Stories about the Old Masters of Optimization

I believe that optimization is in some way "built into nature". In many of their approaches to understand nature, physicists, chemists, biologists, and others assume that the systems they try to comprehend tend to reach a state that is characterized by the optimality of some function. In statistical mechanics, e.g., the consensus is that the systems considered develop in the direction of an energy minimal configuration, called ground state. I also think that, in many of their activities, humans have the desire to be efficient and save resources. I therefore reckon that, beginning with the origin of our species, humans have attempted to be unwasteful whenever strenuous efforts lay ahead. I am very sure that our very ancient forefathers planned travel routes along short or safe paths, organized their hunting endeavors carefully, tried to reduce the work involved in ploughing and harvesting, and meticulously designed the logistics needed for the construction of buildings.

There are no traces that these desires to be efficient were considered a mathematical endeavor. If one looks back at the development of our field, it is the middle of the 20^{th} century when optimization (or mathematical programming, which is the term mostly used until recently) took off. But some of the great old masters have, of course, investigated optimization questions and laid the foundations of several of the subfields of today's optimization theory. It is beyond the scope of this book to survey these contributions in great detail. Instead, I decided to cover only a few historically interesting cases and to mix these with some anecdotes.

The problem of solving linear equations comes up almost everywhere in mathematics; many optimization algorithms need fast subroutines for this task. It is hence not surprising that many algorithms for solving linear equations have been designed throughout history; and it is not so clear who invented what first and which algorithm version should carry which name. The most prominent algorithm is often called Gaussian elimination, although Gauss never claimed to have invented this method. One article in this section highlights the appearance of Gaussian elimination in China more than 2000 years ago.

Another important algorithm is the Newton method. Many algorithms in optimization try to mimic this method in some way with the aim to avoid its unwanted properties and to maintain its quadratic convergence speed. One article tries to clarify whether Newton really invented the algorithm named after him.

It is impossible to omit the birth of the calculus of variations in a book like this. And therefore, the interesting story around the invention of the brachistochrone is outlined. All this happened in 1696 and was induced by a challenge put by Johann Bernoulli to his fellow mathematicians. Similarly, the birth of graph theory in 1736 cannot by skipped. Euler, though, missed to view the Königsberg bridges problem as an optimization problem and thus did not become the father of combinatorial optimization. It is somewhat surprising to learn that it took more than 200 years until an optimization version of Euler's graph problem was considered. This happened in China.

It is outside the scope of this book to sketch the monumental contributions of giants such as Euler and Leibniz. Many voluminous books cover aspects of their work. Three more articles, two on Euler and one on Leibniz, of this section on the old masters are of somewhat anecdotal nature. Two articles discuss the struggle of Euler and Leibniz with "infinity" and one displays a slight human weakness of Euler. Did he cheat a bit in dealing with state authorities?

Martin Grötschel