Renewal in Collegiate Mathematics Education

To the memory of James R. C. Leitzel

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Abstract. The content and pedagogy of college courses in mathematics and science are not well aligned with the desired outcomes of college education. This is due in part to a professoriate that is largely unaware of pedagogical "best practice." Recent research on neurobiology confirms research on the psychology of learning, and both support best practice in pedagogy. The Calculus Reform Movement has developed courses that focus on student-centered learning and show that new knowledge can be translated into effective learning programs. Computer and calculator technologies offer opportunities to rethink a mathematics curriculum heavily weighted with pre-computer techniques, to create learning environments that accord with best practice, and to shift the primary focus in our courses from manipulation to thinking.

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1 Calculus: Reform or Renewal?

"The great obstacle to progress is not ignorance but the illusion of knowledge."[1]

The primary qualification for teaching mathematics in an American university or college is a Ph.D. in mathematics. We take for granted that anyone who has mastered the subject at this level is prepared to teach. If we do what our teachers did, we will be successful — it worked for us. This is not ignorance but a dangerous illusion of knowledge: Good teaching engendered learning in us, so our job is good teaching — learning will follow. If it doesn't, the students must be at fault.

In the mid-1980's there was widespread recognition that something was wrong with this theory, at all levels of mathematics education. Calculus was chosen as the first target for "reform" because it was both the capstone course for secondary education and the entry course for collegiate mathematics. Thus was born the

[1] Daniel Boorstin, former director of the Library of Congress ([2], p. 57).

Calculus Reform Movement, whose history, philosophy, and practice are described in [9], [11], [13].

The first National Science Foundation calculus grants were awarded 10 years ago. Since then we have seen development and implementation of several new approaches to teaching calculus, with widespread acceptance on some campuses, and rejection and backlash on others. Our own approach is to treat calculus as a laboratory science course that emphasizes real-world problems, hands-on activities, discovery learning, writing, teamwork, intelligent use of tools, and high expectations of students.

At the time of development, we had little or no theoretical support for our choice of strategies. In place of theory, we relied on careful empirical work. The following sections develop the theoretical base that we lacked 10 years ago. The results from cognitive psychology were in the literature then but unknown to us and most of the other developers. The results from neurobiology have come to fruition just in this decade, and they confirm the cognitive theories that fit with our empirical observations. Thus, we are replacing the illusion of knowledge with real knowledge about learning and the teaching strategies that engender learning.

In hindsight, "reform" was not a good choice of name. The word has stuck, and most people recognize the course types to which it refers. However, it is an emotionally charged word — in the area of religion, wars have been fought over it. One source of the current controversy is that people with deeply held beliefs feel they are under attack. "Renewal" would be a better descriptor — perhaps we can discuss rationally whether the new aspects are also good, and whether renewal of pedagogical strategies from time to time is itself a good thing to do.

2 Who studies calculus and why

Some 700,000 students enroll in college-level calculus courses in the U. S. in any given year. Of these, 100,000 are in Advanced Placement courses in high schools, 125,000 in two-year colleges, and the rest in four-year colleges or universities [11]. A very small percentage of these students intend to take any mathematics beyond calculus, let alone major in mathematics or do graduate study or become a mathematician. Most of this enrollment is generated either by general education requirements or by prerequisites for subsequent course work. To cite just one example, Duke University has 24 major programs that require one or more semesters of calculus. Even though many students enter with Advanced Placement credits, some 80% of our first-year students take a calculus course. About 2% of each class graduates with a major in mathematics. Thus, most students are not motivated to study calculus except as it serves some other goal — e.g., keeping open options for a major.

American colleges provide liberal, vocational, and/or pre-professional education to students who overwhelmingly see themselves as participants in pre-professional or vocational programs. A small percentage contemplate academic graduate study, but only the tiniest fraction have any concept of liberal education and its potential importance in their lives. Parents usually see things the same way: The objective is for their child to become productive and self-supporting.

Potential employers of graduates at all levels have definite expectations for the skills and abilities of their employees. Collectively, these employers influence support for and accountability from institutions of higher education, public or private. Here is what they want, expressed in seven "skill groups" [1]:

1. The foundation: knowing how to learn
2. Competence: reading, writing, and computation
3. Communication: listening and speaking
4. Adaptability: creative thinking and problem-solving
5. Personal management: self esteem, goal setting and motivation, personal and career development
6. Group effectiveness: interpersonal skills, negotiation, and teamwork
7. Influence: organizational effectiveness and leadership

Students enter college lacking most of these skills, so college must be where they learn them. Indeed, this list defines the goals of higher education in the broad sense: liberal, vocational, and pre-professional. The job of teaching these skills belongs to the entire faculty, including the Mathematics Department — and not just for "computation" and "problem-solving." To get a consistent message from the faculty and to have a good chance of graduating with these skills in place, students must encounter most of them in almost every course.

3 Problems with American collegiate education in mathematics

What was wrong with mathematics education in colleges and universities in the 1980's that led to a perceived need for reform? Many have described the turned-off students and jaded faculty in our classrooms and lecture halls, usually with the intention of blaming someone — teachers at a lower level, society, administrators, or the students themselves. A more constructive description appears in a recent essay [8], a product of discussions among a group of 35 science and mathematics faculty, administrators, foundation officers, and program directors. Their thesis is that there is broad consensus on what constitutes effective science education, but institutional barriers to change have thus far prevented widespread implementation. We quote selected parts of their description of the problem. (The word "science" here is shorthand for "science, mathematics, engineering, and technology.")

> "The traditional approach is to conceive of science education as a process that sifts from the masses of students a select few deemed suitable for the rigors of scientific inquiry. It is a process that resembles what most science faculty remember from their own experiences, beginning with the early identification of gifted students before high school, continuing with the acceleration of those students during grades 9 to 12, fostering in them the disciplined habits of inquiry through their undergraduate majors, and culminating in graduate study and the earning of a Ph.D. Forgotten ... are most students for whom a basic knowledge of science is principally a tool for citizenship, for personal enlightenment,

> for introducing one's own children to science, and for fulfilling employment. Forgotten as well are those students who will become primary and secondary school teachers and, as such, will be responsible for the general quality of the science learning most students bring with them to their undergraduate studies. ...
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> "Although it is widely recognized that an inquiry-based approach to science increases the quality of learning, introductory-level students are often not given to understand what it means to be a scientist at work. ...
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> "... science faculty have at times openly acknowledged their tendency to gear instruction to the top 20 percent of the class — to those students whose native ability and persistence enable them to keep pace with the professor's expectations. The fact that others are falling behind and then dropping out is seen not as a failure of pedagogy but as an upholding of standards."

In short, when we use ourselves as models for our students, we get it all wrong. Hardly any entry-level mathematics and science students are like us. In particular, most students in most calculus courses are in their *last* mathematics course. And these students are the next generation's parents, workers, employers, doctors, lawyers, schoolteachers, and legislators. It matters to us how they regard mathematics.

It's not hard to trace how we got out of touch with the needs of our students. Those of us educated in the Sputnik era were in the target population of that "traditional approach" — just at the end of a time when it didn't matter much that the majority of college graduates (an elite subset of the population) didn't know much about science or mathematics. As we became the next generation of faculty, the demographics of college-going broadened significantly, new money flowed to support science, and broad understanding of science became much more important. The reward structure for faculty was significantly altered in the direction of research — away from teaching — just when we were confronted with masses of students whose sociology was quite different from our own.

This oversimplifies a complex story, but our response was to water down expectations of student performance, while continuing to teach in the only way we knew how. We created second-tier courses (e.g., calculus for business and life sciences), we wrote books that students were not expected to read, and we dropped test questions we didn't dare ask. The goal for junior faculty was to become senior faculty so we wouldn't have to deal with freshman courses. Along the way, we produced high-quality research and excellent research-oriented graduate students to follow in our footsteps. But seldom was there any opportunity or incentive to learn anything about learning — in particular, about how our students learn.

4 Messages from cognitive psychology

In 1987, Chickering and Gamson [2], building on an exhaustive review of "50 years of research on the way teachers teach and students learn," enunciated Seven

Principles of Good Practice in Undergraduate Education:

1. Encourages student-faculty contact.
2. Encourages cooperation among students.
3. Encourages active learning.
4. Gives prompt feedback.
5. Emphasizes time on task.
6. Communicates high expectations.
7. Respects diverse talents and ways of learning.

They also published detailed inventories for faculty and administrators ([2], Appendices B and C) to assess the extent to which a school, its departments, and its faculty do or do not follow these principles. One does not need an inventory to see that much of the traditional teaching practice in mathematics is not in accord with these principles. But it doesn't have to be that way. Indeed, [2] is a handbook for implementing these principles.

Research in cognitive psychology has been sending us consistent messages for a half-century, but few mathematicians were listening until the current decade. As Chickering and Gamson summarize,

> "While each practice can stand on its own, when they are all present, their effects multiply. Together, they employ six powerful forces in education:
> - Activity
> - Cooperation
> - Diversity
> - Expectations
> - Interaction
> - Responsibility."

Another result from cognitive research is the Kolb learning cycle ([6], pp. 128-133). The four stages of this cycle are

- Concrete Experience (CE)
- Reflection/Observation (RO)
- Abstract Conceptualization (AC)
- Active Experimentation (AE)

The ideal learner cycles through these stages in each significant learning experience. The AE stage represents testing in new situations the implications of concepts formed at the AC stage. Depending on the results of that testing, the cycle starts over with a new learning experience or with a revision of the current one. The ideal learning environment is designed to lead the learner through these stages and not allow "settling" in a preferred stage. But there are few ideal learners. Most have preferred learning activities and styles, and they are not all alike. This is one reason why learning experiences work better for everyone in a diverse, cooperative, interactive group.

The action-reflection axis (AE-RO) and the concrete-abstract axis (CE-AC) divide the Kolb cycle into four quadrants associated with the four dominant learning styles ([6], pp. 131-132): *Converger* (AC, AE), *Diverger* (CE, RO), *Assimilator* (AC, RO), and *Accommodator* (CE, AE). Most people are not rooted at a single point in the learning style plane, but rather move around in some subset of this plane, depending on the task at hand. However, most mathematicians spend most of their time in the Assimilator quadrant, whereas the students in a calculus class are likely to come from at least three quadrants. If our pedagogical strategies address only the students who are "like us," we are not likely to succeed in reaching all of them.

5 Messages from modern brain research

This is the Decade of the Brain, an exciting period of advances in neurobiology. This work builds on research with animal models and with epileptics after split-brain surgery, but the most exciting advances have come from imaging techniques — CAT, PET, MRI. We can now study functioning human brains for biological insights into the processes of reasoning, memory, and learning in the normal brain.

An important message of brain research for learning is "selection, not instruction" [4]. Evolutionary theory tells us that at birth we have our entire neural system — and it has not changed significantly in the last 10,000 years. Learning takes place by construction of neural networks. External challenges (sensory inputs) select certain neural connections to become active. Inputs enter the brain through old networks — there aren't any others. Each input can trigger memory if it is not new or learning if it is new. The cognitive term for this process is *constructivism*: The learner builds knowledge on what is already known, but only in response to a challenge. In particular, knowledge is not a commodity that can be transferred from knower to learner.

Selection also means that some potential neural pathways are *not* selected, that is, they become dormant through lack of use. The message for collegiate education: If we want to foster such skills as problem solving, creative thinking, and critical thinking, our task is much easier if educational challenges have been developing these skills from infancy. We have a stake in what happens at all levels before college.

Memory is an intricate collection of neural networks. Most experiences initially form relatively weak neural connections in "working memory," necessarily of short duration. The biochemical connections become stronger with use, weaker with disuse. The stabilized networks of long-term memory are accessed mainly by numerous connections to the emotional centers of the brain, but working memory has hardly any connections to the emotional brain. That is, working memory is not related to emotions — just facts — but formation of long-term memory strongly involves emotion [3], [7]. The message: We need to stimulate emotional connections to our subject matter if we expect it to transfer to long-term memory.

Similarly, there are strong connections between the emotional and rational centers in the brain. Indeed, emotional pathways can sometimes direct rational decision making before the learner is consciously aware of the decision process. It's not hard to see the evolutionary connection here. Since all of these structures are 10,000 years old, they are intimately related to fight-or-flight reactions and other survival strategies [3].

Just as emotion is linked in the brain to learning, memory, and rationality, so are the motor centers of the brain, and by extension, the rest of the body. Body movement facilitates learning — sitting still inhibits learning [5].

We have already linked brain research to constructivism. Now we connect with Kolb's learning cycle. The concrete experience (CE) phase is input to the sensory cortex of the brain: hearing, seeing, touching, body movement. The reflection/observation (RO) phase is internal, mainly right-brain, producing context and relationship, which we need for understanding. Because the right brain is slower than the left, this takes time. The abstract conceptualization (AC) phase is left-brain activity, developing interpretations of our experiences and reflections. These are action plans, explanations to be tested. They place memories and reflections in logical patterns, and they trigger use of language. Finally, the active experimentation (AE) phase calls for external action, for use of the motor brain. Deep learning, based on understanding, is *whole brain* activity. Effective teaching must involve stimulation of all aspects of the learning cycle [12], [14].

6 Technology and learning

In the minds of many, "reform" is strongly associated with introduction of electronic technologies: graphing calculators, symbolic computer systems, the Internet. These technologies have become widely available, increasingly powerful, and increasingly affordable during the same decade as reform efforts. Is this good or bad or neutral for education? The short answer is "yes" — that is, use of technology is good or bad or neutral, depending on who's doing what. There is already an embarrassingly large literature addressing such questions as "Do students learn better with calculators (or Maple, or whatever)?", questions that are just as meaningless as they would have been for earlier technologies, such as blackboards, pencil and paper, slide rules, textbook graphics, or overhead projectors. There are also substantial numbers of thoughtful papers that compare particular classroom technology experiments with traditionally taught classes and measure whatever can be measured. The typical conclusion is that students in the experimental group did as well (or only slightly worse) on traditional skills, and they learned other things as well.

There are also *costs* associated with new technologies, just as there were with older technologies that we now take for granted. We don't know much about cost-effectiveness of new (or old) technologies, because we don't have good ways to measure *effectiveness* of education. Our effectiveness at addressing the goals in Section 2 may not be known until long after the students have left us, and maybe not even then. A more productive line of inquiry is to examine the costs of *not* using technology, in light of the current context of education, of reasonable projections about the world our students will live in, and of what we now know about learning.

Technology is a fact of life for our students — before, during, and after college. Most students entering college now have experience with a graphing calculator, and a growing percentage of students have computer experience as well. Many colleges require computer purchase or at least expect use of technology in a variety of courses. After graduation, it is virtually certain that, whatever the job is, there will be a computer close at hand. And there is no sign that increase in power or decrease in cost will slow down any time in the near future. We know these tools can be used stupidly or intelligently, and intelligent choices often require knowledge of mathematics, so this technological environment is our business. Since most of our curriculum was assembled in a pre-computer age, we need to rethink whether this curriculum still addresses the right issues in the right ways.

But calculus renewal is not primarily about whether we have been teaching the "right stuff." Rather, it is about what students are *learning* and how we can tell. To review, we have seen that the external world (employers) has certain expectations that turn out to be highly consistent with both learning theories and good practice. Neurobiologists have provided the biological basis for accepting sound learning theories and practices, while rejecting unsound ones. What does technology have to do with this?

Looking first at the Kolb cycle, we see that computers and calculators can facilitate the concrete experience (CE) and active experimentation (AE) phases — but *not* the other two phases, which are right brain and left brain activities. Thus, if the activity allows the student to go directly from CE to AE without engaging the brain, it may do more harm than good. Well designed learning activities usually involve the entire cycle. Technology can also support each of the Seven Principles.

7 Technology and curriculum

Developers of new curricula have found most of the traditional content still to be relevant, but not necessarily in the same order or with the same emphases or with the same allotment of time. Here is an example of how technology permits rethinking content and pedagogy in accord with sound theory and good practice.

The *raison d'etre* of calculus is differential equations. Never mind that most calculus students never get there — the interesting problems involve ODE's. Traditionally, understanding ODE's required lots of technique, and that in turn required practically all of Calculus I and II. Now we can pose the problem embodied in a differential equation on Day 1 of a calculus course: The time-rate of change of some important quantity has a certain form — what can we say about the time-evolution of the quantity? We can also draw a picture of the problem: a slope field. The meaning of solution is then clear: We seek a function whose graph fits the slope field. Even the essential content of the existence-uniqueness theorem is intuitively clear — the details can wait for that course in ODE's. By that time, the survivors will have a clear idea of what that course is going to be about and why the details matter.

To be more specific, suppose our question is "What can we say about growth of the human population, past, present, and future?" Students recognize that this is important, and they start to engage with *ideas*. They can make conjectures about growth rates, such as proportionality to the population, and explore where they lead. They can trace solutions using the same technique as for the slope field: That's Euler's Method. Observing that human population is changing more or less "continuously," they are led naturally to the derivative concept and to what's "natural" about the natural exponential function.

There are many models students might pose for population growth, but we don't have to keep guessing. We have 1000 years of more or less reliable data to which we can fit a model. Using logarithmic graphing, we can find that the historic data are *not* exponential. Rather, the growth rate is proportional to the *square* of the population, so the data fit a hyperbola with a vertical asymptote — which occurs within their lifetime (about 2030). Then they really have to think about what all this means. (See [10], Chapter 7 Lab Reading.)

The details involve substantial mathematics — numerical, symbolic, and graphical. Note the echoes of the Kolb cycle: concrete experience with data plots, reflective observation about what the plots mean, abstraction in the symbolic models and their solutions, and active testing of the symbolic solutions against the reality of the data. Then the cycle starts again with the vertical asymptote: What does it mean? How can we fit it into an abstract scheme? How can we test whether our scheme fits with reality?

8 Renewal in calculus courses

It would be foolish to pretend that reformed calculus courses were designed to implement the messages of cognitive psychology or neurobiology. Few of the developers a decade ago had any knowledge of these subjects. Rather, we had some instinctive ideas about what to try. Some of those ideas were reinforced by our experiences and became the basis of our courses. Some ideas didn't work and were quickly forgotten. This is selection at work — but, in order for it to work, we had to challenge our prior knowledge.

Reformers became committed constructivists, even though few of us knew that word (in the cognitive sense). In varying degrees, we discovered empirically all seven principles of good practice. Our best materials encourage students to complete the learning cycle — often. Our best programs incorporate in some measure all seven of the skill groups identified by employers. And we have learned appropriate ways to use technology to serve learning objectives.

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