Mathematics Instruction in the Twenty-first Century

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For the past fifty years, mathematics and science education in the United States, both collegiate and precollegiate, have been criticized as being inadequate and frequently irrelevant. Much of the criticism has focussed on the instruction provided in the elementary and secondary schools and the criticism gets quite intense each time there is an international study of student achievements. Considering the low standing of US students in these comparative studies, it should not be surprising there is discontent with the quality of instruction provided. In the last fifteen years the criticism has been expanded to include mathematics and science instruction at the collegiate level. Here we shall restrict one attention to post-secondary instruction, although what happens in the elementary and secondary schools has a decided impact on the instruction at the next level and the teachers in these schools are educated by the collegiate faculty.

The instructional problems facing the mathematical faculty are not different from that facing science educators in general. Consider the following paragraphs[2] written by Purnell W. Choppin, M.D, President, Howard Hughes Medical Institute.

> "There are two revolutions going on in science education these days. One concerns how teachers teach and students learn, the other concerns technology. As with all revolutions, they carry with them a certain uproar and sense of unease.
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> In the first revolution – changing how students learn science and how teachers teach science – debate has been raging among those who advocate a more inquiry-based, problem-solving approach to education and those who believe that content should prevail above all else. Obviously, the ideal lies somewhere in between. Students must learn how to solve problems, but they must also respect that there is a clearly defined body of facts and principles that guide thinking and the pursuit of truth. And let it not be forgotten that what drives most scientists to their work is not the desire to merely ask questions,

[1] This discussion is from a US perspective, since that is what we know. While the situation varies from country to country, the evolution occuring worldwide in educational systems suggests many of the problems facing US mathematical educators will probably become universal.

[2] *Making New Connections in Science Education*, 1997, Howard Hughes Medical Institute.

> but to find answers about some part of the world that fascinates them and captures their interest. It is the content, after all, that gives meaning to our investigations.
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> The second revolution concerns the set of powerful new computer-based tools available in a growing number of education settings. It is not that these technologies necessarily change how we learn, but they can transform the speed, intensity, and environment in which we master our universe. Grappling with how to integrate these innovations into the classroom and assess their impact is an exercise we can expect to engage in for some time.
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> When these revolutions flow together, and they increasingly do, they force us to re-evaluate so many aspects of education that it can, at times seem overwhelming. We want to know if computer-based instruction can draw out talents in students who have been previously difficult to engage. We want to know if student-centered learning via the Internet will satisfy our desire that content be mastered. And we are concerned about conveying the message that learning is always fun, because sometimes it is very hard work. Science is a rigorous endeavor.
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> Teachers who use technology tell us that its integration into their classrooms has forced a sometimes painful transition in the way they teach, but one that they now feel was worth the effort. In the Institute's undergraduate and precollege education programs we have witnessed firsthand the fruits of these labors; elegant education software, large networks of teachers working together on line, and research products of students who have been guided in the use of these tools for their own learning.
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> The challenge for science educators is to find the balance between content and pedagogy to ensure that we reach the desired educational outcomes for our children – scientific literacy and finely tuned analytical skills."

Certainly the US mathematical educators face the two revolutions identified by Dr. Choppin and have been struggling with how to reconcile the two. Indeed much of the debate that has arisen is around this reconciliation, and much of the criticism is whether the students do develop "finally tuned analytic skills." Mathematical instruction in the US faces a more complicated situation than do the sciences.

The bulk of US collegiate mathematics instruction deals with the calculus and linear algebra, and with still lower level courses in the case of many state universities, especially regional ones. Too many students are admitted with low mathematical credentials – a fault of the institutions, but a political reality given that a college education is the path to well paying positions. While typically the number of faculty in a US mathematics department is large compared to other countries, the ratio of students to faculty is extremely large; large even when one includes the very substantial numbers of part-timers and graduate student assistants. Engineering, all the sciences (biological, physical, social and financial) and many professional schools require competency in the calculus, linear algebra, and frequently statistics. Mathematics has become intrinsic to all these disciplines and they demand competency on the part of students. As a consequence 70-90% of first-year students will enroll in a mathematics course. The top 10-20% of these students will have completed their study of the calculus in high school, so the mathematical ability of those enrolled in the calculus is not the best, and their motivation is quite varied. In contrast to the past, today's students do not accept the concept of deferred gratification; they demand to know that mathematics is relevant to their career goals.

The mathematical instructors have diligently striven to provide meaningful instruction to these masses of students, trying to meet contradictory goals arising from the students and the various nonmathematics faculties. A very fundamental issue for the faculty are the demands that calculus be presented as a basic tool for science and engineering and their own desire to show it as the great intellectual achievement that it is. Thus we find some instructors treat the calculus and linear algebra as an algorithmic tool; others will present it as a theory, but usually only with heuristic justifications; others will provide a highly rigorous Satz-Beweis course in analysis; and still others, responding to the revolutions described by Dr. Choppin, will emphasize real world modeling and problem solving. The use of technology varies greatly from no use to extensive use of symbolic packages. There are those who do all the instruction via the computer, allowing students to go at their own pace and attempting to combine Dr. Choppin's two revolutions.

Clearly there is merit in each approach and to present calculus and linear algebra maximally one needs to involve all these approaches. This can seldom be done since time allocated to these subjects is limited by the various curriculums. As might be expected, the various approaches have strong advocates, which has made for stringent debate. Despite the tensions, a very positive outcome is that pedagogy and content of first and second year collegiate mathematics instruction is being seriously examined and discussed. Teaching has become an important issue even in the most sophisticated and research intensive departments. A major problem is that there has been little in depth longitudinal evaluations of any of the approaches and there has been little use of cognitive studies to justify any approach. Without such, the debate is mostly opinion.

From my perspective, the American faculty are held to too great a responsibility for what the student learns. This is oxymoron. The faculty can present material, but it falls to the student to do the learning. From my observation, the more the responsibility is laid on the student, the more the course has been deemed a success, whatever the approach.

There is mounting pressure that US research university instruction should be based on discovery learning guided by mentoring rather than on the transmission of knowledge,[3] and that before graduating the student should have experience in research. The motivation for this approach comes from the repeated observation that "the teacher who puts his hand on your shoulder is the one who has had an impact on your life." Given the large number of students enrolled and the rather low preparation with which they arrive and the pressure on the faculty to do research, this may be wishful thinking on the part of our academic leaders. But given the outlook of young Americans, it probably will be the only way to attract and retain the very best students in mathematics and science. I would expect a research experience that goes beyond the current undergraduate expository thesis,

[3] *Reinventing Undergraduate Education*, Boyer Commission on the Education of Undergraduates in the Research University, Carnegie Foundation for the Advancement of Teaching, 1998. This report is highly critical of the education undergraduates receive at US Research Universities, and it includes suggestions for improvement.

will become the norm for the honor's students. These developments suggest that the discussion is about to expand to all undergraduate instruction.

The US academic community faces many challenges in the 21^{st} Century. As higher education becomes more universal world-wide, it is likely some of the challenges will come to others.

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