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POSITIVE ALMOST AUTOMORPHIC SOLUTIONS FOR SOME NONLINEAR DELAY INTEGRAL EQUATIONS

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Abstract. This paper is concerned with some nonlinear delay integral equations arising in an epidemic problem. We establish a new existence and uniqueness theorem about positive almost automorphic solutions for delay integral equations. Our theorem is a generalizations of some known results. An example is given to illustrate our results.

1. Introduction

In this paper, we consider the delay integral equation

$$x(t) = \int_{t-\tau(t)}^{t} f(s, x(s))ds, \tag{1.1}$$

which is a model arising in the spread of some infectious disease.

Let us briefly describe the meaning of (1.1) in the context of epidemics. The number $\tau(t)$ can be interpreted as the duration of infectivity, $x(t)$ is the population at time t of infectious individuals, $f(t, x(t))$ is the instantaneous rate of infection, and $f(t, x(t))dt$ is the fraction of individuals infected within the period $[t, t + dt]$.

Since the work of Cooke and Kaplan [3], the delay integral equation (1.1) has been of great interest for many authors (see some nice work, e.g., [12, 10, 13, 16, 8] and references therein). Especially, there is a larger literature about the existence of periodic and almost periodic solutions to (1.1). The existence of positive almost periodic solutions to (1.1) was first investigated by Fink and Gatica [10] in the case of $\tau(t) \equiv \tau$. Afterwards, Torrejón [16] considered the same problem in the case that the delay $\tau(t)$ is state-dependent. This probelm was also studied in [8] by means of Hilbert projective metric.

On the other hand, since Bochner [1] introduced the concept of almost automorphy, almost automorphic functions turns out to be an important generalization of almost periodic functions. Now, almost automorphic functions and their applications have been of great interest for many mathematicians. Recently, the study of existence of almost automorphic solutions to various equations including linear and nonlinear evolution equations, integro-differential equations,

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functional-differential equations, etc., has attracted more and more attention (see, e.g., [9, 11, 14, 15, 2, 5] and the references cited there). We refer the reader to the monographs of N'Guérékata [14, 15] for the basic theory of almost automorphic functions and their applications.

Recently, in [6], the authors discussed the existence of positive almost automorphic solutions to Eq. (1.1) in the case of that $f = \sum_{i=1}^{n} f_i g_i$, where $f_i(t, \cdot)$ is nondecreasing and $g_i(t, \cdot)$ is nonincreasing. Stimulated by this work, in this paper, we will establish a new existence and uniqueness theorem about positive almost automorphic solutions to (1.1). Our theorem generalizes related results in [6] (see Remark 3.4). Also, we give an example to illustrate our results.

This paper is organized as follows. In Section 2, we recall some notions, basic results, and a fixed point theorem in the cone. In Section 3, we prove our existence and uniqueness theorem of positive almost automorphic solutions. In the last section, an example is given to illustrate our results.

2. Preliminaries

Throughout this paper, we denote by $\mathbb{N}$ the set of positive integers, by $\mathbb{R}$ the set of real numbers, by $\mathbb{R}^+$ the set of positive real numbers, and by Ω a closed subset in $\mathbb{R}$. First, let's recall some definitions and notations of almost periodicity and almost automorphy (for more details, see [14, 15]).

Definition 2.1. A continuous function $f : \mathbb{R} \to \mathbb{R}$ is called almost automorphic if for every real sequence (s_m), there exists a subsequence (s_n) such that

$$g(t) = \lim_{n\to\infty} f(t + s_n)$$

is well defined for each $t \in \mathbb{R}$ and

$$\lim_{n\to\infty} g(t - s_n) = f(t)$$

for each $t \in \mathbb{R}$. Denote by $AA(\mathbb{R})$ the set of all such functions.

Remark 2.2. A classical example of automorphic function (not almost periodic) is

$$f(t) = \sin \frac{1}{2 + \cos t + \cos \sqrt{2}t}, \qquad t \in \mathbb{R}.$$

Definition 2.3. A continuous function $f : \mathbb{R}\times\Omega \to \mathbb{R}$ is called almost automorphic in t uniformly for x in compact subsets of Ω if for every compact subset K of Ω and every real sequence (s_m), there exists a subsequence (s_n) such that

$$g(t, x) = \lim_{n\to\infty} f(t + s_n, x)$$

is well defined for each $t \in \mathbb{R}$, $x \in K$ and

$$\lim_{n\to\infty} g(t - s_n, x) = f(t, x)$$

for each $t \in \mathbb{R}$, $x \in K$. Denote by $AA(\mathbb{R} \times \Omega)$ the set of all such functions.

Lemma 2.4. *Assume that f, $g \in AA(\mathbb{R})$. Then the following hold true:*

(a) *The range $\mathcal{R}_f = \{f(t) : t \in \mathbb{R}\}$ is precompact in $\mathbb{R}$, and so f is bounded.*
(b) *$f + g$, $f \cdot g \in AA(\mathbb{R})$.*

(c) *Equipped with the sup norm*

$$\|f\| = \sup_{t\in\mathbb{R}} |f(t)|,$$

$AA(\mathbb{R})$ turns out to be a Banach space.

For a proof of the above lemma, see [14, §2.1]. Next, let us recall some notion about cone (for more details, see [4]) and a fixed point theorem.

Let X be a real Banach space. A closed convex set P in X is called a convex cone if the following conditions are satisfied:

(i) if $x \in P$, then $\lambda x \in P$ for any $\lambda \geq 0$,
(ii) if $x \in P$ and $-x \in P$, then $x = 0$.

A cone P induces a partial ordering $\leq$ in X by

$$x \leq y \quad \text{if and only if} \quad y - x \in P.$$

A cone P is called normal if there exists a constant $k > 0$ such that

$$0 \leq x \leq y \quad \text{implies that} \quad \|x\| \leq k\|y\|,$$

where $\|\cdot\|$ is the norm on X. We denote by P^o the interior of P. A cone P is called a solid cone if $P^o \neq \emptyset$.

Definition 2.5. Let X be a real Banach space and $E \subset X$. An operator $\Phi : E \times E \to X$ is called a mixed monotone operator if $\Phi(x, y)$ is nondecreasing in x and nonincreasing in y, i.e. $x_i, y_i \in E$ (i=1,2), $x_1 \leq x_2$ and $y_1 \geq y_2$ implies that $\Phi(x_1, y_1) \leq \Phi(x_2, y_2)$. An element $x^* \in E$ is called a fixed point of Φ if $\Phi(x^*, x^*) = x^*$.

In the proof of our main results, we will need the following fixed point theorem in a cone, which is a direct corollary of [7, Theorem 2.2].

Theorem 2.6. *Let P be a normal and solid cone in a real Banach space X. Suppose that the operator $A : P^o \times P^o \to P^o$ satisfies*

(A1) *$A : P^o \times P^o \to P^o$ is a mixed monotone operator and there exist a constant $t_0 \in [0, 1)$ and a function $\phi : (t_0, 1) \times P^o \times P^o \to (0, +\infty)$ such that for each $x, y \in P^o$ and $t \in (t_0, 1)$, $\phi(t, x, y) > t$ and*

$$A(tx, t^{-1}y) \geq \phi(t, x, y)A(x, y);$$

(A2) *there exist $x_0, y_0 \in P^o$ such that $x_0 \leq y_0$, $x_0 \leq A(x_0, y_0)$, $A(y_0, x_0) \leq y_0$ and*

$$\inf_{x,y\in[x_0,y_0]} \phi(t, x, y) > t, \quad \forall t \in (t_0, 1).$$

Then A has a unique fixed point x^* in $[x_0, y_0]$. Moreover, for any initial $z_0 \in [x_0, y_0]$, the iterative sequences $z_n = A(z_{n-1}, z_{n-1})$ satisfies

$$\|z_n - x^*\| \to 0, \quad n \to \infty.$$

3. Existence and uniqueness theorem

Throughout the rest of this paper, we assume that f admits a decomposition

$$f(t, x) = \sum_{i=1}^{n} f_i(t, x)g_i(t, x) \tag{3.1}$$

for some $n \in \mathbb{N}$. First, we list some assumptions:

(H1) $f_i, g_i \in AA(\mathbb{R} \times \mathbb{R}^+)$ are nonnegative functions, $i = 1, 2, \dots, n$, and $\tau \in AA(\mathbb{R})$ is a positive function.
(H2) For every $t \in \mathbb{R}$, $f_i(t, \cdot)$ are nondecreasing and $g_i(t, \cdot)$ are nonincreasing in $\mathbb{R}^+$, $i = 1, 2, \dots, n$.
(H3) For each $x \in \mathbb{R}^+$ and each $i \in \{1, 2, \dots, n\}$, $\{f_i(t, \cdot)\}_{t \in \mathbb{R}}$ and $\{g_i(t, \cdot)\}_{t \in \mathbb{R}}$ are equi-continuous at x.
(H4) There exist a constant $t_0 \in [0, 1)$ and positive functions φ_i, ψ_i defined on $(t_0, 1) \times (0, +\infty)$ such that

$$f_i(t, \alpha x) \geq \varphi_i(\alpha, x) f_i(t, x), \quad g_i(t, \alpha^{-1} y) \geq \psi_i(\alpha, y) g_i(t, y),$$
$$\varphi_i(\alpha, x) > \alpha, \quad \psi_i(\alpha, x) > \alpha$$

for all $x, y > 0$, $\alpha \in (t_0, 1)$, $t \in \mathbb{R}$ and $i \in \{1, 2, \dots, n\}$; moreover, for any $0 < a \leq b < +\infty$,

$$\inf_{x, y \in [a, b]} \varphi_i(\alpha, x) \psi_i(\alpha, y) > \alpha, \quad \alpha \in (t_0, 1), \ i = 1, 2, \dots, n.$$

(H5) There exist constants $d \geq c > 0$ such that

$$\inf_{t \in \mathbb{R}} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, c) g_i(s, d) ds \geq c,$$
$$\sup_{t \in \mathbb{R}} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, d) g_i(s, c) ds \leq d.$$

In the proof of the main results, we need the following two lemmas, which were proved in [6].

Lemma 3.1. *If $f \in AA(\mathbb{R} \times \mathbb{R}^+)$, $\{f(t, \cdot)\}_{t \in \mathbb{R}}$ are equi-continuous at every $x \in \mathbb{R}^+$, $x \in AA(\mathbb{R})$ and $x(t) \geq 0$ for every $t \in \mathbb{R}$. Then $f(\cdot, x(\cdot)) \in AA(\mathbb{R})$.*

Lemma 3.2. *Let $f \in AA(\mathbb{R})$ and $\tau \in AA(\mathbb{R})$, then*

$$F(t) = \int_{t-\tau(t)}^{t} f(s) ds \in AA(\mathbb{R}).$$

Now we are ready to present the existence and uniqueness theorem.

Theorem 3.3. *Assume that f has the form of* (3.1) *and* (H1)-(H5) *hold. Then* (1.1) *has exactly one almost automorphic solution x^* with positive infimum. Moreover, for any $x_0 \in AA(\mathbb{R})$ with $c \leq x_0(t) \leq d$ for all $t \in \mathbb{R}$, the iterative sequences*

$$x_k(t) = \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, x_{k-1}(s)) g_i(s, x_{k-1}(s)) ds, \quad k = 1, 2, \dots \tag{3.2}$$

satisfy $\|x_k - x^*\|_{AA(\mathbb{R})} \to 0$ $(k \to +\infty)$.

Proof. Let $P = \{x \in AA(\mathbb{R}) : x(t) \geq 0, \forall t \in \mathbb{R}\}$. It is not difficult to verify that P is a normal and solid cone in $AA(\mathbb{R})$, and

$$P^o = \{x \in AA(\mathbb{R}) : \exists\, \varepsilon > 0 \text{ such that } x(t) > \varepsilon, \forall t \in \mathbb{R}\}.$$

We define a nonlinear operator Φ by

$$\Phi(x, y)(t) = \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, x(s)) g_i(s, y(s)) ds,$$

where $x, y \in P^o$ and $t \in \mathbb{R}$. Then by (H2), Φ is a mixed monotone operator.

Let $x, y \in P^o$. It follows from (H1), (H3) and Lemma 3.1 that

$$f_i(\cdot, x(\cdot)), g_i(\cdot, y(\cdot)) \in AA(\mathbb{R}), \quad i = 1, 2, \dots, n.$$

Combining this with $\tau \in AA(\mathbb{R})$, Lemma 2.4 (b) and Lemma 3.2, we have $\Phi(x, y) \in AA(\mathbb{R})$. Also, since $x, y \in P^o$, there exist $\varepsilon, M > 0$ such that $x(t) \geq \varepsilon$ and $y(t) \leq M$ for all $t \in \mathbb{R}$. Therefore, we have

$$\Phi(x, y)(t) \geq \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, \varepsilon) g_i(s, M) ds, \quad \forall t \in \mathbb{R}. \tag{3.3}$$

On the other hand, by (H5), there exist constants $c, d > 0$ such that

$$\inf_{t \in \mathbb{R}} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, c) g_i(s, d) ds \geq c. \tag{3.4}$$

Suppose that $\varepsilon < c$ and $M > d$ (the other cases are similar and easier to prove). Taking $T \in (t_0, 1)$, there exist nonnegative integer k, l satisfying

$$t_0 < T \leq \frac{\varepsilon}{cT^k} < 1, \quad t_0 < T \leq \frac{d}{MT^l} < 1$$

Set $\overline{c} = \frac{\varepsilon}{cT^k}$ and $\overline{M} = \frac{d}{MT^l}$. Then $\overline{c}, \overline{M} \in (t_0, 1)$. We conclude by (3.3), (3.4) and (H4) that

$$\begin{aligned}
\Phi(x, y)(t) &\geq \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, \varepsilon) g_i(s, M) ds \\
&= \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, T^k c\overline{c}) g_i(s, \frac{d}{T^l \overline{M}}) ds \\
&\geq \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \varphi_i(T, T^{k-1} c\overline{c}) \psi_i(T, \frac{d}{T^{l-1}\overline{M}}) f_i(s, T^{k-1} c\overline{c}) g_i(s, \frac{d}{T^{l-1}\overline{M}}) ds \\
&\geq T^2 \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, T^{k-1} c\overline{c}) g_i(s, \frac{d}{T^{l-1}\overline{M}}) ds \\
&\geq T^{k+l} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, c\overline{c}) g_i(s, \frac{d}{\overline{M}}) ds \\
&\geq T^{k+l} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \varphi_i(\overline{c}, c) \psi_i(\overline{M}, d) f_i(s, c) g_i(s, d) ds \\
&\geq T^{k+l} \overline{c}\overline{M} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, c) g_i(s, d) ds \\
&\geq T^{k+l} \overline{c}\overline{M} c > 0, \quad \forall t \in \mathbb{R}.
\end{aligned}$$

Thus $\Phi(x, y) \in P^o$. Therefore, Φ is from $P^o \times P^o$ to P^o.

Suppose $x, y \in P^o$ and $\alpha \in (t_0, 1)$. Let

$$a(x, y) = \min\{\inf_{s \in \mathbb{R}} x(s), \inf_{s \in \mathbb{R}} y(s)\}, \quad b(x, y) = \max\{\sup_{s \in \mathbb{R}} x(s), \sup_{s \in \mathbb{R}} y(s)\}.$$

Then $0 < a(x,y) \le b(x,y) < +\infty$ and $x(s), y(s) \in [a(x,y), b(x,y)]$ for all $s \in \mathbb{R}$. We define

$$\phi_i(\alpha, x, y) = \inf_{u,v\in[a(x,y),b(x,y)]} \varphi_i(\alpha, u)\psi_i(\alpha, v), \quad i = 1, 2, \dots, n,$$
$$\phi(\alpha, x, y) = \min_{i=1,2,\dots,n} \phi_i(\alpha, x, y).$$

By (H4), it is easy to see that $\phi_i(\alpha, x, y) > \alpha$ $(i = 1, 2, \dots, n)$ for each $x, y \in P^o$ and $\alpha \in (t_0, 1)$, which gives that $\phi(\alpha, x, y) > \alpha$ for each $x, y \in P^o$ and $\alpha \in (t_0, 1)$. Now, We deduce by (H4) that

$$\begin{aligned}
\Phi(\alpha x, \alpha^{-1}y)(t) &= \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, \alpha x(s)) g_i(s, \alpha^{-1} y(s)) ds \\
&\ge \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \varphi_i(\alpha, x(s))\psi_i(\alpha, y(s)) f_i(s, x(s)) g_i(s, y(s)) ds \\
&\ge \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \phi_i(\alpha, x, y) f_i(s, x(s)) g_i(s, y(s)) ds \\
&\ge \phi(\alpha, x, y) \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} f_i(s, x(s)) g_i(s, y(s)) ds \\
&= \phi(\alpha, x, y)\Phi(x, y)(t),
\end{aligned}$$

which means that

$$\Phi(\alpha x, \alpha^{-1}y) \ge \phi(\alpha, x, y)\Phi(x, y)$$

for each $x, y \in P^o$ and $\alpha \in (t_0, 1)$. Thus, the assumption (A1) in Theorem 2.6 is satisfied.

On the other hand, by (H5), we have

$$\Phi(c, d) \ge c, \quad \Phi(d, c) \le d.$$

Also, it follows from (H4) that

$$\begin{aligned}
\inf_{x,y\in[c,d]} \phi(\alpha, x, y) &= \min_{i=1,\dots,n} \inf_{x,y\in[c,d]} \phi_i(\alpha, x, y) \\
&= \min_{i=1,\dots,n} \phi_i(\alpha, c, d) \\
&= \phi(\alpha, c, d) > \alpha,
\end{aligned}$$

for each $\alpha \in (t_0, 1)$. Thus, the assumption (A2) in Theorem 2.6 is satisfied.

Hence, Theorem 2.6 yields that Φ has a unique fixed point x^* in $[c, d]$, and for any $x_0 \in AA(\mathbb{R})$ with $c \le x_0(t) \le d$ for all $t \in \mathbb{R}$, the iterative sequences (3.2) satisfy

$$\|x_k - x^*\|_{AA(\mathbb{R})} \to 0, \quad (k \to +\infty).$$

Next, let us show that x^* is the unique fixed point of Φ in P^o. Suppose $y^* \in P^o$ is a fixed point of Φ. Set

$$\gamma := \sup\{\beta > 0 : \beta^{-1}y^* \ge x^* \ge \beta y^*\}.$$

Then $\gamma^{-1}y^* \ge x^* \ge \gamma y^*$ and $0 < \gamma \le 1$. Suppose $0 < \gamma < 1$. Then there exists a nonnegative integer m and constant $\delta \in (t_0, 1)$ such that

$$t_0 < \delta \le \frac{\gamma}{\delta^m} < 1.$$

Now, we define

$$\alpha := \min\{\delta^m \phi(\frac{\gamma}{\delta^m}, y^*, y^*), \delta^m \phi(\frac{\gamma}{\delta^m}, \frac{\delta^m}{\gamma} y^*, \frac{\gamma}{\delta^m} y^*)\}.$$

Then $\alpha > \gamma$. We deduce by (H4)

$$\begin{aligned} x^* &= \Phi(x^*, x^*) \geq \Phi(\gamma y^*, \gamma^{-1} y^*) \\ &= \Phi(\delta^m \frac{\gamma}{\delta^m} y^*, \delta^{-m} \frac{\delta^m}{\gamma} y^*) \\ &\geq \delta^m \Phi(\frac{\gamma}{\delta^m} y^*, \frac{\delta^m}{\gamma} y^*) \\ &\geq \delta^m \phi(\frac{\gamma}{\delta^m}, y^*, y^*) \Phi(y^*, y^*) \geq \alpha y^*. \end{aligned}$$

Similarly, one can show

$$x^* = \Phi(x^*, x^*) \leq \Phi(\gamma^{-1} y^*, \gamma y^*) \leq \alpha^{-1} y^*.$$

From the definition of γ it follows that $\gamma \geq \alpha > \gamma$, which is a contradiction. Hence, $\gamma = 1$. So $y^* \geq x^* \geq y^*$, that is, $x^* = y^*$. Thus x^* is the unique fixed point of Φ in P^o. □

Remark 3.4. It is not difficult to show that (H1)-(H5) hold provided that all the assumptions in [6, Theorem 3.4] are satisfied. Thus, Theorem 3.3 is a generalization of [6, Theorem 3.4], in which $t_0 = 0$, $\varphi_i(\alpha, \cdot)$ is nondecreasing and $\psi_i(\alpha, \cdot)$ is nonincreasing, $i = 1, 2, \dots, n$.

4. Examples

In this section, we give an example to illustrate our results.

Example 4.1. Let $n = 1$,

$$f_1(t, x) \equiv 1 + \sin^2 \frac{1}{2 + \cos t + \cos \sqrt{2} t}, \quad g_1(t, x) \equiv \frac{x + 1}{2x},$$

and $\tau(t) = 2 + \sin t$. The assumptions (H1), (H2) and (H3) are easily verified. Let

$$\psi_1(\alpha, y) = \frac{\alpha^{-1} y + 1}{\alpha^{-1} y + \alpha^{-1}}, \quad y > 0, \ \alpha \in (0, 1).$$

Then $g_1(t, \alpha^{-1} y) \geq \psi_1(\alpha, y) g_1(t, y)$ and $\psi_1(\alpha, y) > \alpha$ for each $y > 0$, $\alpha \in (0, 1)$ and $t \in \mathbb{R}$. Moreover, it is not difficult to show that $\psi_1(\alpha, \cdot)$ is increasing on $(0, +\infty)$ for each $\alpha \in (0, 1)$. Set $\varphi_1(\alpha, x) \equiv 1$. Then for any $0 < a \leq b < +\infty$,

$$\inf_{x, y \in [a, b]} \varphi_1(\alpha, x) \psi_1(\alpha, y) = \psi_1(\alpha, a) > \alpha, \quad \alpha \in (0, 1).$$

Hence (H4) is satisfied. In addition, (H5) follows from

$$\inf_{t \in \mathbb{R}} \int_{t - \tau(t)}^{t} f_1(s, \frac{1}{2}) g_1(s, 10) ds \geq \frac{11}{20} > \frac{1}{2}$$

and

$$\inf_{t \in \mathbb{R}} \int_{t - \tau(t)}^{t} f_1(s, 10) g_1(s, \frac{1}{2}) ds \leq 9 < 10.$$

Now, by Theorem 3.3, the following delay integral equation

$$x(t) = \int_{t-2-\sin t}^{t} \Big[1 + \sin^2 \frac{1}{2 + \cos s + \cos \sqrt{2}s}\Big] \frac{x(s)+1}{2x(s)} ds$$

has a unique almost automorphic solution with a positive infimum.

Note that, in Example 4.1, $\psi_1(\alpha, \cdot)$ is increasing. Thus, [6, Theorem 3.4] can not be applied to this example.

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