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LYAPUNOV-TYPE INEQUALITIES FOR n-DIMENSIONAL QUASILINEAR SYSTEMS

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Abstract. In this article, inspired by the paper of Yang et al [12], we establish new versions of Lyapunov-type inequalities for a certain class of Dirichlet quasilinear systems.

1. Introduction

In this article, we prove generalized Lyapunov-type inequalities for a special case of the system

$$-\big(r_k(x)\phi_{p_k}(u_k')\big)' = f_k(x)\phi_{\alpha_{kk}}(u_k)\prod_{i=1,\, i\neq k}^{n} |u_i|^{\alpha_{ki}}, \tag{1.1}$$

where $n \in \mathbb{N}$, $\phi_\gamma(u) = |u|^{\gamma-2}u$, $\gamma > 1$, r_k, $f_k \in C([a,b],\mathbb{R})$, $r_k(x) > 0$ for $k = 1, 2, \dots, n$ and $x \in \mathbb{R}$. Let $(u_1(x), u_2(x), \dots, u_n(x))$ be a real nontrivial solution of the system (1.1) such that

$$u_k(a) = u_k(b) = 0 \quad k = 1, 2, \dots, n, \tag{1.2}$$

for $a, b \in \mathbb{R}$ with $a < b$ are consecutive zeros of u_k, and u_k are not identically zero on $[a,b]$, $1 < p_k < \infty$ and α_{ki} are nonnegative constants, for $k, i = 1, 2, \dots, n$.

Lyapunov [6] proved the following remarkable result, for problem (1.1)-(1.2) with $n = 1$, $p_1 = 2$, and $r_1(x) = 1$,

$$-u_1'' = f_1(x)u_1, \tag{1.3}$$

$$u_1(a) = u_1(b) = 0. \tag{1.4}$$

Theorem 1.1. *If $f_1 \in C([a,b],[0,\infty))$ and $u_1(x)$ is a nontrivial solution on $[a,b]$ for the problem (1.3)-(1.4), then the so-called Lyapunov inequality holds,*

$$\frac{4}{b-a} \le \int_a^b f_1(s)ds\,. \tag{1.5}$$

Lyapunov-type inequalities have been studied extensively; see for example the references in this article and their references. Çakmak and Tiryaki [3] obtained the following inequality for system (1.1) with $n = 2$ under the condition $\sum_{k=1}^{2} \frac{\alpha_{ik}}{p_k} = 1$ for $i = 1, 2$.

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Theorem 1.2. *If $f_k \in C([a,b],\mathbb{R})$ for $k=1,2$ and $(u_1(x),u_2(x))$ is a nontrivial solution on $[a,b]$ for the system* (1.1) *with $n=2$, then the inequality*

$$\begin{aligned} 2^{\alpha_{21}+\alpha_{12}} &\le \Big(\int_a^b f_1^+(s)ds\Big)^{\frac{\alpha_{21}}{p_1}}\Big(\int_a^b f_2^+(s)ds\Big)^{\frac{\alpha_{12}}{p_2}} \\ &\times \Big(\int_a^b (r_1(s))^{\frac{1}{1-p_1}}ds\Big)^{\frac{\alpha_{21}(p_1-1)}{p_1}}\Big(\int_a^b (r_2(s))^{\frac{1}{1-p_2}}ds\Big)^{\frac{\alpha_{12}(p_2-1)}{p_2}} \end{aligned} \tag{1.6}$$

holds, where $f_k^+(x)=\max\{0,f_k(x)\}$ is the nonnegative part of $f_k(x)$ for $k=1,2$.

Recently, Çakmak and Tiryaki [4] obtained the following inequality for system (1.1) with $r_k(x)=1$ and $\alpha_{ik}=\alpha_{kk}$, $k,i=1,2,\dots,n$, under the condition $\sum_{k=1}^n \frac{\alpha_{kk}}{p_k}=1$.

Theorem 1.3. *If $f_k \in C([a,b],\mathbb{R})$ for and $(u_1(x),u_2(x),\dots,u_n(x))$ is a nontrivial solution on $[a,b]$ for system* (1.1) *with $r_k(x)=1$ and $\alpha_{ik}=\alpha_{kk}$, $k,i=1,2,\dots,n$, then the inequality*

$$\prod_{k=1}^n [(c_k-a)^{1-p_k}+(b-c_k)^{1-p_k}]^{\alpha_{kk}/p_k} \le \prod_{k=1}^n \Big(\int_a^b f_k^+(s)ds\Big)^{\alpha_{kk}/p_k} \tag{1.7}$$

holds, where $|u_k(c_k)|=\max_{a<x<b}|u_k(x)|$ and $f_k^+(x)=\max\{0,f_k(x)\}$ for $k=1,2,\dots,n$.

Throughout this article, for the sake of brevity, we denote

$$D_k(x)=\frac{[\xi_k(x)\eta_k(x)]^{p_k-1}}{\xi_k^{p_k-1}(x)+\eta_k^{p_k-1}(x)}, \quad E_k(x)=2^{p_k-2}\Big(\frac{\xi_k(x)\eta_k(x)}{\xi_k(x)+\eta_k(x)}\Big)^{p_k-1}, \tag{1.8}$$

$$F_k=2^{-p_k}(\xi_k(x)+\eta_k(x))^{p_k-1}=2^{-p_k}\Big(\int_a^b r_k^{1/(1-p_k)}(s)ds\Big)^{p_k-1}, \tag{1.9}$$

where

$$\xi_k(x)=\int_a^x r_k^{1/(1-p_k)}(s)ds, \quad \eta_k(x)=\int_x^b r_k^{1/(1-p_k)}(s)ds \tag{1.10}$$

for $k=1,2,\dots,n$.

Recently, Yang et al. [12] obtained the following inequality for system (1.1).

Theorem 1.4. *Assume that there exist nontrivial solutions $(e_1,e_2,\dots,e_n)$ of the linear homogeneous system*

$$e_k\big(1-\frac{\alpha_{kk}}{p_k}\big)-\sum_{i=1,\, i\ne k}^n \frac{\alpha_{ik}}{p_k}e_i=0, \tag{1.11}$$

where $e_k\ge 0$ for $k=1,2,\dots,n$ and $\sum_{k=1}^n e_k^2>0$. If $f_k\in C([a,b],\mathbb{R})$ for $k=1,2,\dots,n$ and $(u_1(x),u_2(x),\dots,u_n(x))$ is a nontrivial solution on $[a,b]$ for system (1.1)*, then the inequality*

$$1<\prod_{k=1}^n \Big(F_k\int_a^b f_k^+(s)ds\Big)^{e_k} \tag{1.12}$$

holds, where $f_k^+(x)=\max\{0,f_k(x)\}$ for $k=1,2,\dots n$.

Our motivation comes from the recent papers of Çakmak and Tiryaki [3, 4], Sim and Lee [8], Tang and He [9], and Yang et al. [12]. In this article, we state and prove new generalized Lyapunov-type inequalities for system (1.1) under the condition $\alpha_{ki} = \alpha_{ik}$ for $k, i = 1, 2, \dots, n$.

Since our attention is restricted to the Lyapunov-type inequalities for the quasi-linear systems of differential equations, we shall assume the existence of the nontrivial solution of the system (1.1). For readers interested in the existence of solutions of these type systems, we refer to the paper by Afrouzi and Heidarkhani [1].

Now, we present some inequalities on $D_k(x)$, $E_k(x)$, and F_k for $k = 1, 2, \dots, n$ which are useful in the comparison of our main results. We know that since the function $h(x) = x^{p_k-1}$ is concave for $x > 0$ and $1 < p_k < 2$, Jensen's inequality $h(\frac{\omega+v}{2}) \geq \frac{1}{2}[h(\omega) + h(v)]$ with $\omega = 1/\xi_k(x)$ and $v = 1/\eta_k(x)$ implies

$$D_k(x) \geq E_k(x) \tag{1.13}$$

for $1 < p_k < 2$, $k = 1, 2, \dots, n$. If $p_k > 2$ for $k = 1, 2, \dots, n$, then the function $h(x) = x^{p_k-1}$ is convex for $x > 0$. Thus, the inequality (1.13) is reversed; i.e.,

$$D_k(x) \leq E_k(x) \tag{1.14}$$

for $p_k > 2$, $k = 1, 2, \dots, n$. In addition, since the function $l(x) = x^{1-p_k}$ is convex for $x > 0$ and $p_k > 1$, Jensen's inequality $l(\frac{\omega+v}{2}) \leq \frac{1}{2}[l(\omega) + l(v)]$ with $\omega = \xi_k(x)$ and $v = \eta_k(x)$ implies

$$D_k(x) \leq F_k \tag{1.15}$$

for $k = 1, 2, \dots, n$. By using inequality

$$4AB \leq (A+B)^2 \tag{1.16}$$

with $A = \xi_k(x) > 0$ and $B = \eta_k(x) > 0$ for $k = 1, 2, \dots, n$ in $E_k(x)$, we obtain the inequality

$$E_k(x) \leq F_k \tag{1.17}$$

for $k = 1, 2, \dots, n$.

2. Main results

One of the main results of this paper is the following theorem.

Theorem 2.1. *Assume that there exist nontrivial solutions $(e_1, e_2, \dots, e_n)$ of the linear homogeneous system*

$$e_k\Big(1 - \frac{\alpha_{kk}}{p_k}\Big) - \sum_{i=1,\, i\neq k}^{n} \frac{\alpha_{ki}}{p_k} e_i = 0, \tag{2.1}$$

where $e_k \geq 0$ for $k = 1, 2, \dots, n$ and $\sum_{k=1}^{n} e_k^2 > 0$. If $f_k \in C([a, b], \mathbb{R})$ for $k = 1, 2, \dots, n$ and $(u_1(x), u_2(x), \dots, u_n(x))$ is a nontrivial solution on $[a, b]$ for system (1.1) with $\alpha_{ki} = \alpha_{ik}$ for $k, i = 1, 2, \dots, n$, then the inequality

$$1 < \prod_{k=1}^{n} \Big[\int_a^b f_k^+(s) \prod_{i=1}^{n} D_i^{\alpha_{ki}/p_i}(s)ds \Big]^{e_k} \tag{2.2}$$

holds, where $f_k^+(x) = \max\{0, f_k(x)\}$ for $k = 1, 2, \dots n$.

Proof. Let $u_k(a) = 0 = u_k(b)$ for $k = 1, 2, \dots, n$ where $n \in \mathbb{N}$, $a, b \in \mathbb{R}$ with $a < b$ are consecutive zeros and u_k for $k = 1, 2, \dots, n$ are not identically zero on $[a, b]$. By using $u_k(a) = 0$ and Hölder's inequality, we obtain

$$\begin{aligned} |u_k(x)| &\le \int_a^x |u_k'(s)| ds \\ &\le \Big(\int_a^x r_k^{1/(1-p_k)}(s) ds \Big)^{(p_k-1)/p_k} \Big(\int_a^x r_k(s)|u_k'(s)|^{p_k} ds \Big)^{1/p_k} \\ &= \xi_k^{(p_k-1)/p_k}(x) \Big(\int_a^x r_k(s)|u_k'(s)|^{p_k} ds \Big)^{1/p_k} \end{aligned}$$

for $k = 1, 2, \dots, n$ and $x \in [a, b]$. Thus, we have

$$|u_k(x)|^{p_k} \xi_k^{1-p_k}(x) \le \int_a^x r_k(s)|u_k'(s)|^{p_k} ds \tag{2.3}$$

for $k = 1, 2, \dots, n$ and $x \in [a, b]$. Similarly, by using $u_k(b) = 0$ and Hölder's inequality, we obtain

$$|u_k(x)|^{p_k} \eta_k^{1-p_k}(x) \le \int_x^b r_k(s)|u_k'(s)|^{p_k} ds \tag{2.4}$$

for $k = 1, 2, \dots, n$ and $x \in [a, b]$. Adding (2.3) and (2.4), we have

$$|u_k(x)|^{p_k} \le D_k(x) \int_a^b r_k(s)|u_k'(s)|^{p_k} ds \tag{2.5}$$

for $k = 1, 2, \dots, n$ and $x \in [a, b]$. After that by using a technique similar to the one in [9, Theorem 3.1], it can be showed that the equality case in (2.5) does not hold. Thus, we have

$$|u_k(x)|^{p_k} < A_k D_k(x), \quad x \in (a, b), \tag{2.6}$$

where $A_k = \int_a^b r_k(s)|u_k'(s)|^{p_k} ds$ for $k = 1, 2, \dots, n$. If we take the $\frac{\alpha_{ki}}{p_k}$-th power of both side of inequality (2.6), we obtain

$$|u_k(x)|^{\alpha_{ki}} < A_k^{\alpha_{ki}/p_k} D_k^{\alpha_{ki}/p_k}(x), \tag{2.7}$$

for $k, i = 1, 2, \dots, n$.

Multiplying both sides of (2.7) with $i = k$ by $f_k^+(x) \prod_{i=1,\, i\neq k}^n |u_i(x)|^{\alpha_{ki}}$ for $k = 1, 2, \dots, n$, integrating from a to b, we have

$$\int_a^b f_k(s) \prod_{i=1}^n |u_i(s)|^{\alpha_{ki}} ds < A_k^{\alpha_{kk}/p_k} \int_a^b f_k^+(s) D_k^{\alpha_{kk}/p_k}(s) \prod_{i=1,\, i\neq k}^n |u_i(s)|^{\alpha_{ki}} ds \tag{2.8}$$

for $k = 1, 2, \dots, n$. On the other hand, multiplying the k-th equation of system (1.1) by u_k and integrating from a to b, we get

$$A_k = \int_a^b r_k(s)|u_k'(s)|^{p_k} ds = \int_a^b f_k(s) \prod_{i=1}^n |u_i(s)|^{\alpha_{ki}} ds \tag{2.9}$$

for $k = 1, 2, \dots, n$. By using (2.9) in (2.8), we have

$$A_k < A_k^{\alpha_{kk}/p_k} \int_a^b f_k^+(s) D_k^{\alpha_{kk}/p_k}(s) \prod_{i=1,\, i\neq k}^n |u_i(s)|^{\alpha_{ki}} ds$$

and hence from $\alpha_{ki} = \alpha_{ik}$ for $k, i = 1, 2, \dots, n$

$$A_k < A_k^{\alpha_{kk}/p_k} \int_a^b f_k^+(s) D_k^{\alpha_{kk}/p_k}(s) \prod_{i=1,\, i\neq k}^{n} |u_i(s)|^{\alpha_{ik}} ds \tag{2.10}$$

for $k = 1, 2, \dots, n$. Therefore, by using (2.7) in (2.10), we have

$$A_k^{1-\alpha_{kk}/p_k} < \int_a^b f_k^+(s) D_k^{\alpha_{kk}/p_k}(s) \prod_{i=1,\, i\neq k}^{n} A_i^{\alpha_{ik}/p_i} D_i^{\alpha_{ik}/p_i}(s) ds,$$

and hence

$$A_k^{1-\alpha_{kk}/p_k} < \prod_{i=1,\, i\neq k}^{n} A_i^{\alpha_{ik}/p_i} \int_a^b f_k^+(s) \prod_{i=1}^{n} D_i^{\alpha_{ik}/p_i}(s) ds \tag{2.11}$$

for $k = 1, 2, \dots, n$. Raising the both sides of the inequality (2.11) to the power e_k for each $k = 1, 2, \dots, n$, respectively, and multiplying the resulting inequalities side by side, we obtain

$$\prod_{k=1}^{n} A_k^{e_k(1-\alpha_{kk}/p_k)} < \prod_{k=1}^{n} \Big[\prod_{i=1,\, i\neq k}^{n} A_i^{\alpha_{ik}/p_i} \Big]^{e_k} \prod_{k=1}^{n} \Big[\int_a^b f_k^+(s) \prod_{i=1}^{n} D_i^{\alpha_{ik}/p_i}(s) ds \Big]^{e_k}$$

and hence

$$\prod_{k=1}^{n} A_k^{e_k(1-\alpha_{kk}/p_k)} < \Big[\prod_{k=1}^{n} A_k^{\sum_{i=1,\, i\neq k}^{n} \frac{\alpha_{ki}}{p_k} e_i} \Big] \prod_{k=1}^{n} [\int_a^b f_k^+(s) \prod_{i=1}^{n} D_i^{\alpha_{ik}/p_i}(s) ds]^{e_k}. \tag{2.12}$$

It is easy to see that by using a technique similar to the one in [9, Theorem 3.1], we obtain the inequalities $A_k > 0$ for $k = 1, 2, \dots, n$. Thus, we have

$$\prod_{k=1}^{n} A_k^{\theta_k} < \prod_{k=1}^{n} \Big[\int_a^b f_k^+(s) \prod_{i=1}^{n} D_i^{\alpha_{ki}/p_i}(s) ds \Big]^{e_k}, \tag{2.13}$$

where $\theta_k = e_k(1 - \frac{\alpha_{kk}}{p_k}) - \sum_{i=1,\, i\neq k}^{n} \frac{\alpha_{ki}}{p_k} e_i$ for $k = 1, 2, \dots, n$. By assumption, system (2.1) has nonzero solutions $(e_1, e_2, \dots, e_n)$ such that $\theta_k = 0$ for $k = 1, 2, \dots, n$, where $e_k \geq 0$ for $k = 1, 2, \dots, n$ and at least one $e_j > 0$ for $j = \{1, 2, \dots, n\}$. Choosing one of the solutions $(e_1, e_2, \dots e_n)$, we obtain from (2.13) the inequality (2.2). This completes the proof. □

Another main result of this paper is the following theorem.

Theorem 2.2. *Assume that there exist nontrivial solutions $(e_1, e_2, \dots, e_n)$ of system (2.1). If $f_k \in C([a, b], \mathbb{R})$ for $k = 1, 2, \dots, n$ and $(u_1(x), u_2(x), \dots, u_n(x))$ is a nontrivial solution on $[a, b]$ for the system (1.1) with $\alpha_{ki} = \alpha_{ik}$ for $k, i = 1, 2, \dots, n$, then the inequality*

$$1 < \prod_{k=1}^{n} \Big[\int_a^b f_k^+(s) \prod_{i=1}^{n} E_i^{\alpha_{ki}/p_i}(s) ds \Big]^{e_k} \tag{2.14}$$

holds, where $f_k^+(x) = \max\{0, f_k(x)\}$ for $k = 1, 2, \dots n$.

Proof. Let $u_k(a) = 0 = u_k(b)$ for $k = 1, 2, \dots, n$ where $n \in \mathbb{N}$, $a, b \in \mathbb{R}$ with $a < b$ are consecutive zeros and u_k for $k = 1, 2, \dots, n$ are not identically zero on $[a, b]$. As in the proof of Theorem 2.1, we have inequalities (2.3) and (2.4). Multiplying the inequalities (2.3) and (2.4) by $\eta_k^{p_k-1}(x)$ and $\xi_k^{p_k-1}(x)$, $k=1,2,\dots,n$, respectively, we obtain

$$\eta_k^{p_k-1}(x)|u_k(x)|^{p_k} \le \eta_k^{p_k-1}(x)\xi_k^{p_k-1}(x)\int_a^x r_k(s)|u_k'(s)|^{p_k}ds \tag{2.15}$$

and

$$\xi_k^{p_k-1}(x)|u_k(x)|^{p_k} \le \xi_k^{p_k-1}(x)\eta_k^{p_k-1}(x)\int_x^b r_k(s)|u_k'(s)|^{p_k}ds \tag{2.16}$$

for $k=1,2,\dots,n$ and $x\in[a,b]$. Thus, adding the inequalities (2.15) and (2.16), we have

$$|u_k(x)|^{p_k}(\xi_k^{p_k-1}(x)+\eta_k^{p_k-1}(x)) \le (\xi_k(x)\eta_k(x))^{p_k-1}\int_a^b r_k(s)|u_k'(s)|^{p_k}ds \tag{2.17}$$

for $k=1,2,\dots,n$ and $x\in[a,b]$. It is easy to see that the functions $\xi_k^{p_k-1}(x)+\eta_k^{p_k-1}(x)$ take the minimum values at $c_k\in(a,b)$ such that $\xi_k(c_k)=\eta_k(c_k)$ for $k=1,2,\dots,n$. Thus, we obtain

$$|u_k(x)|^{p_k}(\xi_k^{p_k-1}(c_k)+\eta_k^{p_k-1}(c_k)) \le (\xi_k(x)\eta_k(x))^{p_k-1}\int_a^b r_k(s)|u_k'(s)|^{p_k}ds \tag{2.18}$$

for $k=1,2,\dots,n$. Since $\xi_k(c_k)+\eta_k(c_k)=\xi_k(x)+\eta_k(x)$, $\forall x, c_k\in(a,b)$, and

$$\xi_k(c_k)=\frac{\xi_k(x)+\eta_k(x)}{2}=\frac{1}{2}\int_a^b r_k^{1/(1-p_k)}(s)ds,$$

we have

$$\begin{aligned} &|u_k(x)|^{p_k}[2^{2-p_k}(\xi_k(x)+\eta_k(x))^{p_k-1}] \\ &= |u_k(x)|^{p_k}[2\xi_k^{p_k-1}(c_k)] \\ &\le (\xi_k(x)\eta_k(x))^{p_k-1}\int_a^b r_k(s)|u_k'(s)|^{p_k}ds \end{aligned} \tag{2.19}$$

and hence

$$|u_k(x)|^{p_k} \le E_k(x)\int_a^b r_k(s)|u_k'(s)|^{p_k}ds \tag{2.20}$$

for $k=1,2,\dots,n$ and $x\in[a,b]$. After that by using a technique similar to the one in [9, Theorem 3.1], it can be showed that the equality case in (2.20) does not hold. Thus, we have

$$|u_k(x)|^{p_k} < A_kE_k(x), \quad x\in(a,b), \tag{2.21}$$

where $A_k=\int_a^b r_k(s)|u_k'(s)|^{p_k}ds$ for $k=1,2,\dots,n$. The rest of the proof is the same as in the proof of Theorem 2.1, and hence is omitted. ☐

Remark 2.3. It is easy to see from the inequality (1.13) that if we take $1<p_k<2$ for $k=1,2,\dots,n$, then inequality (2.14) is better than (2.2) in the sense that (2.2) follows from (2.14), but not conversely. Similarly, from the inequality (1.14), if $p_k>2$ for $k=1,2,\dots,n$, then inequality (2.2) is better than (2.14) in the sense that (2.14) follows from (2.2), but not conversely.

By using the inequality (1.15) in Theorem 2.1 or (1.17) in Theorem 2.2, we obtain the following result.

Corollary 2.4. *Assume that there exist nontrivial solutions $(e_1, e_2, \dots, e_n)$ of system (2.1). If $f_k \in C([a,b],\mathbb{R})$ for $k = 1,2,\dots,n$ and $(u_1(x), u_2(x), \dots, u_n(x))$ is a nontrivial solution on $[a,b]$ for system (1.1) with $\alpha_{ki} = \alpha_{ik}$ for $k,i = 1,2,\dots,n$, then*

$$1 < \prod_{k=1}^{n} \Big(F_k \int_a^b f_k^+(s)ds \Big)^{e_k}. \tag{2.22}$$

Remark 2.5. Note that Theorem 2.1 or 2.2 yields a new Lyapunov-type inequality which is not covered by Theorem 1.4 given by Yang et al [12]. It is easy to see that Corollary 2.4 coincides with Theorem 1.4 under the condition $\alpha_{ki} = \alpha_{ik}$ for $k,i = 1,2,\dots,n$.

Remark 2.6. Since $|f(x)| \geq f^+(x)$, the functions $f_k^+(x)$ for in the above results can also be replaced by $|f_k(x)|$ for $k = 1,2,\dots,n$.

Now, we give an application of the obtained Lyapunov-type inequalities for the eigenvalue problem

$$\begin{gathered} -(r_k(x)\phi_{p_k}(u_k'))' = \lambda_k h(x)\phi_{\alpha_{kk}}(u_k) \prod_{i=1,\, i\neq k}^{n} |u_i|^{\alpha_{ki}} \\ u_k(a) = u_k(b) = 0, \end{gathered} \tag{2.23}$$

where $h(x) > 0$. Thus, if there exist nontrivial solutions $(e_1, e_2, \dots, e_n)$ of linear homogeneous system (2.1), then we have

$$\Big\{ \Big(\prod_{k=1}^{n-1} \lambda_k^{e_k} \Big) \prod_{k=1}^{n} \Big[\int_a^b h(s) \prod_{i=1}^{n} D_i^{\alpha_{ki}/p_i}(s) ds \Big]^{e_k} \Big\}^{-\frac{1}{e_n}} < \lambda_n$$

or

$$\Big\{ \Big(\prod_{k=1}^{n-1} \lambda_k^{e_k} \Big) \prod_{k=1}^{n} \Big[\int_a^b h(s) \prod_{i=1}^{n} E_i^{\alpha_{ki}/p_i}(s) ds \Big]^{e_k} \Big\}^{-\frac{1}{e_n}} < \lambda_n.$$

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