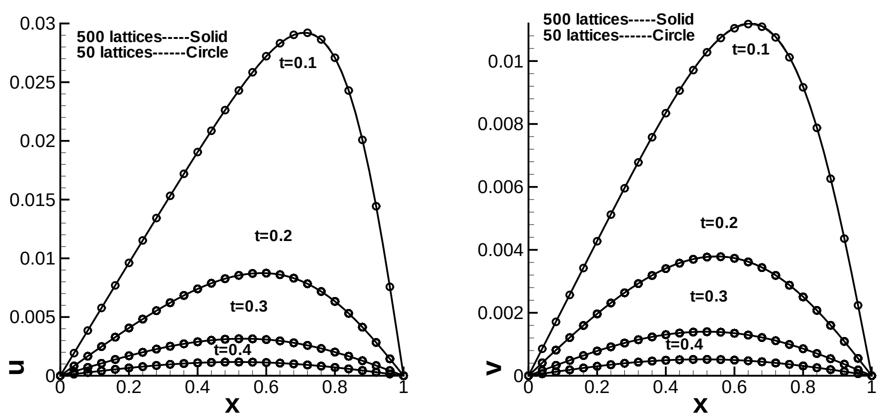

FIGURE 5. Numerical solution (u, v) at different time levels for $\alpha = \beta = 10$ and $\eta = \xi = 200$.

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