Mathematics Education: Reform or Renewal?

George E. Andrews

In considering the relationship between traditional methods of instruction and reform proposals, we should first take into account the environment in which instruction takes place.

It is a sad fact that students in the U.S. today are on the average: (1) not well prepared mathematically before entering college, (2) not studying very hard, and (3) distracted by many absorbing extracurricular activities.

The reports of unpreparedness are legion. From the recent TIMSS report and related sources, we find that primary and secondary mathematics education in the U.S. is not doing well. Indeed this problem has prompted the N.C.T.M. to produce (and then revise) a set of national standards for mathematics education at the primary and secondary levels. Even those of us who have criticized this project do recognize that it was undertaken in response to a real need.

Also there is much evidence that generally students do not study enough. The Pace Report suggests that a majority study less than 15 hours a week. A comparable study at my own university (Penn State) confirms this depressing statistic. No published report I know indicates that students are putting in close to the expected 30 hours a week that the old saying "two hours outside of class for each hour in class" suggests.

These factors must also be viewed against an even more disturbing aspect of undergraduate life in the U.S.: alcohol consumption. Graham Spanier (President of Penn State) is not someone with whom I always agree. However, he is making an effort to draw attention to this problem, and he paints a troubling picture:

"A survey conducted by the Harvard School of Public Health in 1993 reported nationally, 44 percent of all college students were binge drinkers, defined as consuming five or more drinks in a sitting for men and four or more drinks in a sitting for women during a two week period.

About half of these binge drinkers, or about one in five students overall, were frequent binge drinkers, drinking heavily three or more times in two weeks.

About two in five students drank without binging.

Only about one in six – 16 percent – were non-drinkers.

There are unmistakeable consequences of such behavioral patterns. Among the Harvard Study respondents, frequent binge drinkers were 25 times more likely than non-binge drinkers to report having had five or more problems such as doing something they regretted, missing a class, forgetting where they were, getting behind in school work,... and so on...

While only a fraction of one percent of the Harvard Study respondents considered themselves to be problem drinkers, 39 percent said they drink to get drunk."

Given these gargantuan problems, what then should we do for the improvement of undergraduate mathematics education? The answers provided by the bulk of the reform movement are:

Increased Use of:
(1) calculators
(2) computer laboratories
(3) group projects
(4) term papers

Decreased Use of:
(1) lectures
(2) paper and pencil work and drill

At first glance, these answers seem inappropriate to say the least. We are faced with a large group of poorly prepared students with poor work habits who are prone to difficulties with alcohol, and we proceed to set up a system where "the students will learn by constructing calculus for themselves."

However, the current popularity of reform, indeed its great appeal to many administrators, may have more to do with certain unstated (perhaps unconscious) secondary effects than with its ability to improve mathematics education.

Whatever the virtues of each of the first four items (and we often hear much about their virtues), it is clear that each will serve to MASK THE PROBLEMS alluded to earlier.

If you can't multiply 8 times 7, if you don't know that $1/2 = .5$, if you can't divide 1000 by 10, the calculator is your salvation. If you've learned none of the small bits of information that serve to reinforce and accompany the development of mathematical maturity, have no fear; the calculator will come to your rescue. If some sorehead tells you that you need to know a few of these things so that you won't think 1000 divided by 10 is 10000 because you typed "*" when you meant "/", don't worry; just remember to type carefully.

The same can be said of computer laboratories, with the additional comment that these machines seem to obviate any necessity of gaining facility with the fundamentals of calculus. So what if I don't know the derivative of x^3; DERIVE tells me that it is $3x^2$.

Group projects will, of course, assist the ill-prepared and not-so-hard-working who will be able to do better than they otherwise might because they can be carried along by the stronger members of the group.

Term papers are partly a response to the frequent observation that students not only can't calculate, they also can't write. Again strugglers will have resources available from a variety of sources to help hide their deficiencies.

Since we are awash in enthusiasm for calculators, computer laboratories, group projects and term papers, it would be a serious mistake to ignore their quite obvious potential for abuse. This is important in a world where there is heavy pressure on administrators to cover over problems they can't solve directly. It is especially important when the public will buy the idea that "reform" and "innovation" can be expected to solve these problems eventually.

Does all this mean that all is right with the world of traditional instruction? No, unfortunately! While we did not create the environment I described earlier, we traditionalists have behaved less than responsibly, or at least have not squarely faced our dilemmas.

Do we as faculty members sometimes shortchange our students because mathematical research is so much more fun than office hours or lecture preparation? You know the answer.

Do we put in front of our freshmen in the U.S. teaching assistants who are uncomfortable with English? Of course! We reply in response that we are hard pressed to avoid this because the pool of mathematically competent, native English speakers in the U.S. is small. Nonetheless, this is a serious problem that can only be mitigated if it is addressed directly.

Do we value research by an order of magnitude over teaching? Anyone who says "no" is not living in the real world. Think about the colleagues in your department who were courted by better universities. Was it their teaching that Princeton was after? Give me a break! What message does this absolutely certain fact of life send to everyone? Don't misunderstand me. The tremendous value we place on our research is positive overall; among other things it generally protects us from the relativism that has undermined standards and pedagogy in so many disciplines. However, although you and I may agree that excellence in research is the core of our profession, we have a duty to address the resulting implications for teaching. And teaching must be done well if we are to flourish.

Let me conclude with the words of Paul Halmos taken from his article "The Calculus Turmoil":

"Yes, there is a disease, but calculus is neither its cause nor its main symptom. We mathematicians can do our small bit to cure it, but not by rewriting calculus books. All that we can do, all that we are professionally able to do, is to insist on raising the quality of primary and secondary education by establishing and maintaining a high quality in college courses, by insisting on and strictly enforcing severe prerequisites, and by encouraging and properly training prospective grade school and high school teachers. That we can do, and I hope we will."

Halmos is clearly calling for renewal rather than reform, and I could not agree more.

George E. Andrews
Department of Mathematics
Pennsylvania State University
410 McAllister Building
University Park, PA 16802-6401
USA
andrews@math.psu.edu