Continuous Optimization Stories

Sometimes mathematicians coin terms and define relations between them that are "somewhat inconsistent". Nonlinear programming is one such ill-defined term, since linear programming is considered a special case of nonlinear programming. Even someone not exceling in logic may find this strange. I therefore try to use continuous optimization instead of nonlinear programming, although I am aware that combinatorial optimization can be viewed as nonlinear programming but not necessarily as continuous optimization. Optimization may, in fact, be in need of a new consistent nomenclature. The Mathematical Programming Society has already made a small step by renaming itself into Mathematical Optimization Society in order to avoid confusions with computer programming.

My original intention for the contents of this chapter was to highlight the contributions to optimization of mathematicians that have some relation to Berlin. Due to intensive discussions with potential authors, the section developed differently and now contains wonderful survey articles on a wide range of exciting developments in continuous optimization. It begins with the history of the gradient method, discusses the origins of the KKT theorem, the Nelder-Mead simplex algorithm, various aspects of subgradient techniques and nonsmooth optimization, updating techniques, the influence of the Cold War on the maximum principle, and the arrival of infinite-dimensional optimization.

As the ISMP 2012 takes place in Berlin, I feel obliged, however, to provide at least some condensed information about mathematics and mathematicians who contributed to optimization and spent some time in Berlin. (I also use this opportunity to thank my wife for providing me with many of the details. She wrote a book [1], directed at a non-mathematical readership, that covers the history of all aspects of mathematics in Berlin.)

We have already encountered *Gottfried Wilhelm Leibniz*. Mathematics in Berlin began with him. He initiated the foundation of the predecessor of what is today called Berlin-Brandenburgische Akademie der Wissenschaften (BBAW). The academy was officially opened in 1700 and has experienced many name changes in its more than 300 years of existence. Leibniz was the first academy president. Optimization would not exist without his development of calculus (there were other founders as well) and, in particular, his notational inventions. The integral sign is one of these.

Pierre Louis Moreau de Maupertuis came to Berlin in 1740, stayed for 17 years and was also president of the academy. Maupertuis developed a "principle of least action" that states that in all natural phenomena a quantity called 'action' tends to be minimized. His work was highly controversial, though.

Leonhard Euler is the star of mathematics of the 18$^{\text{th}}$ century. Born in 1707 in Basel, he accepted an offer from the academy in St. Petersburg in 1727 and came to Berlin in 1741, he stayed until 1766 to return to St. Petersburg, where he died in 1783. Most of his gigantic mathematical production was carried out at the academy in Berlin.

Another giant of optimization, *Joseph Louis Lagrange*, whose name is encoded in many terms of our field, spent 20 of his most productive years in Berlin. In 1766 he became Euler's successor as the director of the mathematical class of the academy.

Carl Gustav Jacobi, a mathematical household name, whom we encountered in this book in connection with the Hungarian method (Introduction to Discrete Optimization Stories), was born in 1804 in Potsdam, was privately tutored until the age of 12 and graduated at age 13. In 1821 he was allowed to start studying at Berlin University, passed his teacher examination at the age of 19 and obtained his PhD and habilitation in 1825. He became professor in Königsberg in 1826 and returned to Berlin in 1844 as a member of the academy. He died in 1851 in Berlin.

Johann Peter Gustav Lejeune Dirichlet was mentioned in this book in the discussion of the LLL algorithm (Introduction to Linear Programming Stories). He was the first outstanding mathematician at Berlin University whose foundation in 1810 was initiated by Wilhelm von Humboldt. This university carried the name Friedrich-Wilhelms-Universität from 1828 to 1945 and was renamed Humboldt-Universität in 1949, after the brothers Wilhelm and Alexander von Humboldt. Dirichlet was born in Düren in 1805, came to Berlin in 1827 and stayed until 1855 when he accepted an offer from Göttingen to succeed Gauss. He died in 1859.

Karl Theodor Weierstraß (1815–1897), also written Weierstrass, was one of the dominating figures of the 19$^{\text{th}}$ century mathematics in Berlin. He is known to every mathematician for bringing highest standards of rigor to analysis (e.g., the (ϵ, δ)-definition of continuity); many theorems carry his name. Every calculus student learns a result formulated by Weierstraß, namely, that every continuous function from a compact space to the real numbers attains its maximum and minimum. The Weierstraß Institut für Angewandte Analysis und Stochastik is named after him. His grave is shown in Fig. 2.

My wife and I live close to Waldfriedhof Heerstraße, a beautiful cemetery near the Olympic Stadium in Berlin. One day, my wife showed me the joint grave of *Hermann Minkowski* (1864–1909) and his brother Oskar (1858–1931). I was very astonished that the Minkowski brothers had an Ehrengrab (honorary grave maintained by the city of Berlin), see Fig. 1. I knew that Minkowski had studied in Berlin (under Weierstraß and Kummer) and had worked in Königsberg, Zürich, and finally in Göttingen where he died. (Minkowski is

Figure 1: Minkowski's grave
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Figure 2: Weierstrass' grave
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my academic great great grandfather.) Minkowski will forever be known as the person who coined the name spacetime, but for optimizers his work on convexity that arose via his studies of the geometry of numbers, an area he created, is of particular importance. This work is excellently surveyed in [2] and in chapter 0 (and several other chapters) of the handbook [3]. The idea to edit this book on optimization history, in fact, arose when my wife and I tried to find out more about Minkowski's grave. One remark only: The city of Berlin decided on March 22, 1994 to declare the graves of Karl Weierstraß and Hermann Minkowski as honorary graves.

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References

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[2] P. M. Gruber and J. M. Wills (eds.), *Handbook of Convex Geometry*, Vol. A and B, North Holland, 1993.

[3] T. H. Kjeldsen, History of Convexity and Mathematical Programming: Connections and Relationships in Two Episodes of Research in Pure and Applied Mathematics of the 20th Century, in: R. Bhatia (ed.) et al., *Proceedings of the International Congress of Mathematicians (ICM 2010), Hyderabad, India, August 19–27, 2010. Vol. IV: Invited lectures*, World Scientific, Hackensack; Hindustan Book Agency, New Delhi, 2011, pp. 3233–3257.