History of Mathematics in China: A Factor in World History and a Source for New Questions

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In the last decades much research has been devoted to the *Jiuzhang suanshu* or *The nine chapters on mathematical procedures* (hereafter abbreviated *The nine chapters*), a book which played a crucial role in the mathematical traditions written in Chinese characters, quite comparable to that of Euclid's *Elements* of geometry in the West. Compiled during the Han dynasty (206 B.C.E. – 221 C.E.), around the beginning of the common era, after the unification of the Chinese empire, the book was to become a "Classic" from which most subsequent Chinese mathematicians drew inspiration. It constitutes the earliest known Chinese source devoted to mathematics to have been handed down by a written tradition. With the discovery, in a grave, of a *Book on mathematical procedures* from the first half of the 2nd century B. C. E., archeologists have recently started to unearth documents that survived in an entirely different way. When they become available, we may expect our understanding of mathematics in early China to be radically changed, especially as regards the background of the composition of *The nine chapters* during the Han dynasty and the modalities of its compilation.

As with all other writings which were granted the status of "Classics" in China, commentaries were composed on *The nine chapters*, some of which were selected to be handed down together with the book. This is how commentaries ascribed to Liu Hui (third century) and Li Chunfeng (seventh century) survived until today.

This paper presents some recent observations on the book itself and its commentaries[1]. It then discusses how the mathematical results obtained in ancient China can be embedded in a world history of mathematics. The examples selected

[1]Since 1984, Professor Guo Shuchun and myself have been collaborating on a critical edition and a French translation of *The nine chapters* and its commentaries within the framework of an agreement between the Academia Sinica (Beijing, China) and the CNRS (France) *18. My ideas on the topic certainly benefited from this joint work, and I am pleased to express my gratitude towards Prof. Guo. Given the limits of this paper, I can unfortunately not do justice to all publications on the subject. The reader is referred to the bibliography in *18. I list below only critical editions of the text published recently *20, 23*, and the references for ideas sketched here. It is my pleasure to thank B. Belhoste, F. Bray, B. Chandler and J. Peiffer for very helpful discussions.

give various reasons why only an international approach to history of mathematics can provide an adequate framework to capture the historical processes which have constituted mathematical lores around the world. Finally, some new questions for the study of mathematical activity raised by research on *The nine chapters* are discussed.

I. Algorithms and their proofs in Early Imperial China

The nine chapters consist of problems and general algorithms with which to solve them. Their terms regularly evoke concrete questions with which the bureaucracy of the Han dynasty was faced, and, more precisely, questions that were the responsability of the "Grand Minister of Agriculture" (*dasinong*), such as remunerating civil-servants, managing granaries or enacting standard grain measures. Moreover, the sixth of *The nine chapters* takes its name from an economic measure actually advocated by a Grand Minister of Agriculture, Sang Hongyang (152-82 B.C.E.), to levy taxes in a fair way, a program for which the Classic provides mathematical procedures. These echoes between the duties of specific sectors of the bureaucracy and some of the mathematical problems tally with the fact that several scholars known in Han times for their ability in mathematics are also recorded as having at some point worked for this very administration. One of them, Geng Shouchang, is one of the two to whom Liu Hui's preface ascribes the composition of *The nine chapters*, whereas the other, Zhang Cang, also dealt with accounting and finance at high levels of the bureaucracy. Hence mathematics seem to have historically developed in Han dynasty China in relation with an administration in charge of economic matters *15. On another hand, some problems of *The nine chapters* were read by later scholars in ancient China to relate to astronomical questions *19. These practitionners hence identified within the book a reflection of an interaction between astronomy and mathematics, long stressed as crucial for the way in which the latter developed in China. Sources also record that both Zhang Cang and Geng Shouchang worked in astronomy.

However, the problems that evidence shows were quoted in the context of astronomical discussions may be perceived as recreational by some readers of today, because of the terms in which they are cast. The historian is thus warned against the assumption that the category of "mathematical problem" remained invariant in time, and is instead invited to describe the practice of problems with respect to which a text was written, before setting out to read it *16. In our case, despite the fact that *The nine chapters* usually present a problem within a particular concrete context, the first readers that we can observe, namely the commentators, read it as exemplifying a set of problems sharing a similar structure and solved by the same algorithm. They felt free to have a problem "circulate" between different contexts, without reformulating it either in other concrete terms or in abstract ones. Such a historical reconstruction guards us from mistaking a problem as merely particular or practical, when Chinese scholars read it as general and meaningful beyond its own context, or mistaking it as merely recreational when it was put to use in concrete situations. This is a crucial point, since it prevents us from jumping to the conclusion that mathematics in China was merely practical, simply because ancient Chinese texts attest to ways of managing the relationships between abstraction and generality, between pure and practical mathematics, which are different from those we expect.

If a problem was not presented abstractly, that seemingly did not affect the value of generality attached to it. But the commentators expected that the algorithm given for solving it be general, if not abstract. For Liu Hui would criticize an algorithm provided, if it appeared to be less general than it could be and if it made use of inessential particular circumstances in the problem *16. In such cases, and within the framework of the same problem, he would restate a more general algorithm. Presumably, an algorithm's efficiency should extend as widely as possible beyond the scope of the problem for which it was formulated. Generality was thus expected for the operations rather than the situations themselves, and the commentators read the algorithm as determining the domain of problems which a particular one was exemplifying. Moreover, the Classic displays a rational architecture in nine chapters, based on the constitution of the algorithms, and not on the themes of the problems *12. This again highlights the authors' main emphasis on operations. *The nine chapters* thus articulate, within a theoretical framework, problems still bearing the marks of the contexts in which they were put to use or for which specific algorithms were developed. In the authors' opinion, the flavour of practice seemingly did not deprive theory of its glamour.

Mathematical knowledge was cast under the form of algorithms, for arithmetical as well as geometrical matters (computing the area of a circle or the volume of a pyramid). Inspired by Donald Knuth, who suggested reading Old-Babylonian clay-tablets from the point of view of algorithmic theory, Wu Wenjun initiated a new approach to ancient Chinese mathematical sources along similar lines *29. The properties that the algorithms in *The nine chapters* display confirm that they constituted a basis for mathematical effort. For instance, algorithms given for square and cube root extractions of integers and fractions bring into play the place-value decimal numeration system representing numbers on a counting board on which mathematics was practised: the sophistication of resources to which their description testifies – assignment of variables, conditionals, iterations – implies that lists of operations as such were compared, rewritten to be unified *1. This conclusion, drawn by observing how the algorithms are described, fits with what was noted above: an algorithm should be written so as to work for as many situations as possible. Moreover, should the algorithm not have exhausted the integer N when the units of the root are obtained, the Classic prescribed that the result must be given as "side of N", i. e. $\sqrt{N}$ *27, 22, 4.

Again, the algorithm to solve systems of n simultaneous linear equations with n unknowns, amounting to "Gauss elimination method" *22, 21*, puts into play a place-value notation for the equations on the counting board, and its description displays the same properties as listed above *9. It brings in marked numbers ("positive", "negative") and "missing" coefficients, as well as rules for computing with them, to achieve the utmost efficiency *7. First introduced in the flow of the computations, such numbers were then reused to represent any linear equation on the board and have the algorithm cover all possible such systems of linear equations. This way of instituting the general linear equation evokes how quadratic equations appear in *The nine chapters*: the algorithm for square root equation deprived of a first step, as well as the state of the counting board at this point of the computation, were granted autonomy, the latter yielding the concept of quadratic equation, the former, the algorithm to compute "its" root *7. Both cases attest to the same specific way of defining new objects: algorithms operate on configurations of numbers on the board, and, in both cases, some of their temporary states as well as the part of the algorithm flowing from them received the status of autonomous mathematical objets. Algebraic equations were to develop in China in that way, exclusively as numerical operations depending on n-th root extraction, until the 13th century.

The positive and negative numbers introduced, however, differ in nature from the quadratic irrationals mentioned above: the results could not be such marked numbers, which also betrays that they differ from the modern concepts. They functioned rather as algorithmic marks, exclusively within the context of systems of linear equations, and it was as such that in the 13th century they were exported into a second mathematical domain: used to represent the coefficients of any algebraic equation, they provided the basis for extending the Ruffini-Horner algorithm to obtain "the" root in the most general case *7. The treatment of algebraic equations was thus completed within the framework in which these equations had appeared in *The nine chapters*.

A last group of algorithms, the "rules of false double position", which have disappeared from today's mathematics, betray in yet another way the Classic's interest in algorithms encompassing the widest range of situations possible. A common list of operations is obtained to solve problems of two intrinsincally different types. It takes different meanings when applied to these different cases, but formal identity of the solving procedures served as a basis for a unique algorithm in *The nine chapters*. Again the result of a formal work on operations themselves, such a property epitomizes the development, within this algorithmic framework, of a kind of algebra *3.

In contrast with *The nine chapters* themselves, the commentaries explicitly set out to prove the correctness of the algorithms provided by the Classic. Since they systematically dealt with algorithms, their proofs developed within a context differing from what can be found in Greek texts of Antiquity, where mathematicians addressed establishing the truth of statements. The description of these proofs, besides acquainting us further with the conception of algorithms in ancient China, brings to light what constituted an original practice of proof *3, 14. When proving that the given algorithm for the area of the circle or the volume of the pyramid is correct, Liu Hui brings into play infinitesimal reasonings, using inscribed polygons for the circle, and smaller and smaller similar solids for the pyramid *28, 22, 21. Their detailed structural similarity indicates that these reasonings may have been ruled by patterns or fulfilled constraints *10. Concluding his proof that the algorithm for the circle (i.e. "multiplying half the circumference by half the diameter, one obtains the area of the circle") works, Liu Hui stresses that this algorithm, correct when it involves the actual dimensions of the circle, allows no computation. This singular situation induces him to make explicit a distinction crucial for us to understand how an algorithm was conceived, since he contrasts the algorithm as *prescription for computation* –to produce a value–, from the algorithm as *relation* *of transformation between magnitudes*, essential for the proofs *10. As regards the area of the circle, where the two do not run in parallel, this causes a division in the proof. Liu Hui first addresses the latter aspect, before turning to the former and examining how computations can provide approximations. More generally, the problem after which an algorithm is stated offers a context of interpretation of its operations as relations of transformation, which the commentators may bring into play in the proof *16. In the course of proving that an algorithm works, problems occur in another way. Willing to establish that an algorithm actually yields the sought-for unknown, Liu Hui may first himself produce a list of computations performing the same task as follows: he decomposes it into a sequence of auxiliary tasks in which he recognizes known problems and concatenates the algorithms for their solution. The second part of the proof then consists in transforming the algorithm obtained into equivalent ones, until he gets to the algorithm he was originally considering. To this end, Liu Hui applies rules of rewriting to lists of operations, which include: deleting inverse operations such as division and multiplication; reversing the order of operations; merging multiplications and divisions together; inverting algorithms. This kind of formal transformations, operating on an algorithm as such, attests to the development of a form of *algebraic proof again within an algorithmic framework*. The key point here is that Liu Hui relates the validity of this form of proof to the fact that various kinds of numbers were introduced by the Classic (fractions and quadratic irrationals) to provide divisions and root extractions with exact results *17. This makes the opposition between multiplication and division operate with full generality and efficiency in mathematics *12. The interest in pairs of opposed but complementary operations echoes the numerous quotations from the *Yijing (Classic of changes)* in the commentaries. Considering, further, that Liu Hui refers to algorithms – the core of mathematical activity – as embodying change (*bianhua*) within mathematics, we may conjecture that philosophical inquiries into change in ancient China influenced mathematical research or benefited from meditating on mathematics *14, 15.

II. A Factor in World History

Embedding Chinese sources in the world corpus of mathematical writings discloses that their authors shared topics of interest and results with other communities on the planet. This raises various kinds of question. *The nine chapters* share with the earliest extant Indian mathematical writing (6th c.) basic common knowledge, among which is the use of a place-value decimal numeration system. Such evidence allows no conclusion as to where this knowledge originated, a question which the state of the remaining sources may prevent us from ever answering. Instead, it suggests that, from early on, communities practising mathematics in both areas must have established substantial communication.

Later on, Arab scholars became interested in this scientific world through India. This is documented. However, several elements common to Chinese and Arabic sources from the 9th century onwards, and so far not found in the known Indian sources, seem to indicate that there were also direct contacts between the Chinese- and Arabic-speaking intellectual communities. One of these, the topic of a treatise by Qusta ibn-Luqa (9th c.) before spreading westwards, was the set of rules of false double position. Interacting with their new intellectual context, these rules were proved with Euclidean geometry, which required drawing a diagram representing the relation between what goes in and out of an algorithm *13. Some centuries later, several similarities occur between Chinese and Arabic sources. As-Samawa'l (1172) extracted a 5-th root with a Ruffini-Horner algorithm *25*, as Jia Xian (11th c.) did *6*, and considered polynomials written in a place-value notation for the powers of the indeterminate, which is similar to the notation for polynomials found in sources from Northern China in the 13th cent. *On equations*, by Sharaf-al-Din al Tusi (12th c.), articulates improving approaches to quadratic and cubic equations previously developed in Arabic with finding roots using tabular, numerical algorithms cognate to those traditionally used in China *8* and later to be used by Viète *24. Of course, the earliest evidence available today proves nothing about where a result was obtained. Such a conclusion might be contradicted by finding new manuscripts. However, another type of conclusion can more safely be drawn: some Chinese and Arabic mathematical communities must have been in close enough contact to share a whole group of results *6. These contacts were probably not very intimate, since we have no evidence that Euclidean geometry as widely practised then in the Arab world received mathematicians' attention in China before the arrival of European missionaries at the end of 16th century. Conversely, we so far have found no echo in Arabic sources of the algorithms for solving systems of linear equations which were continuously used in China.

The history of algebraic equations, however, raises many general issues other than the question of "transmission". The sources prior to Tusi's *On equations* in which we recognize such equations and modes of resolution, be they Babylonian, Greek, Chinese, Indian, or Arabic, attest in fact to different concepts and practices, presenting, despite the transformations they underwent, stable features over long periods of time *8. Hence different mathematical traditions elaborated in diverse ways an object that today's readers recognize as the same. The description of these different elaborations, all the more precise when it involves comparing the various treatments to distinguish them, displays the conceptual variety likely to affect what we would conceive of as a unique mathematical object. Considering these sources as a whole, we also see that the approach to equations devised in China can be found in no other corpus of ancient texts. As a result, this gives us a precious piece of historical information which enables us to tackle questions of transmission with greater precision. In another respect, some of these sources display concepts of equation that in turn become ingredients that other sources articulate in their own treatment of equation. For instance, Tusi inherited al-Khwarizmi's theory of quadratic equations (9th century), itself a framework based on blending two different ingredients: Babylonian algorithms solving particular equations by radicals and Diophantos (ca. 2nd c.)' *Arithmetics'* handling of equations as statements of equality involving an unknown. Onto this, Tusi articulated Khayyam (11th c.)'s geometrical theory of cubic equations – an elaboration merging al-Khwarizmi's concept and framework for equations with Greek approaches using conics to problems only later conceived of as equations *24* – and numerical algorithms echoing with Chinese sources. Tusi's *On equations* attests to new developments concerning equations not only because it systematically provided the numerical algorithms with proofs and conceptually improved the geometrical approach to equations *26*, but also because it bears witness to a synthesis of different concepts and approaches to equations. The mathematical work required to perform this synthesis also needs to be stressed and studied for itself as, more generally, one kind of the processes forming mathematical knowledge *5. It demanded that mathematical bridges be built between different concepts, thus the concepts were melted into a unique one. Such a work, however, may become invisible today to those who inherited concepts depending on this synthesis. Cumulative progress is by no means the only process accounting for the constitution of mathematical knowledge. Non-linear processes took place, the study of which requires that all traditions be taken into account and that the old and yet too widespread global framework of the history of mathematics, drawing a line between the Greeks and the so-called "Renaissance", be revised. The new picture may well bring to light the crucial part played by Arabic-speaking mathematicians of the Middle Ages in bringing together traditions from everywhere, carrying out synthesis of this type and elaborating on them, thereby changing the nature of mathematics *5.

The rules of false double position show the limits of an account in terms of cumulative progress from another perspective. If Western sources in which they could be found after the 9th century enriched them with proofs in the Euclidean manner, they lost the subtelty of *The nine chapters*, since they no longer presented algorithms able to solve problems of two kinds. In fact, these Western sources (Arabic writings and European commercial arithmetics of the Middle Ages) contained only problems of one kind. However, transmission was not smoother in China, where, at the end of 16th century, after a period of mathematical decline, the algorithms were used only for the second kind of problems. When Jesuits brought European mathematical writings to China, Chinese scholars found themselves confronted with cognate algorithms applied to different kinds of problem and coming from two different sources, and could not figure out their relation. They gathered them together again and concluded that Western mathematics was superior *13, 9!

III. New questions

Even though *The nine chapters* contain concepts and results, the international circulation of which shows their potential universality, the Classic adheres in several ways to the local cultural contexts within which it was produced and handed down. We saw above how the emphasis placed on algorithms and on opposed operations might relate to a wider interest in change. In another respect, the status of "Classic" granted to the book refers to a category of writings typical of the history of Chinese literature. It implies that the book was to be treated in a special way and called for peculiar modes of reading from its commentators. In yet another respect, specific literary techniques were used in writing *The nine chapters*: The algorithms for square and cube root extraction were described, sentence by sentence, in parallel with each other, thus bringing to light the ways in which the operations could be considered as analogous to each other. This relates to the fact that, more generally, Chinese texts abound in such parallel sentences which correspond to each other character by character and are read as expressing a correspondence between their topics *2. Chinese mathematical writings hence demonstrate that they stick to, and benefit from, a given set of common scholarly practices. The historian must take into account the adherence to scholarly practices in order to interpret mathematical texts in a better way. Conversely, because of the characteristics of the subject, mathematical writings could provide a useful observatory to describe widespread scholarly practices *2, 11, 16. This delineates a research program for the study of mathematics as part of a wider cultural context.

These facts cast light on a more general phenomenon: *The nine chapters* attest to a specific concrete work environment for practising mathematics. Ways of dealing with problems and algorithms, of using visual aids, of handling the counting board also demonstrate specific features in the way mathematicians used them in ancient China. On another hand, characteristic interests (in algorithms with an emphasis on generality, or in opposed operations) coalesce with specific features of the mathematical objects (recurring place-value notations *11*, singular concepts like equations as numerical operations, designed as a temporary state in a flow of computation) to form the image of a particular mathematical world. This raises two kinds of problems. How was the work environment designed and used to pursue specific interests? How far can we correlate the questions raised and the means designed to tackle them with the specificities of the notations, concepts and results produced? The case of the counting board yields interesting elements in both respects. A surface handled according to strict and particular rules, this board offers positions where numbers can be placed and transformed in order to carry out computations. For example, multiplication and division both require three positions (top, middle, bottom). The key point is that these positions appear to be subjected to either opposed or the same sequences of events when multiplying or dividing *12. A kind of object, consisting of a position and the sequence of events to which it is subjected in the concrete course of a computation, draws our attention. This kind of object enabled one to display in a specific way the opposition between operations on the board. But it is also involved in working out the similarity between square and cube root extractions, or root extractions and division *2. There, the same names are conferred to positions affected by similar sequences of events: the nature of some basic concepts seems to originate from observing mathematical reality as given shape through computations on the board. The Ruffini-Horner algorithms found in 11th century China make sense in relation to this context too: this way of carrying out root extraction reduces the algorithm to repeating, on a succession of positions, the same sequences of events as found in either a division or a multiplication *6. This again emphasizes the interest in finding out a list of operations the applicability of which is as broad as possible. Moreover such positions enable root extraction to appear as composed of alternately opposed operations, and they eventually formed the place-value notation for algebraic equations and polynomials as they appear in 13th century texts *11. The hypothesis that algorithms were worked out on the board through such objects as positions and the sequence of events on them thus ties together features which we underlined: the interest in generality of the algorithms, in opposed operations, in results such as Ruffini-Horner algorithms, and the recurring of place-value notations over centuries *11. It links the specific practice of mathematics to the concepts and results on which mathematicians focused in ancient China. However, this is not peculiar to China: more generally, the products of mathematical activity that eventually become universal are worked out by resorting to specific forms of practice, partly inherited, partly reworked according to the problems addressed or the conditions available. Clearly, describing such regimes of mathematical activity and their relations to the mathematical results produced is an agenda for which ancient China provides a unique contribution.

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